Chapter 04.00F

Physical Problem for Industrial Engineering Simultaneous Linear Equations

Problem Statement

A company that manufactures small toys recently received a contract from a fast-food company, to manufacture three toys, at a low cost, to be added to kid's lunches. The company has to manufacture toys for boys (toy B), girls (toy G) and a generic version (toy U). Furthermore, based on the demand and demographics, the fast food company has specified that 5% more girl's toys than boy's toys should be produced, and that there is no constraint is specified on the number of generic toys. The components of each toy (B, G, and U) must be injection molded out of plastic (Process 1) and then assembled (Process 2). After the toys have been designed, it is determined that the following production times will be needed on each toy:

- Toy B will require 2 minutes for injection molding 6 toys and 1 minute for assembling all 6 toys.
- Toy G will require 2 minutes for injection molding 12 toys and 8 minutes for assembling all 12 toys.
- Toy U will require 4 minutes for injection molding 6 toys and 2 minutes for assembling all 6 toys.

Note that because of daily scheduled maintenance of the injection molding machine, it can only run for a maximum of 756 out of 1440 minutes a day, whereas the assembly line works 3 shifts a day with scheduled breaks for a maximum of 1260 out of 1440 minutes per day. An industrial engineer working for the toy company is asked to determine the production schedule that maximizes, on a daily basis, the use of both the injection molding machine and the assembly line.

Background The variables need to solve the problem are listed in Table 1

Table 1. Variables for these different toys.

Variable	Toy B	Toy G	Toy U
Time (minutes) required in Process 1 per toy	B_1	G_1	U_1
Time (minutes) required in Process 2 per toy	B_2	G_2	U_2
Total manufactured per day	X_B	X_G	X_U

The total time required to produce toys in process 1 (injection molding) has to be equal to the maximum minutes per day that process 1 can run, that is,

$$B_1X_B + G_1X_G + U_1X_U = M_1$$

where M_1 is the maximum minutes that process 1 can run per day. Similarly, for process 2 (assembly),

$$B_2X_B + G_2X_G + U_2X_U = M_2$$

where M_2 is the maximum minutes that process 2 can run per day. Finally, the constraint of 5% more girl's toys than boy's toys is expressed as

$$1.05X_B = X_G \qquad \text{or}$$

$$1.05X_B - X_G = 0$$

The previous three simultaneous linear equations can be expressed in matrix form as follows

$$\begin{bmatrix} B_1 & G_1 & U_1 \\ B_2 & G_2 & U_2 \\ 1.05 & -1 & 0 \end{bmatrix} \begin{Bmatrix} X_B \\ X_G \\ X_U \end{Bmatrix} = \begin{Bmatrix} M_1 \\ M_2 \\ 0 \end{Bmatrix}$$

Solution

The input variables to the preceding simultaneous linear equations are

$$B_1 = \frac{2}{6} = \frac{1}{3} \text{ toy per minute}$$

$$B_2 = \frac{1}{6} \text{ toy per minute}$$

$$G_1 = \frac{2}{12} = \frac{1}{6} \text{ toy per minute}$$

$$G_2 = \frac{8}{12} = \frac{2}{3} \text{ toy per minute}$$

$$U_1 = \frac{4}{6} = \frac{2}{3} \text{ toy per minute}$$

$$U_2 = \frac{2}{6} = \frac{1}{3} \text{ toy per minute}$$

$$M_1 = 756 \text{ minutes per day}$$

$$M_2 = 1260 \text{ minutes per day}$$

Substituting onto the matrix representation of the simultaneous linear equations yields

$$\frac{1}{6}\begin{bmatrix} 2 & 1 & 4 \\ 1 & 4 & 2 \\ 6.3 & -6 & 0 \end{bmatrix}\begin{Bmatrix} X_B \\ X_G \\ X_U \end{Bmatrix} = \begin{Bmatrix} 756 \\ 1260 \\ 0 \end{Bmatrix}$$

One needs to solve these simultaneous linear equations to find the number of boys, girl and unisex toys for maximizing the manufacturing facility yield.

SIMULTANEOUS LINEAR EQUATIONS	
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