Gauss-Siedel Method

Civil Engineering Majors

Authors: Autar Kaw

http://numericalmethods.eng.usf.edu

Transforming Numerical Methods Education for STEM Undergraduates

Gauss-Seidel Method

http://numericalmethods.eng.usf.edu

Gauss-Seidel Method

An ***iterative*** method.

Basic Procedure:

-Algebraically solve each linear equation for x_i

-Assume an initial guess solution array

-Solve for each x_i and repeat

-Use absolute relative approximate error after each iteration to check if error is within a pre-specified tolerance.

Gauss-Seidel Method

Why?

The Gauss-Seidel Method allows the user to control round-off error.

Elimination methods such as Gaussian Elimination and LU Decomposition are prone to prone to round-off error.

Also: If the physics of the problem are understood, a close initial guess can be made, decreasing the number of iterations needed.

Gauss-Seidel Method

Algorithm

A set of *n* equations and *n* unknowns:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2$$

$$\vdots \qquad \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ldots + a_{nn}x_n = b_n$$

If: the diagonal elements are non-zero

Rewrite each equation solving for the corresponding unknown

ex:

First equation, solve for x_1

Second equation, solve for x_2

Gauss-Seidel Method

Algorithm

Rewriting each equation

$$x_1 = \frac{c_1 - a_{12}x_2 - a_{13}x_3 \ldots\ldots - a_{1n}x_n}{a_{11}}$$ ← From Equation 1

$$x_2 = \frac{c_2 - a_{21}x_1 - a_{23}x_3 \ldots\ldots - a_{2n}x_n}{a_{22}}$$ ← From equation 2

$$\vdots \qquad \vdots \qquad \vdots$$

$$x_{n-1} = \frac{c_{n-1} - a_{n-1,1}x_1 - a_{n-1,2}x_2 \ldots\ldots - a_{n-1,n-2}x_{n-2} - a_{n-1,n}x_n}{a_{n-1,n-1}}$$ ← From equation n-1

$$x_n = \frac{c_n - a_{n1}x_1 - a_{n2}x_2 - \ldots\ldots - a_{n,n-1}x_{n-1}}{a_{nn}}$$ ← From equation n

Gauss-Seidel Method

Algorithm

General Form of each equation

$$x_1 = \frac{c_1 - \sum_{\substack{j=1 \\ j\neq 1}}^{n} a_{1j} x_j}{a_{11}}$$

$$x_{n-1} = \frac{c_{n-1} - \sum_{\substack{j=1 \\ j\neq n-1}}^{n} a_{n-1,j} x_j}{a_{n-1,n-1}}$$

$$x_2 = \frac{c_2 - \sum_{\substack{j=1 \\ j\neq 2}}^{n} a_{2j} x_j}{a_{22}}$$

$$x_n = \frac{c_n - \sum_{\substack{j=1 \\ j\neq n}}^{n} a_{nj} x_j}{a_{nn}}$$

Gauss-Seidel Method

Algorithm

General Form for any row 'i'

$$x_i = \frac{c_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} x_j}{a_{ii}}, i = 1,2,\ldots,n.$$

How or where can this equation be used?

Gauss-Seidel Method

Solve for the unknowns

Assume an initial guess for [X]

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}$$

Use rewritten equations to solve for each value of x_i.

Important: Remember to use the most recent value of x_i. Which means to apply values calculated to the calculations remaining in the **current** iteration.

Gauss-Seidel Method

Calculate the Absolute Relative Approximate Error

$$\left|\in_a\right|_i = \left|\frac{x_i^{new} - x_i^{old}}{x_i^{new}}\right| \times 100$$

So when has the answer been found?

The iterations are stopped when the absolute relative approximate error is less than a prespecified tolerance for all unknowns.

Example: Cylinder Stresses

To find the maximum stresses in a compounded cylinder, the following four simultaneous linear equations need to solved.

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 4.2857\times10^{7} & -5.4619\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

Example: Cylinder Stresses

In the compound cylinder, the inner cylinder has an internal radius of $a = 5''$, and outer radius $c = 6.5''$, while the outer cylinder has an internal radius of $c = 6.5''$ and outer radius, $b=8''$. Given E = 30×10^6 psi, v = 0.3, and that the hoop stress in outer cylinder is given by

$$\sigma_\theta = \frac{E}{1-\nu^2}\left[c_3(1+\nu)+c_4\left(\frac{1-\nu}{r^2}\right)\right]$$

find the stress on the inside radius of the outer cylinder.
Find the values of c_1, c_2, c_3 and c_4 using Gauss-Seidel Method.

Assume an initial guess of

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -0.005 \\ 0.001 \\ 0.0002 \\ 0.03 \end{bmatrix}$$

and conduct two iterations.

Example: Cylinder Stresses

Rewriting each equation

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 4.2857\times10^{7} & -5.4619\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -7.887\times10^{3} \\ 0 \\ 0.007 \\ 0 \end{bmatrix}$$

$$c_1 = \frac{-7.887\times10^{3} - \left(-9.2307\times10^{5}\right)c_2 - 0c_3 - 0c_4}{4.2857\times10^{7}}$$

$$c_2 = \frac{0 - 4.2857\times10^{7}c_1 - \left(-4.2857\times10^{7}\right)c_3 - 5.4619\times10^{5}c_4}{-5.4619\times10^{5}}$$

$$c_3 = \frac{0.007 - \left(-6.5\right)c_1 - \left(-0.15384\right)c_2 - 0.15384c_4}{6.5}$$

$$c_4 = \frac{0 - 0c_1 - 0c_2 - 4.2857\times10^{7}c_3}{-3.6057\times10^{5}}$$

Example: Cylinder Stresses

<u>Iteration 1</u>

Substituting initial guesses into the equations

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -0.005 \\ 0.001 \\ 0.0002 \\ 0.03 \end{bmatrix}$$

$$c_1 = \frac{-7.887\times10^{3}\left(9.2307\times10^{5}\right)\times0.001}{4.2857\times10^{7}} = -1.6249\times10^{-4}$$

$$c_2 = \frac{0 \quad 4.2857\times10^{7}\times\left(-1.6249\times10^{-4}\right)-\left(-4.2857\times10^{7}\right)\times0.0002-5.4619\times10^{5}\times0.03}{-5.4619\times10^{5}} = 1.5569\times10^{-3}$$

$$c_3 = \frac{0.007-\left(-6.5\right)\times\left(-1.6249\times10^{-4}\right)-\left(-0.15384\right)\times1.5569\times10^{-3}-0.15384\times0.03}{6.5} = 2.4125\times10^{-4}$$

$$c_4 = \frac{0-4.2857\times10^{7}\times2.4125\times10^{-4}}{-3.6057\times10^{5}} = 2.8675\times10^{-2}$$

Example: Cylinder Stresses

Finding the absolute relative approximate error

$$\left|\in_a\right|_i = \left|\frac{c_i^{new} - c_i^{old}}{c_i^{new}}\right| \times 100$$

$$\left|\in_a\right|_1 = \left|\frac{-1.6249\times10^{-4} - (-0.005)}{-1.6249\times10^{-4}}\right| \times 100 = 2977.1\%$$

$$\left|\in_a\right|_2 = \left|\frac{1.5569\times10^{-3} - 0.001}{1.5569\times10^{-3}}\right| \times 100 = 35.770\%$$

$$\left|\in_a\right|_3 = \left|\frac{2.4125\times10^{-4} - 0.002}{2.4125\times10^{-4}}\right| \times 100 = 17.098\%$$

$$\left|\in_a\right|_4 = \left|\frac{2.8675\times10^{-2} - 0.03}{2.8675\times10^{-2}}\right| \times 100 = 4.6223\%$$

At the end of the first iteration

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -1.6249\times10^{-4} \\ 1.5569\times10^{-3} \\ 2.4125\times10^{-4} \\ 2.8675\times10^{-2} \end{bmatrix}$$

The maximum absolute relative approximate error is 2977.1%

Example: Cylinder Stresses

<u>Iteration 2</u>

Using $\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -1.6249\times10^{-4} \\ 1.5569\times10^{-3} \\ 2.4125\times10^{-4} \\ 2.8675\times10^{-2} \end{bmatrix}$

$$c_1 = \frac{-7.887\times10^{3} - (-9.2307\times10^{5})\times1.559\times10^{-3}}{4.2857\times10^{7}}$$

$$= -1.5050\times10^{-4}$$

$$c_2 = \frac{0 - 4.2857\times10^{7}\times(-1.5050\times10^{-4}) - (-4.2857\times10^{7})\times2.4125\times10^{-4} - 5.4619\times10^{5}\times2.8675\times10^{-2}}{-5.4619\times10^{5}}$$

$$= -2.0639\times10^{-3}$$

$$c_3 = \frac{0.007 - (-6.5)\times(-1.5050\times10^{-4}) - (-0.15384)\times -2.0639\times10^{-3} - 0.15384\times2.8675\times10^{-2}}{6.5}$$

$$= 1.9892\times10^{-4}$$

$$c_4 = \frac{0 - 4.2857\times10^{7}\times1.9892\times10^{-4}}{-3.6057\times10^{5}}$$

$$= 2.3643\times10^{-2}$$

Example: Cylinder Stresses

Finding the absolute relative approximate error for the second iteration

$$\left|\in_a\right|_1 = \left|\frac{-1.5050\times10^{-4} - \left(-1.6249\times10^{-4}\right)}{1.5050\times10^{\;\;4}}\right|\times100 = 7.9702\%$$

$$\left|\in_a\right|_2 = \left|\frac{-2.0639\times10^{-3} - 1.5569\times10^{-3}}{-2.0639\times10^{-3}}\right|\times100 = 175.44\%$$

$$\left|\in_a\right|_3 = \left|\frac{1.9892\times10^{-4} - 2.4125\times10^{-4}}{1.9892\times10^{-4}}\right|\times100 = 21.281\%$$

$$\left|\in_a\right|_4 = \left|\frac{2.3643\times10^{-2} - 2.8675\times10^{-2}}{2.3643\times10^{-2}}\right|\times100 = 21.281\%$$

Example: Cylinder Stresses

At the end of the second iteration:

$$\begin{bmatrix} c_1 \\ c_2 \\ c_3 \\ c_4 \end{bmatrix} = \begin{bmatrix} -1.5050\times10^{-4} \\ -2.0639\times10^{-3} \\ 1.9892\times10^{-4} \\ 2.3643\times10^{-2} \end{bmatrix}$$

The maximum absolute relative approximate error is 175.44%

At the end of the second iteration the stress on the inside radius of the outer cylinder is calculated.

$$\sigma_\theta = \frac{E}{1-\nu^2}\left[c_3(1+\nu)+c_4\left(\frac{1-\nu}{r^2}\right)\right]$$

$$= \frac{30\times10^6}{1-(0.3)^2}\left[1.9892\times10^{-4}(1+0.3)+2.3643\times10^{-2}\left(\frac{1-0.3}{(6.5)^2}\right)\right]$$

$$= 21439 \text{ psi}$$

Example: Cylinder Stresses

Conducting more iterations, the following values are obtained

Iteration	c_1	$\lvert\in_a\rvert_1 \%$	c_2	$\lvert\in_a\rvert_2 \%$	c_3	$\lvert\in_a\rvert_3 \%$	c_4	$\lvert\in_a\rvert_4 \%$
1	$-1.6249 \quad 10^{-4}$	2977.1	$1.5569 \quad 10^{-3}$	35.770	$2.4125 \quad 10^{-4}$	17.098	$2.8675 \quad 10^{-2}$	4.6223
2	$-1.5050 \quad 10^{-4}$	7.9702	$-2.0639 \quad 10^{-3}$	175.44	$1.9892 \quad 10^{-4}$	21.281	$2.3643 \quad 10^{-2}$	21.281
3	$-2.2848 \quad 10^{-4}$	34.132	$-9.8931 \quad 10^{-3}$	79.138	$5.4716 \quad 10^{-5}$	263.55	$6.5035 \quad 10^{-3}$	263.55
4	$-3.9711 \quad 10^{-4}$	42.464	$-2.8949 \quad 10^{-2}$	65.826	$-1.5927 \quad 10^{-4}$	134.53	$-1.8931 \quad 10^{-2}$	134.35
5	$-8.0755 \quad 10^{-4}$	50.825	$-6.9799 \quad 10^{-2}$	58.524	$-9.3454 \quad 10^{-4}$	82.957	$-1.1108 \quad 10^{-1}$	82.957
6	$-1.6874 \quad 10^{-3}$	52.142	$-1.7015 \quad 10^{-1}$	58.978	$-2.0085 \quad 10^{-3}$	53.472	$-2.3873 \quad 10^{-1}$	53.472

Notice: The absolute relative approximate errors are not decreasing

Gauss-Seidel Method: Pitfall

What went wrong?

Even though done correctly, the answer is not converging to the correct answer

This example illustrates a pitfall of the Gauss-Siedel method: not all systems of equations will converge.

Is there a fix?

One class of system of equations always converges: One with a *diagonally dominant* coefficient matrix.

Diagonally dominant: [A] in [A] [X] = [C] is diagonally dominant if:

$$\left|a_{\mathrm{ii}}\right| \geq \sum_{\substack{j=1 \\ j \neq \mathrm{i}}}^{n} \left|a_{ij}\right| \quad \text{for all 'i'} \qquad \text{and} \qquad \left|a_{ii}\right| > \sum_{\substack{j=1 \\ j \neq i}}^{n} \left|a_{ij}\right| \quad \text{for at least one 'i'}$$

Gauss-Seidel Method: Pitfall

Diagonally dominant: The coefficient on the diagonal must be at least equal to the sum of the other coefficients in that row and at least one row with a diagonal coefficient greater than the sum of the other coefficients in that row.

Which coefficient matrix is diagonally dominant?

$$[A] = \begin{bmatrix} 2 & 5.81 & 34 \\ 45 & 43 & 1 \\ 123 & 16 & 1 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 124 & 34 & 56 \\ 23 & 53 & 5 \\ 96 & 34 & 129 \end{bmatrix}$$

Most physical systems do result in simultaneous linear equations that have diagonally dominant coefficient matrices.

Example: Cylinder Stresses

Examination of the coefficient matrix reveals that it is not diagonally dominant and cannot be rearranged to become diagonally dominant

$$\begin{bmatrix} 4.2857\times10^{7} & -9.2307\times10^{5} & 0 & 0 \\ 4.2857\times10^{7} & -5.4619\times10^{5} & -4.2857\times10^{7} & 5.4619\times10^{5} \\ -6.5 & -0.15384 & 6.5 & 0.15384 \\ 0 & 0 & 4.2857\times10^{7} & -3.6057\times10^{5} \end{bmatrix}$$

This particular problem is an example of a system of linear equations that cannot be solved using the Gauss-Seidel method.

Other methods that would work:

1. Gaussian elimination

2. LU Decomposition

Gauss-Seidel Method: Example 2

Given the system of equations

$$12x_1 + 3x_2 - 5x_3 = 1$$
$$x_1 + 5x_2 + 3x_3 = 28$$
$$3x_1 + 7x_2 + 13x_3 = 76$$

The coefficient matrix is:

$$[A] = \begin{bmatrix} 12 & 3 & -5 \\ 1 & 5 & 3 \\ 3 & 7 & 13 \end{bmatrix}$$

With an initial guess of

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Will the solution converge using the Gauss-Siedel method?

Gauss-Seidel Method: Example 2

Checking if the coefficient matrix is diagonally dominant

$$[A]=\begin{bmatrix}12 & 3 & -5\\ 1 & 5 & 3\\ 3 & 7 & 13\end{bmatrix}$$

$$\left|a_{11}\right| = \left|12\right| = 12 \geq \left|a_{12}\right| + \left|a_{13}\right| = \left|3\right| + \left|-5\right| = 8$$

$$\left|a_{22}\right| = \left|5\right| = 5 \geq \left|a_{21}\right| + \left|a_{23}\right| = \left|1\right| + \left|3\right| = 4$$

$$\left|a_{33}\right| = \left|13\right| = 13 \geq \left|a_{31}\right| + \left|a_{32}\right| = \left|3\right| + \left|7\right| = 10$$

The inequalities are all true and at least one row is *strictly* greater than:

Therefore: The solution should converge using the Gauss-Siedel Method

Gauss-Seidel Method: Example 2

Rewriting each equation

$$\begin{bmatrix} 12 & 3 & -5 \\ 1 & 5 & 3 \\ 3 & 7 & 13 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 28 \\ 76 \end{bmatrix}$$

$$x_1 = \frac{1 - 3x_2 + 5x_3}{12}$$

$$x_2 = \frac{28 - x_1 - 3x_3}{5}$$

$$x_3 = \frac{76 - 3x_1 - 7x_2}{13}$$

With an initial guess of

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$x_1 = \frac{1 - 3(0) + 5(1)}{12} = 0.50000$$

$$x_2 = \frac{28 - (0.5) - 3(1)}{5} = 4.9000$$

$$x_3 = \frac{76 - 3(0.50000) - 7(4.9000)}{13} = 3.0923$$

Gauss-Seidel Method: Example 2

The absolute relative approximate error

$$\left|\in_a\right|_1 = \left|\frac{0.50000 - 1.0000}{0.50000}\right| \times 100 = 100.00\%$$

$$\left|\in_\mathrm{a}\right|_2 = \left|\frac{4.9000 - 0}{4.9000}\right| \times 100 = 100.00\%$$

$$\left|\in_\mathrm{a}\right|_3 = \left|\frac{3.0923 - 1.0000}{3.0923}\right| \times 100 = 67.662\%$$

The maximum absolute relative error after the first iteration is 100%

Gauss-Seidel Method: Example 2

After Iteration #1

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.5000 \\ 4.9000 \\ 3.0923 \end{bmatrix}$$

Substituting the x values into the equations

$$x_1 = \frac{1 - 3(4.9000) + 5(3.0923)}{12} = 0.14679$$

$$x_2 = \frac{28 - (0.14679) - 3(3.0923)}{5} = 3.7153$$

$$x_3 = \frac{76 - 3(0.14679) - 7(4.900)}{13} = 3.8118$$

After Iteration #2

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.14679 \\ 3.7153 \\ 3.8118 \end{bmatrix}$$

Gauss-Seidel Method: Example 2

Iteration #2 absolute relative approximate error

$$\left|\in_a\right|_1 = \left|\frac{0.14679 - 0.50000}{0.14679}\right| \times 100 = 240.61\%$$

$$\left|\in_a\right|_2 = \left|\frac{3.7153 - 4.9000}{3.7153}\right| \times 100 = 31.889\%$$

$$\left|\in_a\right|_3 = \left|\frac{3.8118 - 3.0923}{3.8118}\right| \times 100 = 18.874\%$$

The maximum absolute relative error after the first iteration is 240.61%

This is much larger than the maximum absolute relative error obtained in iteration #1. Is this a problem?

Gauss-Seidel Method: Example 2

Repeating more iterations, the following values are obtained

Iteration	a_1	$\left\|\in_a\right\|_1 \%$	a_2	$\left\|\in_a\right\|_2 \%$	a_3	$\left\|\in_a\right\|_3 \%$
1	0.50000	100.00	4.9000	100.00	3.0923	67.662
2	0.14679	240.61	3.7153	31.889	3.8118	18.876
3	0.74275	80.236	3.1644	17.408	3.9708	4.0042
4	0.94675	21.546	3.0281	4.4996	3.9971	0.65772
5	0.99177	4.5391	3.0034	0.82499	4.0001	0.074383
6	0.99919	0.74307	3.0001	0.10856	4.0001	0.00101

The solution obtained $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.99919 \\ 3.0001 \\ 4.0001 \end{bmatrix}$ is close to the exact solution of $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$.

Gauss-Seidel Method: Example 3

Given the system of equations

$$3x_1 + 7x_2 + 13x_3 = 76$$

$$x_1 + 5x_2 + 3x_3 = 28$$

$$12x_1 + 3x_2 - 5x_3 = 1$$

With an initial guess of

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Rewriting the equations

$$x_1 = \frac{76 - 7x_2 - 13x_3}{3}$$

$$x_2 = \frac{28 - x_1 - 3x_3}{5}$$

$$x_3 = \frac{1 - 12x_1 - 3x_2}{-5}$$

Gauss-Seidel Method: Example 3

Conducting six iterations, the following values are obtained

Iteration	a_1	$\left\|\in_a\right\|_1 \%$	A_2	$\left\|\in_a\right\|_2 \%$	a_3	$\left\|\in_a\right\|_3 \%$
1	21.000	95.238	0.80000	100.00	50.680	98.027
2	−196.15	110.71	14.421	94.453	−462.30	110.96
3	−1995.0	109.83	−116.02	112.43	4718.1	109.80
4	−20149	109.90	1204.6	109.63	−47636	109.90
5	$2.0364 \quad 10^{5}$	109.89	−12140	109.92	$4.8144 \quad 10^{5}$	109.89
6	$-2.0579 \quad 10^{5}$	109.89	$1.2272 \quad 10^{5}$	109.89	$-4.8653 \quad 10^{6}$	109.89

The values are not converging.

Does this mean that the Gauss-Seidel method cannot be used?

Gauss-Seidel Method

The Gauss-Seidel Method can still be used

The coefficient matrix is not diagonally dominant

$$[A]=\begin{bmatrix}3 & 7 & 13\\ 1 & 5 & 3\\ 12 & 3 & -5\end{bmatrix}$$

But this is the same set of equations used in example #2, which did converge.

$$[A]=\begin{bmatrix}12 & 3 & -5\\ 1 & 5 & 3\\ 3 & 7 & 13\end{bmatrix}$$

If a system of linear equations is not diagonally dominant, check to see if rearranging the equations can form a diagonally dominant matrix.

Gauss-Seidel Method

Not every system of equations can be rearranged to have a diagonally dominant coefficient matrix.

Observe the set of equations

$$x_1 + x_2 + x_3 = 3$$

$$2x_1 + 3x_2 + 4x_3 = 9$$

$$x_1 + 7x_2 + x_3 = 9$$

Which equation(s) prevents this set of equation from having a diagonally dominant coefficient matrix?

Gauss-Seidel Method

Summary

-Advantages of the Gauss-Seidel Method

-Algorithm for the Gauss-Seidel Method

-Pitfalls of the Gauss-Seidel Method

Gauss-Seidel Method

Questions?

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/gauss_seidel.html

THE END

http://numericalmethods.eng.usf.edu