Chapter 03.01
Solution of Quadratic Equations

After reading this chapter, you should be able to:

1. *find the solutions of quadratic equations,*
2. *derive the formula for the solution of quadratic equations,*
3. *solve simple physical problems involving quadratic equations.*

What are quadratic equations and how do we solve them?

A quadratic equation has the form

$$ax^2 + bx + c = 0, \text{ where } a \neq 0$$

The solution to the above quadratic equation is given by

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

So the equation has two roots, and depending on the value of the discriminant, $b^2 - 4ac$, the equation may have real, complex or repeated roots.

If $b^2 - 4ac < 0$, the roots are complex.
If $b^2 - 4ac > 0$, the roots are real.
If $b^2 - 4ac = 0$, the roots are real and repeated.

Example 1

Derive the solution to $ax^2 + bx + c = 0$.

Solution

$$ax^2 + bx + c = 0$$

Dividing both sides by a, $(a \neq 0)$, we get

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

Note if $a = 0$, the solution to

$$ax^2 + bx + c = 0$$

is

$$x = -\frac{c}{b}$$

Rewrite

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0$$

as

$$\left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\left(x + \frac{b}{2a}\right)^2 = \frac{b^2}{4a^2} - \frac{c}{a}$$

$$= \frac{b^2 - 4ac}{4a^2}$$

$$x + \frac{b}{2a} = \pm\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$= \pm\frac{\sqrt{b^2 - 4ac}}{2a}$$

$$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Example 2

A ball is thrown down at 50 mph from the top of a building. The building is 420 feet tall. Derive the equation that would let you find the time the ball takes to reach the ground.

Solution

The distance s covered by the ball is given by

$$s = ut + \frac{1}{2}gt^2$$

where

u = initial velocity (ft/s)

g = acceleration due to gravity (ft/s^2)

t = time (s)

Given

$$u = 50\,\frac{\text{miles}}{\text{hour}} \times \frac{1\,\text{hour}}{3600\,\text{s}} \times \frac{5280\ \text{ft}}{1\,\text{mile}}$$

$$= 73.33\,\frac{\text{ft}}{\text{s}}$$

$$g = 32.2\,\frac{\text{ft}}{\text{s}^2}$$

$$s = 420\,\text{ft}$$

we have

$$420 = 73.33t + \frac{1}{2}(32.2)t^2$$

$$16.1t^2 + 73.33t - 420 = 0$$

The above equation is a quadratic equation, the solution of which would give the time it would take the ball to reach the ground. The solution of the quadratic equation is

$$t = \frac{-73.33 \pm \sqrt{73.33^2 - 4\times 16.1\times(-420)}}{2(16.1)}$$

$$= 3.315, -7.870$$

Since $t > 0$, the valid value of time t is $3.315\ \text{s}$.

NONLINEAR EQUATIONS	
Topic	Solution of quadratic equations
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