Problem Set
Multiple Choice Test

Chapter 01.01
Introduction to Numerical Methods
COMPLETE SOLUTION SET

1. Solving an engineering problem requires four steps. In order of sequence the four steps are
   (A) formulate, model, solve, implement
   (B) formulate, solve, model, implement
   (C) formulate, model, implement, solve
   (D) model, formulate, implement, solve

Solution
The correct answer is (A).

The four steps of solving an engineering problem are:
1) Formulate the problem (same as describing the problem)
2) Mathematically model the problem
3) Solve the mathematical model
4) Implement the results in engineering practice

2. One of the roots of the equation $x^3 - 3x^2 + x - 3 = 0$ is

(A) -1

(B) 1

(C) $\sqrt{3}$

(D) 3

Solution

The correct answer is (D).

$$x^3 - 3x^2 + x - 3 = 0$$
$$x^2(x-3) + 1(x-3) = 0$$
$$(x^2+1)(x-3) = 0$$

Therefore, $x = 3$ is a solution to the above equation.

3. The solution to the set of equations

$$25a + b + c = 25$$
$$64a + 8b + c = 71$$
$$144a + 12b + c = 155$$

most nearly is $(a,b,c) =$

(A) (1,1,1)
(B) (1,-1,1)
(C) (1,1,-1)
(D) does not have a unique solution.

Solution

The correct answer is (C).

$$25a + b + c = 25 \quad (1)$$
$$64a + 8b + c = 71 \quad (2)$$
$$144a + 12b + c = 155 \quad (3)$$

Subtracting Equation (1) from Equation (2) gives

$$39a + 7b = 46 \quad (4)$$

Subtracting Equation (1) from Equation (3) gives

$$119a + 11b = 130 \quad (5)$$

From Equation (4),

$$a = \frac{46 - 7b}{39} \quad (6)$$

Substituting the value of a from Equation (6) in Equation (5) gives

$$119\left(\frac{46 - 7b}{39}\right) + 11b = 130$$
$$140.36 - 21.359 + 11b = 130$$
$$-10.358b = -10.36$$
$$b = \frac{-10.36}{-10.359}$$
$$= 1.0001$$

From Equation (4),

$$a = \frac{46 - 7(1.0001)}{39}$$
$$= 0.99998$$

From Equation (1),

$$c = 25 - 25a - b$$
$$= 25 - 25(0.99998) - 1.0001$$
$$= -0.99960$$

So

$$(a,b,c) = (0.99998, 1.0001, -0.99960)$$
$$\approx (1,1,-1)$$

4. The exact integral of $\int_0^{\frac{\pi}{4}} 2\cos 2x dx$ is most nearly

(A) -1.000
(B) 1.000
(C) 0.000
(D) 2.000

Solution
The correct answer is (B).

$$\int_0^{\frac{\pi}{4}} 2\cos 2x dx$$
$$= \left[2\frac{\sin(2x)}{2}\right]_0^{\frac{\pi}{4}}$$
$$= [\sin(2x)]_0^{\frac{\pi}{4}}$$
$$= \sin\left(2\left(\frac{\pi}{4}\right)\right) - \sin(2(0))$$
$$= \sin\left(\frac{\pi}{2}\right) - \sin(0)$$
$$= 1 - 0$$
$$= 1$$

5. The value of $\frac{dy}{dx}(1.0)$, given $y = 2\sin(3x)$ most nearly is

(A) -5.9399
(B) -1.980
(C) 0.31402
(D) 5.9918

Solution

The correct answer is (A).

$$y = 2\sin(3x)$$
$$\frac{dy}{dx} = 2(3\cos(3x))$$
$$= 6\cos(3x)$$
$$\frac{dy}{dx}(1.0) = 6\cos(3(1.0))$$ (Remember the argument of trig functions is radians)
$$= 6(-0.98999)$$
$$= -5.9399$$

6. The form of the exact solution of the ordinary differential equation

$$2\frac{dy}{dx}+3y=5e^{-x},\ y(0)=5 \text{ is}$$

(A) $Ae^{-1.5x} + Be^{x}$

(B) $Ae^{-1.5x} + Be^{-x}$

(C) $Ae^{1.5x} + Be^{-x}$

(D) $Ae^{-1.5x} + Bxe^{-x}$

Solution

The correct answer is (B).

$$2\frac{dy}{dx}+3y=5e^{-x},\ y(0)=5$$

The characteristic equation for the homogeneous part of the solution is

$$2m^1 + 3m^0 = 0$$
$$2m + 3 = 0$$
$$m = -1.5$$

The homogeneous part of the solution hence is

$$y_H = Ae^{-1.5x}$$

The particular part of the solution is

$$y_P = Be^{-x}$$

So the form of the solution to the ordinary differential equation is

$$y = y_H + y_P$$
$$= Ae^{-1.5x} + Be^{-x}$$