

# Direct Method of Interpolation

Industrial Engineering Majors

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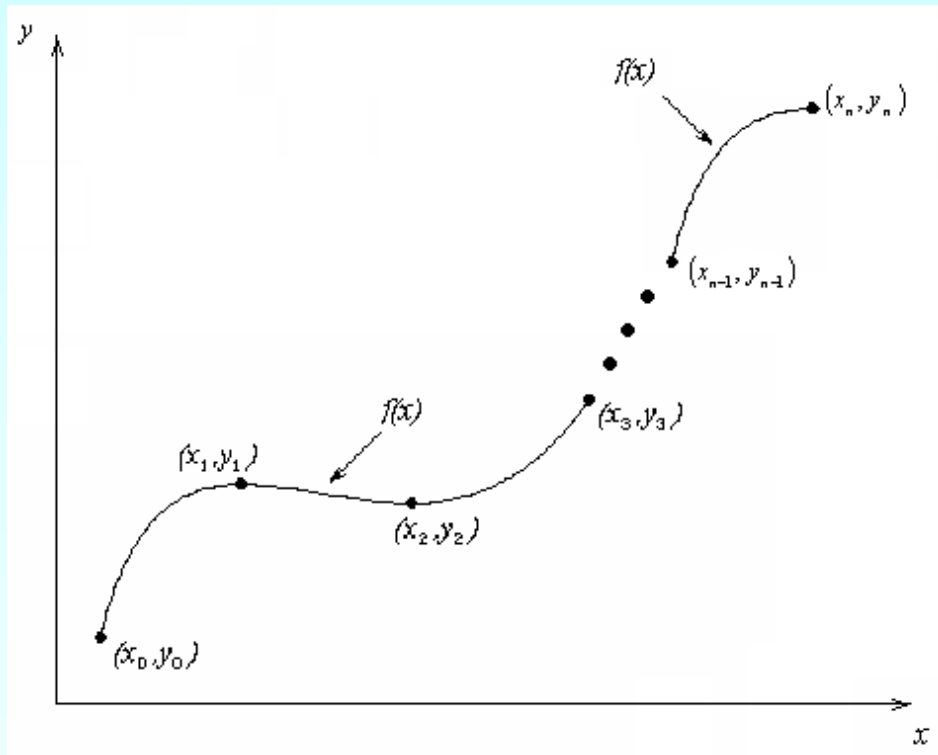
Transforming Numerical Methods Education for STEM Undergraduates

# Direct Method of Interpolation

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# What is Interpolation ?

Given  $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ , find the value of 'y' at a value of 'x' that is not given.



**Figure 1** Interpolation of discrete.

# Interpolants

Polynomials are the most common choice of interpolants because they are easy to:

- Evaluate
- Differentiate, and
- Integrate

# Direct Method

Given 'n+1' data points  $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$ , pass a polynomial of order 'n' through the data as given below:

$$y = a_0 + a_1x + \dots + a_nx^n .$$

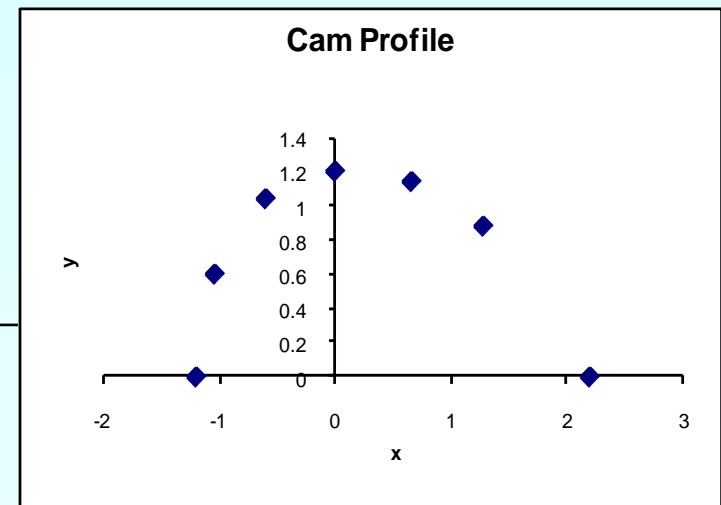
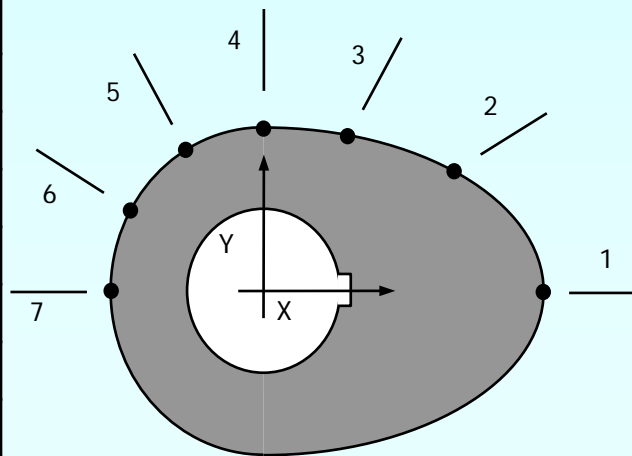
where  $a_0, a_1, \dots, a_n$  are real constants.

- Set up 'n+1' equations to find 'n+1' constants.
- To find the value 'y' at a given value of 'x', simply substitute the value of 'x' in the above polynomial.

# Example

A curve needs to be fit through the given points to fabricate the cam. If the cam follows a straight line profile between  $x = 1.28$  to  $x = 0.66$ , what is the value of  $y$  at  $x=1.1$  ? Find using the direct method and linear interpolation.

Point	$x$ (in.)	$y$ (in.)
1	2.20	0.00
2	1.28	0.88
3	0.66	1.14
4	0.00	1.20
5	-0.60	1.04
6	-1.04	0.60
7	-1.20	0.00



# Linear Interpolation

$$y(x) = a_0 + a_1 x$$

$$y(1.28) = a_0 + a_1(1.28) = 0.88$$

$$y(0.66) = a_0 + a_1(0.66) = 1.14$$

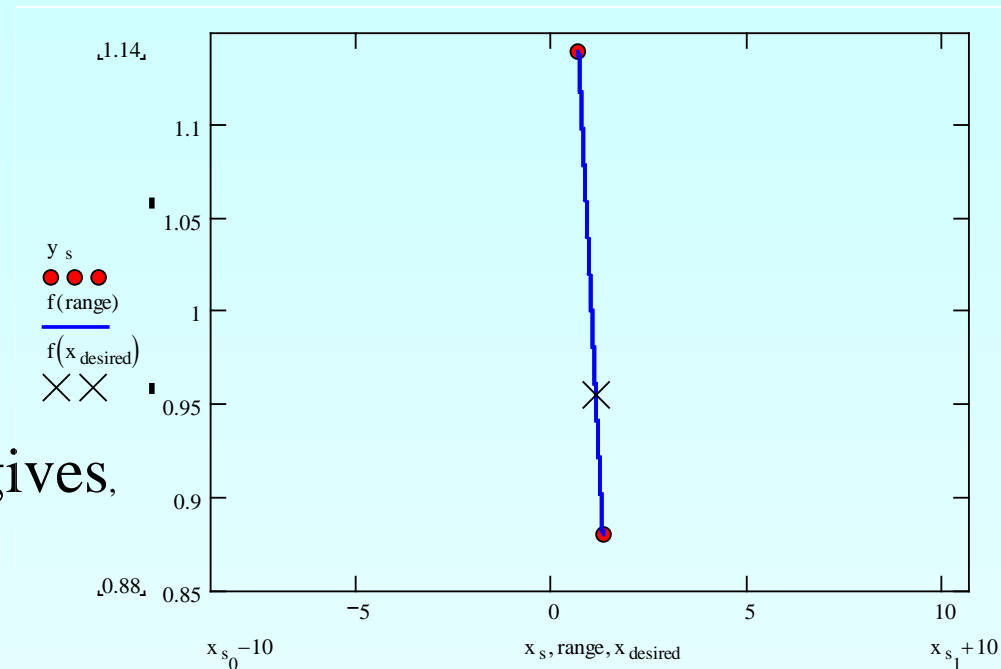
Solving the above two equations gives,

$$a_0 = 1.4168 \quad a_1 = -0.41935$$

Hence

$$y(x) = 1.4168 - 0.41935x, \quad 0.66 \leq x \leq 1.28.$$

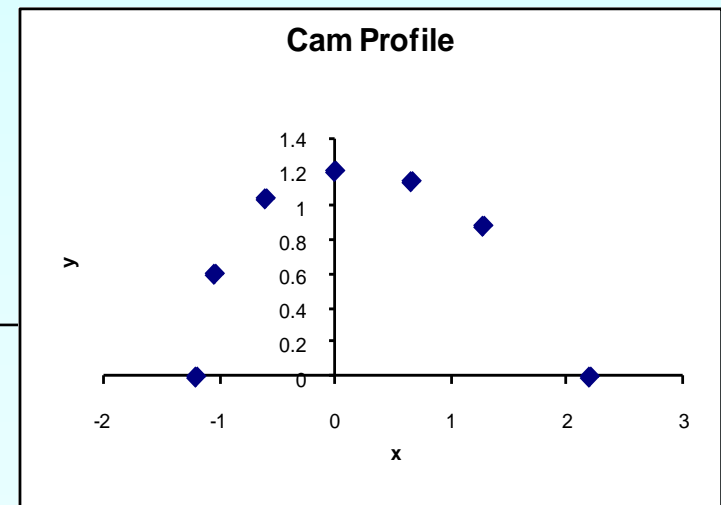
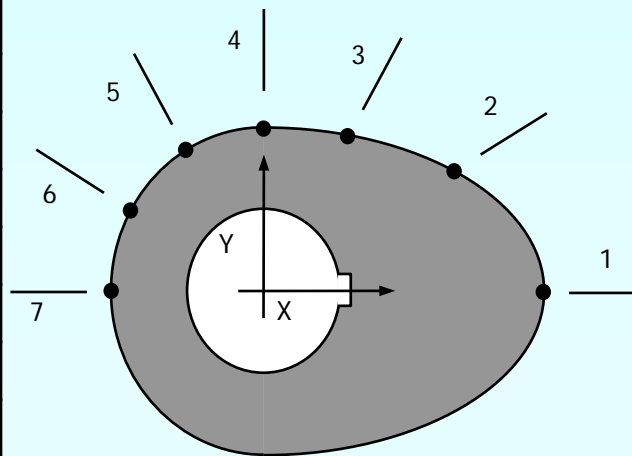
$$y(1.10) = 1.4168 - 0.41935(1.10) = 0.95548 \text{ in.}$$



# Example

A curve needs to be fit through the given points to fabricate the cam. If the cam follows a straight line profile between  $x = 1.28$  to  $x = 0.66$ , what is the value of  $y$  at  $x=1.1$  ? Find using the direct method and quadratic interpolation.

Point	$x$ (in.)	$y$ (in.)
1	2.20	0.00
2	1.28	0.88
3	0.66	1.14
4	0.00	1.20
5	-0.60	1.04
6	-1.04	0.60
7	-1.20	0.00





# Quadratic Interpolation

$$y(x) = a_0 + a_1x + a_2x^2$$

$$y(2.20) = a_0 + a_1(2.20) + a_2(2.20)^2 = 0$$

$$y(1.28) = a_0 + a_1(1.28) + a_2(1.28)^2 = 0.88$$

$$y(0.66) = a_0 + a_1(0.66) + a_2(0.66)^2 = 1.14$$

Solving the above three equations gives

$$a_0 = 1.1221 \quad a_1 = 0.25734 \quad a_2 = -0.34881$$

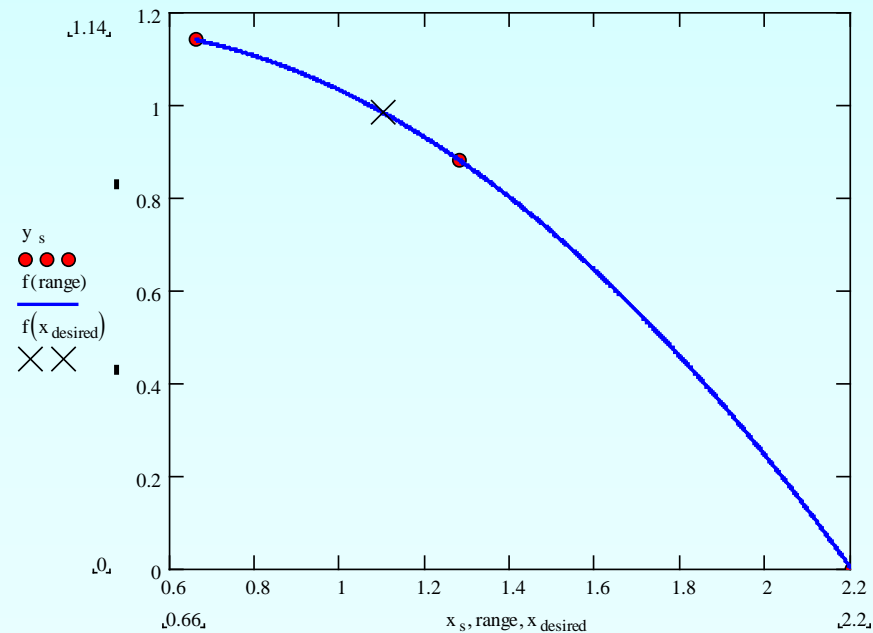
# Quadratic Interpolation (contd)

$$y(x) = 1.1221 + 0.25734x - 0.34881x^2, \quad 0.66 \leq x \leq 2.20$$

$$\begin{aligned} y(1.10) &= 1.1221 + 0.25734(1.10) - 0.34881(1.10)^2 \\ &= 0.98311 \text{ in.} \end{aligned}$$

The absolute relative approximate error obtained between the results from the first and second order polynomial is

$$\begin{aligned} |\epsilon_a| &= \left| \frac{0.98311 - 0.95548}{0.98311} \right| \times 100 \\ &= 2.8100\% \end{aligned}$$



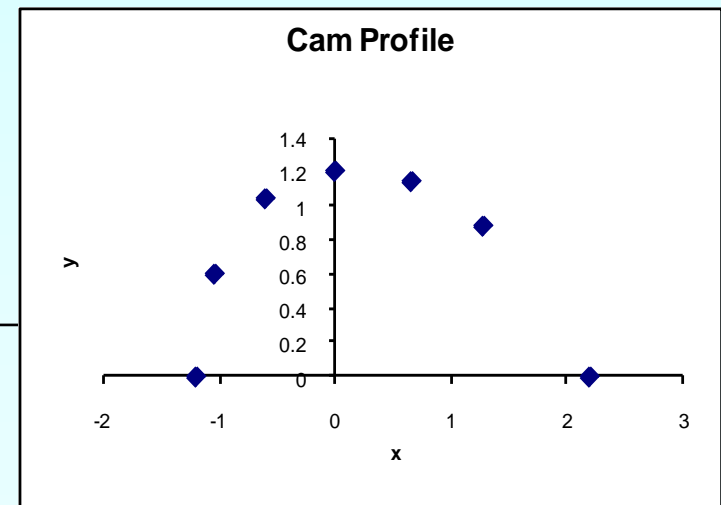
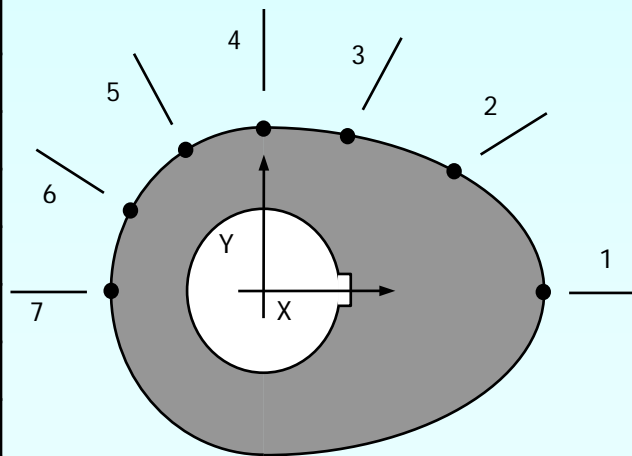
# Comparison Table

Order of Polynomial	1	2
Value of $y$ at $x = 1.10$	0.98311	0.95548
Absolute Relative Approximate Error	-----	2.8100 %

# Example

A curve needs to be fit through the given points to fabricate the cam. If the cam follows a straight line profile between  $x = 1.28$  to  $x = 0.66$ , what is the value of  $y$  at  $x=1.1$  ? Find using the direct method and a sixth order polynomial.

Point	$x$ (in.)	$y$ (in.)
1	2.20	0.00
2	1.28	0.88
3	0.66	1.14
4	0.00	1.20
5	-0.60	1.04
6	-1.04	0.60
7	-1.20	0.00



# Sixth Order Interpolation

$$y(x) = a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6$$

$$y(2.20) = 0.00 = a_0 + a_1(2.20) + a_2(2.20)^2 + a_3(2.20)^3 + a_4(2.20)^4 + a_5(2.20)^5 + a_6(2.20)^6$$

$$y(1.28) = 0.88 = a_0 + a_1(1.28) + a_2(1.28)^2 + a_3(1.28)^3 + a_4(1.28)^4 + a_5(1.28)^5 + a_6(1.28)^6$$

$$y(0.66) = 1.14 = a_0 + a_1(0.66) + a_2(0.66)^2 + a_3(0.66)^3 + a_4(0.66)^4 + a_5(0.66)^5 + a_6(0.66)^6$$

$$y(0.00) = 1.20 = a_0 + a_1(0.00) + a_2(0.00)^2 + a_3(0.00)^3 + a_4(0.00)^4 + a_5(0.00)^5 + a_6(0.00)^6$$

$$y(-0.60) = 1.04 = a_0 + a_1(-0.60) + a_2(-0.60)^2 + a_3(-0.60)^3 + a_4(-0.60)^4 + a_5(-0.60)^5 + a_6(-0.60)^6$$

$$y(-1.04) = 0.60 = a_0 + a_1(-1.04) + a_2(-1.04)^2 + a_3(-1.04)^3 + a_4(-1.04)^4 + a_5(-1.04)^5 + a_6(-1.04)^6$$

$$y(-1.20) = 0.00 = a_0 + a_1(-1.20) + a_2(-1.20)^2 + a_3(-1.20)^3 + a_4(-1.20)^4 + a_5(-1.20)^5 + a_6(-1.20)^6$$

# Sixth Order Interpolation (contd)

Writing the seven equations in matrix form, we have

$$\begin{bmatrix} 1 & 2.20 & 2.20^2 & 2.20^3 & 2.20^4 & 2.20^5 & 2.20^6 \\ 1 & 1.28 & 1.28^2 & 1.28^3 & 1.28^4 & 1.28^5 & 1.28^6 \\ 1 & 0.66 & 0.66^2 & 0.66^3 & 0.66^4 & 0.66^5 & 0.66^6 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -0.60 & 0.60^2 & -0.60^3 & 0.60^4 & -0.60^5 & 0.60^6 \\ 1 & -1.04 & 1.04^2 & -1.04^3 & 1.04^4 & -1.04^5 & 1.04^6 \\ 1 & -1.20 & 1.20^2 & -1.20^3 & 1.20^4 & -1.20^5 & 1.20^6 \end{bmatrix} \begin{bmatrix} a_0 \\ a_1 \\ a_2 \\ a_3 \\ a_4 \\ a_5 \\ a_6 \end{bmatrix} = \begin{bmatrix} 0.00 \\ 0.88 \\ 1.14 \\ 1.20 \\ 1.04 \\ 0.60 \\ 0.00 \end{bmatrix}$$

# Sixth Order Polynomial (contd)

Solving the above seven equations gives

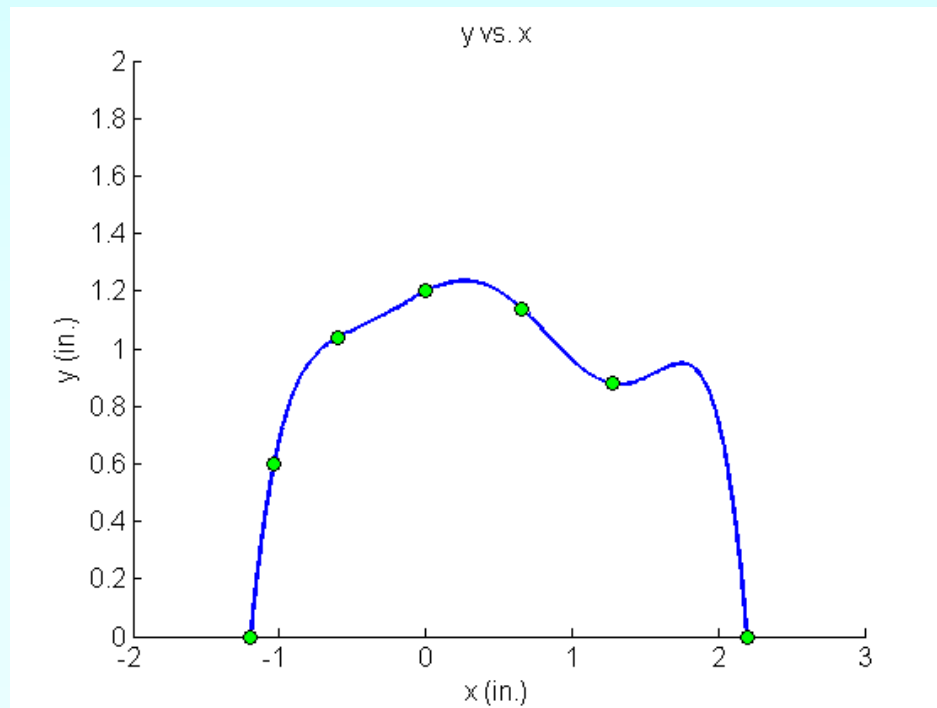
$$a_0 = 1.2 \quad a_2 = -0.27255 \quad a_4 = 0.072013 \quad a_6 = -0.17103$$

$$a_1 = 0.25112 \quad a_3 = -0.56765 \quad a_5 = 0.45241$$

$$\begin{aligned} y(x) &= a_0 + a_1x + a_2x^2 + a_3x^3 + a_4x^4 + a_5x^5 + a_6x^6 \\ &= 1.2 + 0.25112x - 0.27255x^2 - 0.56765x^3 \\ &\quad + 0.072013x^4 + 0.45241x^5 - 0.17103x^6, \quad -1.20 \leq x \leq 2.20 \end{aligned}$$

# Sixth Order Polynomial (contd)

$$y(x) = 1.2 + 0.25112x - 0.27255x^2 - 0.56765x^3 + 0.072013x^4 + 0.45241x^5 - 0.17103x^6, \quad -1.20 \leq x \leq 2.20$$





# Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

[http://numericalmethods.eng.usf.edu/topics/direct\\_method.html](http://numericalmethods.eng.usf.edu/topics/direct_method.html)

**THE END**

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