Chapter 02.00E

Physical Problem for Differentiation Electrical Engineering

Electrical systems are used for a wide-array of applications in the commercial and industrial world. Many of them perform physical work using motors, compressors, solenoids, and similar components. In almost all cases, these devices employ electro-magnetic principles and are thus inductive in nature. This presents an electrical load that result in a power factor lagging. Whenever the power factor of a load is not 1.0 (leading or lagging), it results in a discrepancy between the apparent power delivered to the load and the real power consumed by the load.

For most commercial and industrial power consumers, as opposed to residential, the electrical power utility bases its rate charge on the apparent power delivered. This often takes the form of a significant penalty for power factors much below 1.0 (e.g. 0.8). To help correct the power factor it is common to add capacitor banks in parallel with a load that can shift a lagging power factor toward 1.0. Under ideal circumstances, the capacitors could be connected and disconnected from the circuit at any time. Under practical conditions, however, this is not advisable.

The amount of energy in a capacitor is directly related to the voltage across the capacitor as shown in the familiar equation

$$E_c = \frac{1}{2}CV_c^2.$$

When a switch tries to disconnect the capacitor, the energy resists this disconnection, and the result is often sparking and gapping as the switch opens. This not only is a significant source of noise in the circuit, but it also results in damage to the switch that greatly reduces its effective service life. The solution to this problem is quite simple; just open the switch when the stored energy is zero. This, of course, occurs when the voltage across the capacitor is zero.

Unfortunately, it is not just a matter of monitoring the voltage and activating the switch when the voltage is zero. The normal frequency for AC power systems is 60 Hz (50 Hz in Europe). This means that there are 120 zero crossings per second and the typical mechanical switch is simply not fast enough to react “instantaneously” at the zero crossing. To deal with this a smart capacitor bank switch is needed. It monitors the voltage and using numerical methods anticipates the point of zero crossing and initiates the switch action enough in advance to have it occur as close to the zero crossing as possible.

This can be done using a first-order approximation of the Taylor Series as shown by the following equations as well as Figure 1.

$$f(t+\Delta t) \cong f(t) + f'(t)\Delta t = 0$$

$$\Delta t \cong -\frac{f(t)}{f'(t)}$$

If this formula looks a bit familiar, it is the same one used as the basis for Newton's Method as it is commonly used in root finding algorithms.

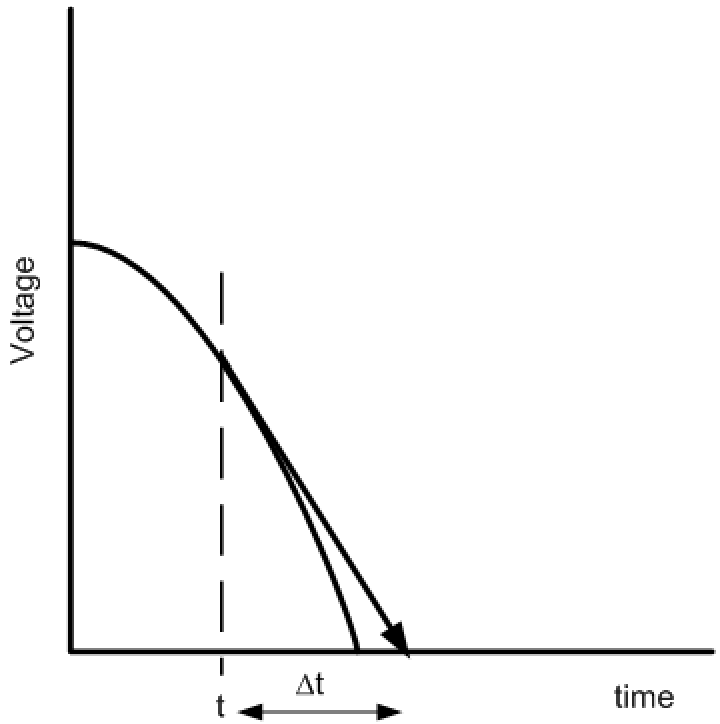

Figure 1 Illustration of Zero-Crossing Prediction Using First-Order Taylor Series

Suppose a system containing a capacitor bank has been constructed and the data of Table 1 was sampled during operation. Using a numerical differentiating algorithm determine the derivative at each point and use the Taylor Series approximation to determine the value of Δt which gives the future estimate of the zero crossing. Keep in mind that when Δt is determined to be negative then the voltage has already crossed the zero point and it will be necessary to wait for the next half cycle of the waveform.

Most numerical methods for taking a derivative (e.g. the Central Difference Method) rely on samples taken both before and after the period in question. In this problem that is not practical since our desire it to anticipate the future zero crossing at which the power should be turned off. Repeat your prior analysis using a differentiation algorithm that is one-sided, that is, one that only uses prior samples to determine the present value of the voltage's derivative.

Table 1 Voltage Samples

Time	Voltage	Time	Voltage
1	0.62161	13	-0.210796
2	0.362358	14	0.087499

3	0.070737	15	0.377978
4	-0.227202	16	0.634693
5	-0.504846	17	0.834713
6	-0.737394	18	0.96017
7	-0.904072	19	0.999859
8	-0.989992	20	0.950233
9	-0.98748	21	0.815725
10	-0.896758	22	0.608351
11	-0.725932	23	0.346635
12	-0.490261	24	0.053955

QUESTIONS

1. What effect does the order of your differentiation algorithm have on the ability to predict correctly the point of zero crossing? Is there an order at which there are diminishing returns on the ability to identify the point of zero crossing?
2. Using a higher-order approximation to the Taylor Series will also improve the prediction of the zero crossing, especially when it is a bit distant. Use a second-order model of the Taylor Series that requires both the first- and second-order derivative to determine Δt.
3. In a real AC system, there could easily be quite a large amount of noise on the sampled voltages. Are there any reasonable algorithms for minimizing the effects this noise has on the results?
4. The typical AC system in the United States uses a 60 Hz frequency (50 Hz in Europe). What is a reasonable sampling rate to use for the voltage sensor? How much does the required length for Δt impact your answer. Did you account for the problem of question (c) in your answer?
5. As in the data of Table 1, the samples have not been synchronized well with the cycles of the AC waveform (that is why you do not see 1 or -1 among the samples). Can you suggest a mechanism for bringing the samples in better synchronization with the cycles?
6. Most industrial smart-sensors for this application use a prediction and correction method to detect the zero crossing, that is, they predict the crossing and then see how close they are for a few cycles prior to the actual disconnecting action. How might such an algorithm be structured?
7. Would there be any advantage in using an interpolation or curve fitting algorithm (e.g. regression to a sinusoid) to better select the instant of true zero voltage?
8. In a system with multiple loads (e.g. a three-phase system), the best instant of shut off for one phase of the capacitor banks may not be the best for the others. Is there any way to determine the point of best compromise or is it simply better to shut each of them off at the zero crossing and tolerate the temporary imbalance to the three-phase system?

Topic	DIFFERENTIATION
Sub Topic	Physical Problem for Electrical Engineering for Differentiation
Summary	The vast majority of commercial and industrial loads are inductive in character. Many electrical consumers find it advantageous to use capacitor banks to adjust their power factor toward 1.0 to avoid extensive rate penalties from their power utility. As load conditions change, it becomes necessary to add or remove capacitors from the bank. The most opportune time to do this is when the voltage across the capacitor is zero to avoid arcing and damages to the switches. Since the switches are slower than the AC waveforms, it is necessary to use a derivative of the voltage to anticipate the zero crossing.
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