Binary Representation

Major: All Engineering Majors

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http://numericalmethods.eng.usf.edu

Numerical Methods for STEM undergraduates

How a Decimal Number is Represented

$$257.76 = 2\times10^{2} + 5\times10^{1} + 7\times10^{0} + 7\times10^{-1} + 6\times10^{-2}$$

Base 2

$$(1011.0011)_2 = \begin{pmatrix} (1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0) \\ + (0\times 2^{-1} + 0\times 2^{-2} + 1\times 2^{-3} + 1\times 2^{-4}) \end{pmatrix}_{10}$$

$$= 11.1875$$

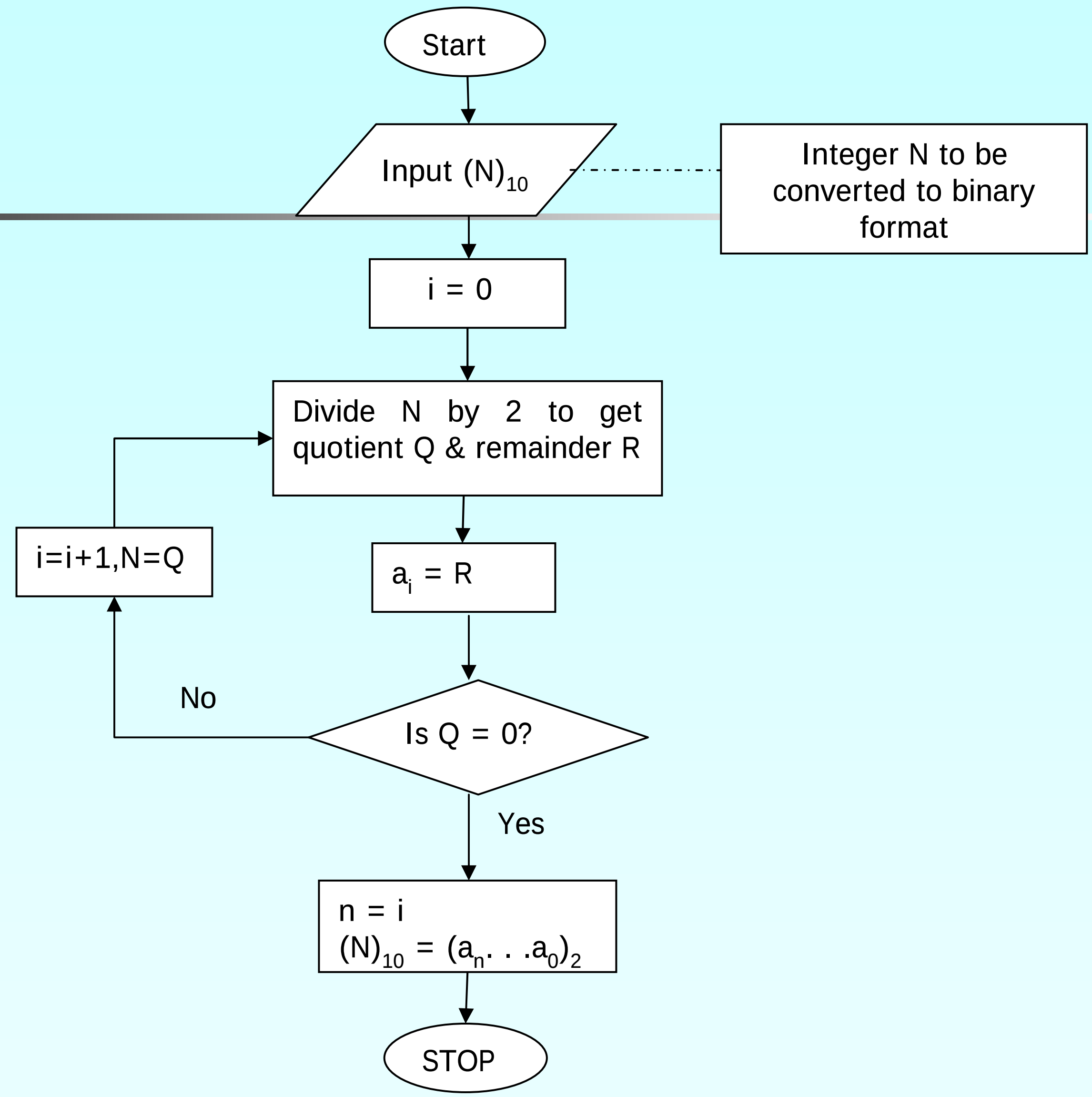
Convert Base 10 Integer to binary representation

Table 1 Converting a base-10 integer to binary representation.

	Quotient	Remainder
11/2	5	$1 = a_0$
5/2	2	$1 = a_1$
2/2	1	$0 = a_2$
1/2	0	$1 = a_3$

Hence

$$(11)_{10} = (a_3 a_2 a_1 a_0)_2$$
$$= (1011)_2$$


Start
Input $(N)_{10}$
Integer N to be converted to binary format
i = 0
Divide N by 2 to get quotient Q & remainder R
a_i = R
Is Q = 0?
No
i=i+1,N=Q
Yes
n = i
$(N)_{10} = (a_n \ldots a_0)_2$
STOP

Fractional Decimal Number to Binary

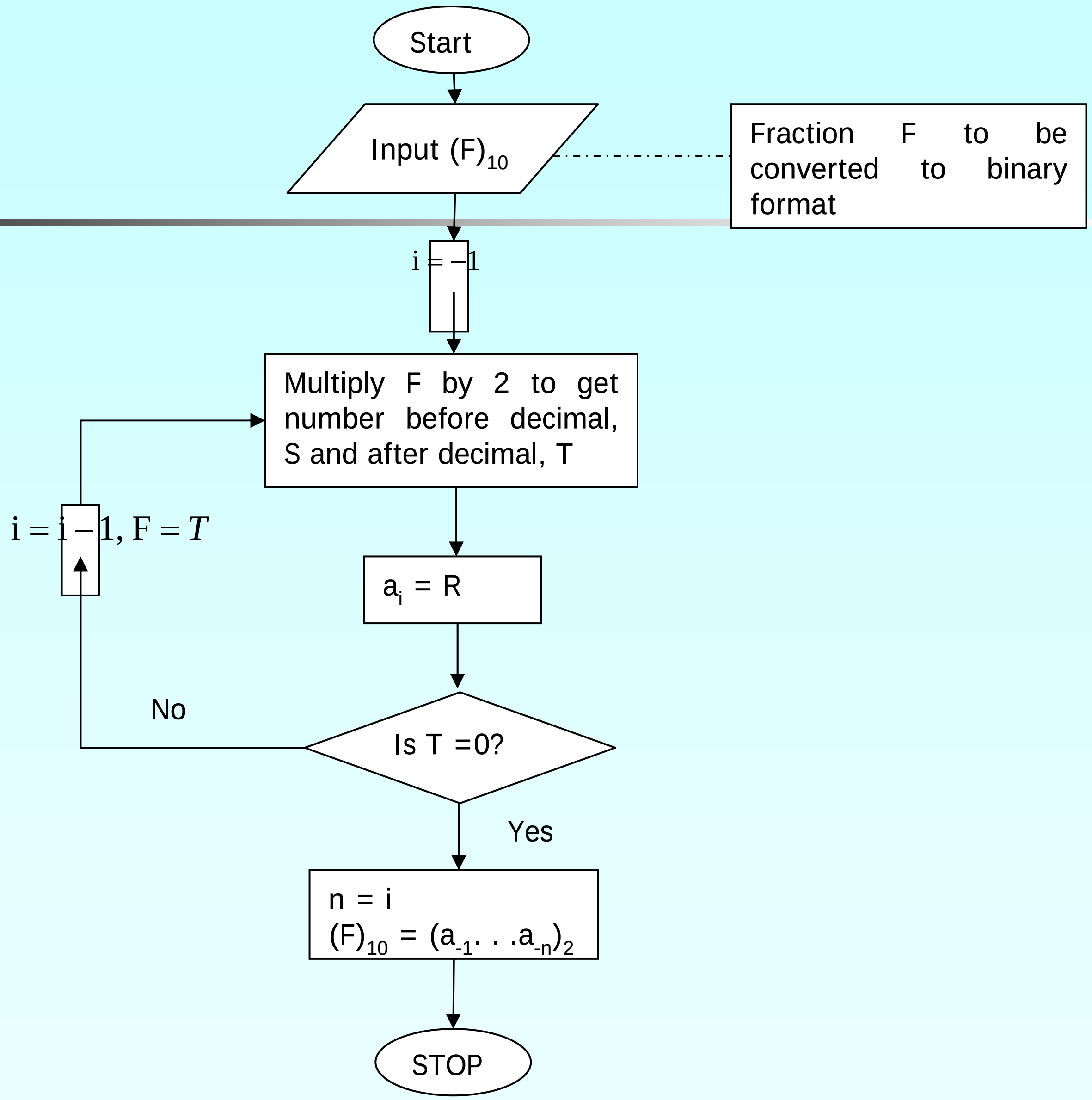
Table 2. Converting a base-10 fraction to binary representation.

	Number	Number after decimal	Number before decimal
0.1875×2	0.375	0.375	$0 = a_{-1}$
0.375×2	0.75	0.75	$0 = a_{-2}$
0.75×2	1.5	0.5	$1 = a_{-3}$
0.5×2	1.0	0.0	$1 = a_{-4}$

Hence

$$(0.1875)_{10} = (a_{-1}a_{-2}a_{-3}a_{-4})_2$$

$$= (0.0011)_2$$


Start
Input (F)10
Fraction F to be converted to binary format
i = −1
Multiply F by 2 to get number before decimal, S and after decimal, T
i = i − 1, F = T
ai = R
No
Is T =0?
Yes
n = i
(F)10 = (a-1. . .a-n)2
STOP

Decimal Number to Binary

$$(11.1875)_{10} = (\quad ?.? \quad)_2$$

Since

$$(11)_{10} = (1011)_2$$

and

$$(0.1875)_{10} = (0.0011)_2$$

we have

$$(11.1875)_{10} = (1011.0011)_2$$

All Fractional Decimal Numbers Cannot be Represented Exactly

Table 3. Converting a base-10 fraction to approximate binary representation.

	Number	Number after decimal	Number before Decimal
0.3×2	0.6	0.6	$0=a_{-1}$
0.6×2	1.2	0.2	$1=a_{-2}$
0.2×2	0.4	0.4	$0=a_{-3}$
0.4×2	0.8	0.8	$0=a_{-4}$
0.8×2	1.6	0.6	$1=a_{-5}$

$$(0.3)_{10} \approx (a_{-1}a_{-2}a_{-3}a_{-4}a_{-5})_2 = (0.01001)_2 = 0.28125$$

Another Way to Look at Conversion

Convert $(11.1875)_{10}$ to base 2

$$
\begin{aligned}
(11)_{10} &= 2^3 + 3 \\
&= 2^3 + 2^1 + 1 \\
&= 2^3 + 2^1 + 2^0 \\
&= 1\times 2^3 + 0\times 2^2 + 1\times 2^1 + 1\times 2^0 \\
&= (1011)_2
\end{aligned}
$$

$$(0.1875)_{10} = 2^{-3} + 0.0625$$
$$= 2^{-3} + 2^{-4}$$
$$= 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}$$
$$= (.0011)_2$$

$$(11.1875)_{10} = (1011.0011)_2$$