1. Truncation error is caused by approximating
   (A) irrational numbers
   (B) fractions
   (C) rational numbers
   (D) exact mathematical procedures.

Solution

*The correct answer is (D).*

Truncation error is related to approximating mathematical procedures. Examples include using a finite number of terms of a Taylor series to approximate transcendental and trigonometric functions, the use of a finite number of areas to find the integral of a function, etc.
2. A computer that represents only 4 significant digits with chopping would calculate 66.666\(\times\)33.333 as

(A) 2220  
(B) 2221  
(C) 2221.17778  
(D) 2222

**Solution**

The correct answer is (B).

\[
66.66 \approx 66.66 \\
33.33 \approx 33.33 \\
66.66 \times 33.33 = 2221.7778 \\
\approx 2221
\]
3. A computer that represents only 4 significant digits with rounding would calculate 66.666*33.333 as
   (A) 2220
   (B) 2221
   (C) 2221.17778
   (D) 2222

**Solution**

*The correct answer is (D).*

\[
66.666 \approx 66.67 \\
33.333 \approx 33.33 \\
66.67 \times 33.33 = 2222.1111 \\
\approx 2222
\]
4. The truncation error in calculating \( f'(2) \) for \( f(x) = x^2 \) by \( f'(x) \approx \frac{f(x+h) - f(x)}{h} \) with \( h = 0.2 \) is

(A) -0.2  
(B) 0.2  
(C) 4.0  
(D) 4.2

**Solution**

The correct answer is (A).

The approximate value of \( f'(2) \) is

\[
f'(x) \approx \frac{f(x+h) - f(x)}{h}
\]

\[
x = 2, \ h = 0.2
\]

\[
f'(2) \approx \frac{f(2+0.2) - f(2)}{0.2}
\]

\[
= \frac{f(2.2) - f(2)}{0.2}
\]

\[
= \frac{2.2^2 - 2^2}{0.2}
\]

\[
= \frac{4.84 - 4}{0.2}
\]

\[
= 4.2
\]

The true value of \( f'(2) \) is

\[
f(x) = x^2
\]

\[
f'(x) = 2x
\]

\[
f'(2) = 2 \times 2
\]

\[
= 4
\]

Thus, the true error is

\[
E_t = \text{True Value - Approximate Value}
\]

\[
= 4 - 4.2
\]

\[
= -0.2
\]
5. The truncation error in finding \[ \int_{-3}^{9} x^3 \, dx \] using LRAM (left end point Riemann approximation method) with equally portioned points \(-3 < 0 < 3 < 6 < 9\) is

(A) 648  
(B) 756  
(C) 972  
(D) 1620

**Solution**

*The correct answer is (C).*

**Graph of \(f(x) = x^3\) for LRAM Approximation**

\[
LRAM = f(-3) \times 3 + f(0) \times 3 + f(3) \times 3 + f(6) \times 3 \\
= (-3)^3 \times 3 + (0)^3 \times 3 + (3)^3 \times 3 + (6)^3 \times 3 \\
= -81 + 0 + 81 + 648 \\
= 648
\]

\[
\int_{-3}^{9} x^3 \, dx = \left[ \frac{x^4}{4} \right]_{-3}^{9} \\
= \left[ \frac{9^4 - (-3)^4}{4} \right] \\
= 1620
\]

Truncation Error = True Value – Approximate Value (if there is no round-off error)

\[
= 1620 - 648 \\
= 972
\]
6. The number \( \frac{1}{10} \) is registered in a fixed 6 bit-register with all bits used for the fractional part. The difference is accumulated every \( \frac{1}{10} \)th of a second for one day. The magnitude of the accumulated difference is

(A) 0.082
(B) 135
(C) 270
(D) 5400

Solution

The correct answer is (D).

<table>
<thead>
<tr>
<th>Number</th>
<th>Number after decimal</th>
<th>Number before decimal</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.1×2</td>
<td>0.2</td>
<td>0</td>
</tr>
<tr>
<td>0.2×2</td>
<td>0.4</td>
<td>0</td>
</tr>
<tr>
<td>0.4×2</td>
<td>0.8</td>
<td>0</td>
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<tr>
<td>0.8×2</td>
<td>1.6</td>
<td>1</td>
</tr>
<tr>
<td>0.6×2</td>
<td>1.2</td>
<td>1</td>
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<td>1.2</td>
<td>1</td>
</tr>
</tbody>
</table>

\( (0.1)_{10} \cong (0.000110011)_2 \)

Hence

\( (0.1)_{10} \cong (0.000110)_2 \) in a six bit fixed register.

\( (0.000110)_2 = 0 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} + 1 \times 2^{-5} + 0 \times 2^{-6} \)

\( = 0.09375 \)

The difference (true error) between 0.1 and 0.09375 is

\( = 0.1 - 0.09375 \)

\( = 0.00625 \)

The accumulated difference in a day is then

\( = 0.00625 \times 10 \times 60 \times 60 \times 24 \)

\( = 5400 \)