Chapter 06.00E

Physical Problem for Regression Electrical Engineering

Problem Statement

All electrical devices exhibit non-linear behavior to some extent. This is caused by many factors including material and thermal properties. For most of the basic components, a simple linear model is used that is sufficient for most operating conditions. This is the case for resistors, capacitors, and inductors. Semiconductor devices, however, are specifically chosen for their non-linear behaviors. This allows them to perform gating, switching, and amplifying operations. Consider the case of a simple diode, like the 1N4001, whose forward-bias VI-characteristic is shown in Figure 1.
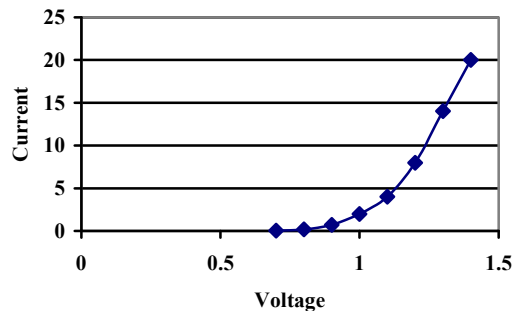

Figure 1 VI Forward-Bias Characteristic of a 1N4001 Diode [1]

In many applications it is suitable to model the forward-bias behavior of the diode as a simple voltage drop of about 0.7 V [2, pp. 70,1]. Other types of diodes, such as LEDs, have voltage drops on the order of 2.0 to 2.5 V. This modeling is not very realistic because it fails to account for the increasing voltage drop as current flow through the diode increases as is shown in Figure 1. If our analysis requires us to know how much current a diode can supply, then the ideal model is clearly inadequate.

It would be, of course, possible to model the diode using some form of interpolated curve, but that would almost certainly force any kind of analysis to be able to only use numerical tools. A much better solution would be to develop a model using simpler linear components

that could be used in place of the diode. This is typically done using a DC supply and a series resistor as shown in Figure 2 [2, pp. 72-5].
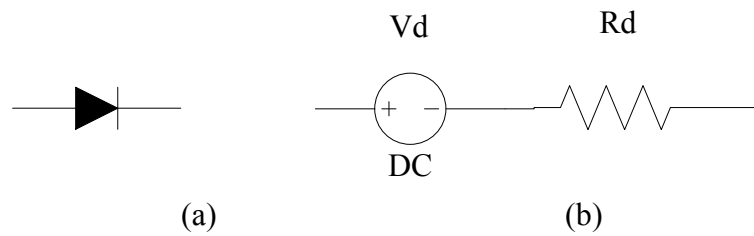
Vd   Rd

(a)   (b)

Figure 2. (a) Ideal Diode and (b) Forward Bias Model

The question then becomes, what are the appropriate values for V_d and R_d? The answer will vary from diode to diode, but good values can be determined using the numerical method of linear regression. The model of Figure 2b in effect replaces the characteristic of Figure 1 with that of Figure 3. Remember that for this to work we are still assuming the diode is sufficiently forward-biased to turn it on which means that the voltage across the diode must exceed V_d.
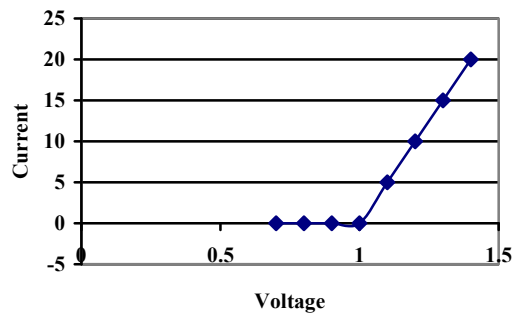
Figure 3. VI Characteristic of Forward Bias Model

Choosing suitable values for V_d and R_d will depend on the potential use for the diode. In a small signal application (low current) it would be more appropriate to choose current values from the original characteristic curve that are well-below 5 A. and for large-signal applications it would be a better fit to use current values of 5 A and above. To study this problem, let us take a closer look at the actual data used to generate the characteristic in Figure 1. This is shown in Table 1.

The simplest way to determine the V_d and R_d from the selected data is to generate a trend line for the region in question. This can be done using linear regression since a linear regression trend line has the effect of minimizing the mean-square error between the modeled line and the actual data. Using the standard linear regression model using β_0 and β_1 parameters yields the following equation for the trend line.

$$I \cong \beta_1 V + \beta_0$$

Once β_0 and β_1 are known V_d and R_d can be computed as follows:

$$V_d = -\frac{\beta_0}{\beta_1}$$

$$R_d = \frac{1}{\beta_1}$$

Voltage, V	Current, A	Small Signal	Large Signal
0.0	0.00	N	N
0.6	0.01	Y	N
0.7	0.05	Y	N
0.8	0.20	Y	N
0.9	0.70	Y	Y
1.0	2.00	Y	Y
1.1	4.00	Y	Y
1.2	8.00	N	Y
1.3	14.00	N	Y
1.4	20.00	N	Y

Table 1. VI Characteristic of a Forward-Biased Diode

The effects of this choice are shown in Figure 4. When the data from the small-signal column of Table 1 is used to generate the regression line, it results in the model of Figure 4a and when the data from the large-signal column of Table 2 is used, the result is the model of Figure 4b.
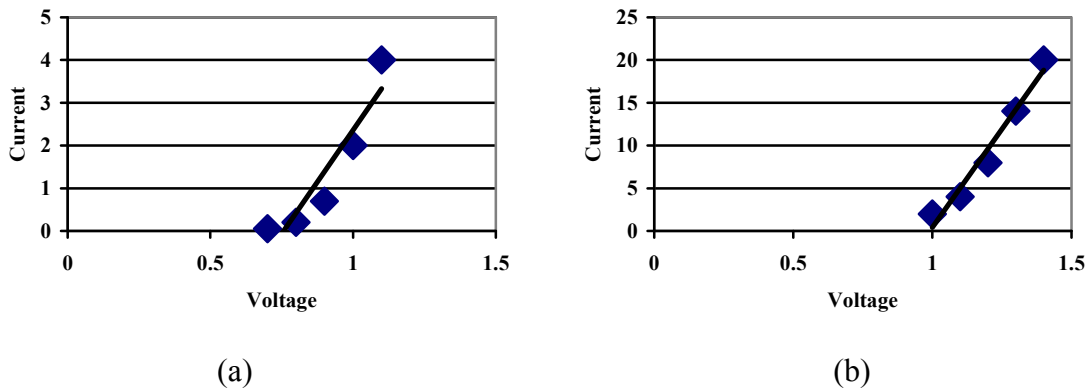

(a)   (b)

Figure 4 (a) Small-Signal Model and (b) Large-Signal Model for a Diode

The small-signal model agrees with the typical diode modeling where V_d is approximately 0.7V and will model the diode fairly well for currents below about 4 A. For a higher-current circuit the large-signal model with V_d approximately 1 V is a better choice, but will not model the data as well when the current is 2 A or below.

Complete the analysis by determining more exact values for V_d and R_d for both of these models. Use the small signal and large signal columns of Table 1 to select the data to use in your analysis.

QUESTIONS
1. This analysis can be extended to handle other diodes. Find the data sheet for a 1N4150 or an LED, which shows the VI-Characteristic and derive suitable small- and large-signal forward-bias models.
2. A similar analysis can be used to model the reverse-bias behavior of a diode. Use the datasheet from (1) to develop an appropriate model.
3. Model the reverse-bias characteristics for a zener diode (e.g. the 1N4678 to 1N4717 family).
4. The use of linear regression only reduces the modeling/approximating error for the sample data points provided. In this problem, we actually have a two-piece linear model (the flat portion up to V_d when the diode turns on and the forward-bias model involving R_d after the diode is on). This may not adequately model the entire forward-bias characteristic. Using your model values for V_d and R_d as a start, examine the overall characteristic and pick values for V_d and R_d that minimize the mean-square error over the entire forward bias region. This will have to be a combination of the error caused by having V_d to the right of the actual turn-on point as well as the error in the turned-on region. How close is this model to our simpler approach that modeled each piece separately?

References

[1]   1N4001 Data Sheet, Fairchild Semiconductor, http://www.fairchildsemi.com/ds/1N/1N4007.pdf, accessed May 12, 2005.
[2]   Malvino, A., Electronic Principles, Sixth Edition, Glencoe McGraw-Hill, 1999.

REGRESSION	
Topic	Physical Problem for Regression
Summary	Many electrical devices have non-linear behavior. It is often advantageous to substitute a model for the device using simpler linear components. In the case of a diode, this is often done with a DC supply and a series resistor. To determine an appropriate value for the resistor, regression is used.
Major	Electrical Engineering
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