Problem Set#1
Multiple Choice Test
Chapter 01.06 Propagation Errors
COMPLETE SOLUTION SET

1. If $A = 3.56 \pm 0.05$ and $B = 3.25 \pm 0.04$, the values of $A + B$ are
   (A) $6.81 \le A + B \le 6.90$
   (B) $6.72 \le A + B \le 6.90$
   (C) $6.81 \le A + B \le 6.81$
   (D) $6.71 \le A + B \le 6.91$

Solution
The correct answer is (B).

$$A = 3.50 \pm 0.05$$

Hence

$$3.56 - 0.05 \le A \le 3.56 + 0.05$$
$$3.51 \le A \le 3.61$$

$$B = 3.25 \pm 0.04$$

Hence

$$3.25 - 0.04 \le B \le 3.25 + 0.04$$
$$3.21 \le B \le 3.29$$

Hence

$$3.51 + 3.21 \le A + B \le 3.61 + 3.29$$
$$6.72 \le A + B \le 6.90$$

2. A number A is correctly rounded to 3.18 from a given number B. Then $\left|A-B\right| \leq C$, where C is

(A) 0.005
(B) 0.01
(C) 0.18
(D) 0.09999

Solution

The correct answer is (A).

Since A is rounded off to 3.18, the number can be $3.17XYZ\ldots$ where X is a number between 5 and 9 or $3.18XYZ$ where X is a number between 0 and 4. Hence,

$\left|A-B\right| \leq C$ makes $C = 0.005$

3. Two numbers *A* and *B* are approximated as *C* and *D*, respectively. The relative error in $C \times D$ is given by

(A) $\left|\left(\frac{A-C}{A}\right)\right| \times \left|\left(\frac{B-D}{B}\right)\right|$

(B) $\left|\left(\frac{A-C}{A}\right)\right| + \left|\left(\frac{B-D}{B}\right)\right| + \left|\left(\frac{A-C}{A}\right)\right| \times \left|\left(\frac{B-D}{B}\right)\right|$

(C) $\left|\left(\frac{A-C}{A}\right)\right| + \left|\left(\frac{B-D}{B}\right)\right| - \left|\left(\frac{A-C}{A}\right)\right| \times \left|\left(\frac{B-D}{B}\right)\right|$

(D) $\left(\frac{A-C}{A}\right) - \left(\frac{B-D}{B}\right)$

Solution

The correct answer is (C).

$$\text{Rel}\left(C \times D\right) = \frac{A \times B - C \times D}{A \times B}$$

True Error = True Value – Approximate Value
Approximate Value = True Value – True Error

$$C = A - \alpha$$
$$D = B - \beta$$

Where α and β are the true errors in the representation of *A* and *B*, respectively.

$$\text{Rel}\left(CD\right) = \frac{AB - (A-\alpha)(B-\beta)}{AB}$$
$$= \frac{AB - AB + B\alpha + A\beta - \alpha\beta}{AB}$$

AB cancels which yields,

$$\text{Rel }(CD) = \frac{\alpha}{A} + \frac{\beta}{B} - \frac{\alpha}{A}\frac{\beta}{B}$$
$$= \text{Rel}(A) + \text{Rel}(B) + \text{Rel}(A)\text{Rel}(B)$$
$$= \left|\left(\frac{A-C}{A}\right)\right| + \left|\left(\frac{B-D}{B}\right)\right| - \left|\left(\frac{A-C}{A}\right)\right| \times \left|\left(\frac{B-D}{B}\right)\right|$$

4. The formula for normal strain in a longitudinal bar is given by $\in = \frac{F}{AE}$ where

F = normal force applied
A = cross-sectional area of the bar
E = Young's modulus

If $F = 50 \pm 0.5\,\text{N}$, $A = 0.2 \pm 0.002\,\text{m}^2$, and $E = 210\times 10^9 \pm 1\times 10^9\ \text{Pa}$, the maximum error in the measurement of strain is

(A) 10^{-12}
(B) 2.95×10^{-11}
(C) 1.22×10^{-9}
(D) 1.19×10^{-9}

Solution

The correct answer is (B).

The total error for strain is given by

$$|\Delta \in| = \left|\frac{\partial \in}{\partial F}\Delta F\right| + \left|\frac{\partial \in}{\partial A}\Delta A\right| + \left|\frac{\partial \in}{\partial E}\Delta E\right|$$

The partial derivatives are then

$$\frac{\partial \in}{\partial F} = \frac{1}{AE},\quad \frac{\partial \in}{\partial A} = -\frac{F}{A^2E},\quad \frac{\partial \in}{\partial E} = -\frac{F}{AE^2}$$

The total error for strain is then

$$|\Delta \in| = \left|\left(\frac{1}{AE}\right)\Delta F\right| + \left|\left(\frac{-F}{A^2E}\right)\Delta A\right| + \left|\left(\frac{-F}{AE^2}\right)\Delta E\right|$$

$$|\Delta \in| = \left|\left(\frac{1}{(0.2)(210\times 10^9)}\right)(0.5)\right| + \left|\left(\frac{-50}{(0.2)^2(210\times 10^9)}\right)(0.002)\right| + \left|\left(\frac{-50}{(0.2)(210\times 10^9)}\right)(1\times 10^9)\right|$$

$$|\Delta \in| = 1.19\times 10^{-11} + 1.19\times 10^{-11} + 5.67\times 10^{-12} = 2.95\times 10^{-11}$$

5. A wooden block is measured to be 60mm by a ruler and the measurements are considered to be good to 1/4th of a millimeter. Then in the measurement 60mm, we have .________. significant digits

(A) 0
(B) 1
(C) 2
(D) 3

Solution
The correct answer is (C).

We are given the uncertainty is within 1/4th of a millimeter so at least 2 significant digits are accurate.

6. In the calculation of the volume of a cube of nominal size $5''$, the uncertainty in the measurement of each side is 10%. The uncertainty in the measurement of the volume would be

(A) 5.477%
(B) 10.00%
(C) 17.32%
(D) 30.00%

Solution

The correct answer is (D).

For this problem, $V = a^3$ where a is the length of the side of the cube.

$$|\Delta V| = \left|\frac{dV}{da}\Delta a\right|$$

$$= \left|3a^2 \Delta a\right|$$

$$|\Delta V| = \left|3\frac{a^3}{a}\Delta a\right|$$

$$|\Delta V| = \left|3\frac{V}{a}\Delta a\right|$$

$$\left|\frac{\Delta V}{V}\right| = \left|3\frac{\Delta a}{a}\right|$$

Plugging in numbers yields

$$\left|\frac{\Delta V}{V}\right| = 3 \times 0.1$$

$$= 0.3$$

$$= 30\%$$