Problem Set#1
Multiple Choice Test
Chapter 01.03 Sources of Error
COMPLETE SOLUTION SET

1. Truncation error is caused by approximating
   (A) irrational numbers
   (B) fractions
   (C) rational numbers
   (D) exact mathematical procedures.

Solution
The correct answer is (D).

Truncation error is related to approximating mathematical procedures. Examples include using a finite number of terms of a Taylor series to approximate transcendental and trigonometric functions, the use of a finite number of areas to find the integral of a function, etc.

2. A computer that represents only 4 significant digits with chopping would calculate 66.666*33.333 as

(A) 2220
(B) 2221
(C) 2221.17778
(D) 2222

Solution

The correct answer is (B).

$$66.666 \approx 66.66$$
$$33.333 \approx 33.33$$

$$\begin{aligned} 66.66 \times 33.33 &= 2221.7778 \\ &\approx 2221 \end{aligned}$$

3. A computer that represents only 4 significant digits with rounding would calculate 66.666*33.333 as

(A) 2220
(B) 2221
(C) 2221.17778
(D) 2222

Solution

The correct answer is (D).

$$66.666 \approx 66.67$$
$$33.333 \approx 33.33$$

$$\begin{aligned} 66.67 \times 33.33 &= 2222.1111 \\ &\approx 2222 \end{aligned}$$

4. The truncation error in calculating $f'(2)$ for $f(x) = x^2$ by $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$ with $h = 0.2$ is

(A) -0.2
(B) 0.2
(C) 4.0
(D) 4.2

Solution

The correct answer is (A).

The approximate value of $f'(2)$ is

$$f'(x) \approx \frac{f(x+h) - f(x)}{h}$$

$$x = 2,\ \ h = 0.2$$

$$\begin{aligned} f'(2) &\approx \frac{f(2+0.2) - f(2)}{0.2} \\ &= \frac{f(2.2) - f(2)}{0.2} \\ &= \frac{2.2^2 - 2^2}{0.2} \\ &= 4.2 \end{aligned}$$

The true value of $f'(2)$ is

$$\begin{aligned} f(x) &= x^2 \\ f'(x) &= 2x \\ f'(2) &= 2 \times 2 \\ &= 4 \end{aligned}$$

Thus, the true error is

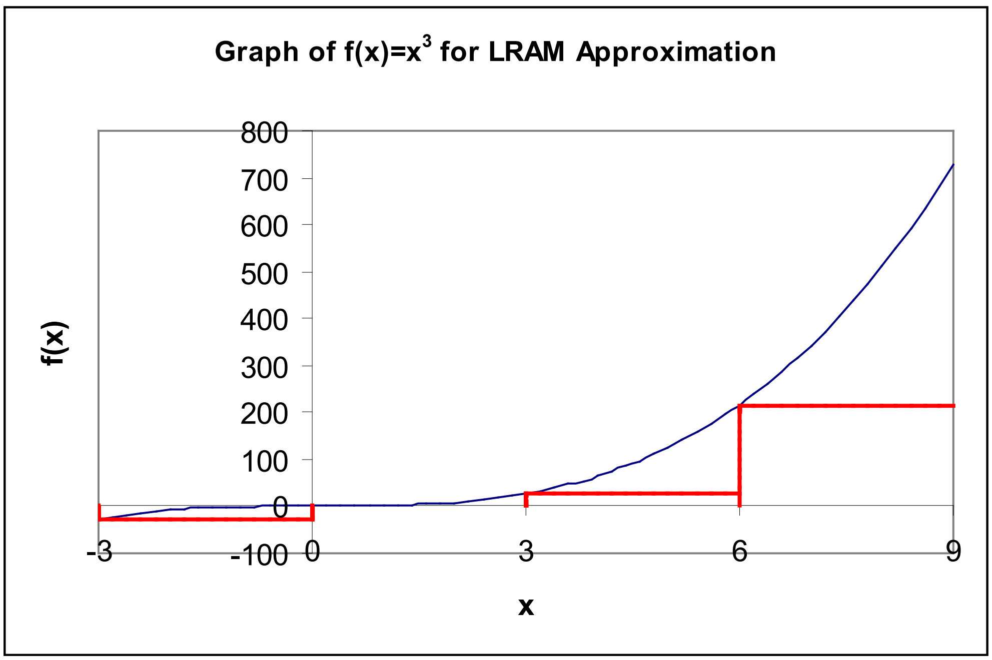
$$\begin{aligned} E_t &= \text{True Value - Approximate Value} \\ &= 4 - 4.2 \\ &= -0.2 \end{aligned}$$

5. The truncation error in finding $\int_{-3}^{9} x^3 dx$ using LRAM (left end point Riemann approximation method) with equally portioned points $-3 < 0 < 3 < 6 < 9$ is

(A) 648
(B) 756
(C) 972
(D) 1620

Solution

The correct answer is (C).



$$LRAM = f(-3)\times 3 + f(0)\times 3 + f(3)\times 3 + f(6)\times 3$$
$$= (-3)^3 \times 3 + (0)^3 \times 3 + (3)^3 \times 3 + (6)^3 \times 3$$
$$= -81 + 0 + 81 + 648$$
$$= 648$$

$$\int_{-3}^{9} x^3 = \left[\frac{x^4}{4}\right]_{-3}^{9}$$
$$= \left[\frac{9^4 - (-3)^4}{4}\right]$$
$$= 1620$$

Truncation Error = True Value – Approximate Value (if there is no round-off error)

$$= 1620 - 648$$
$$= 972$$

6. The number $\frac{1}{10}$ is registered in a fixed 6 bit-register with all bits used for the fractional part. The difference is accumulated every $\frac{1}{10}$ th of a second for one day. The magnitude of the accumulated difference is

(A) 0.082
(B) 135
(C) 270
(D) 5400

Solution

The correct answer is (D).

	Number	Number after decimal	Number before decimal
0.1×2	0.2	0.2	0
0.2×2	0.4	0.4	0
0.4×2	0.8	0.8	0
0.8×2	1.6	0.6	1
0.6×2	1.2	0.2	1
0.2×2	0.4	0.4	0
0.4×2	0.8	0.8	0
0.8×2	1.6	0.6	1
0.6×2	1.2	0.2	1

$$(0.1)_{10} \cong (0.000110011)_2$$

Hence

$(0.1)_{10} \cong (0.000110)_2$ in a six bit fixed register.

$$\begin{aligned}(0.000110)_2 &= 0 \times 2^{-1} + 0 \times 2^{-2} + 0 \times 2^{-3} + 1 \times 2^{-4} + 1 \times 2^{-5} + 0 \times 2^{-6} \\ &= 0.09375\end{aligned}$$

The difference (true error) between 0.1 and 0.09375 is

$$\begin{aligned}&= 0.1 - 0.09375 \\ &= 0.00625\end{aligned}$$

The accumulated difference in a day is then

$$\begin{aligned}&= 0.00625 \times 10 \times 60 \times 60 \times 24 \\ &= 5400\end{aligned}$$