Multiple-Choice Test

Chapter 01.07
Taylors Series Revisited

1. The coefficient of the x^5 term in the Maclaurin polynomial for $\sin(2x)$ is
   (A) 0
   (B) 0.0083333
   (C) 0.016667
   (D) 0.26667

2. Given $f(3)=6$, $f'(3)=8$, $f''(3)=11$, and that all other higher order derivatives of $f(x)$ are zero at $x=3$, and assuming the function and all its derivatives exist and are continuous between $x=3$ and $x=7$, the value of $f(7)$ is
   (A) 38.000
   (B) 79.500
   (C) 126.00
   (D) 331.50

3. Given that $y(x)$ is the solution to $\frac{dy}{dx}=y^3+2$, $y(0)=3$ the value of $y(0.2)$ from a second order Taylor polynomial written around $x=0$ is
   (A) 4.400
   (B) 8.800
   (C) 24.46
   (D) 29.00

4. The series $\sum_{n=0}^{\infty}(-1)^n \frac{x^{2n}}{(2n)!}4^n$ is a Maclaurin series for the following function
   (A) $\cos(x)$
   (B) $\cos(2x)$
   (C) $\sin(x)$
   (D) $\sin(2x)$

5. The function

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

is called the error function. It is used in the field of probability and cannot be calculated exactly for finite values of x. However, one can expand the integrand as a Taylor polynomial and conduct integration. The approximate value of $erf(2.0)$ using the first three terms of the Taylor series around $t = 0$ is

(A) -0.75225
(B) 0.99532
(C) 1.5330
(D) 2.8586

6. Using the remainder of Maclaurin polynomial of n^{th} order for $f(x)$ defined as

$$R_n(x) = \frac{x^{n+1}}{(n+1)!} f^{(n+1)}(c), \quad n \geq 0, \ 0 \leq c \leq x$$

the least order of the Maclaurin polynomial required to get an absolute true error of at most 10^{-6} in the calculation of $\sin(0.1)$ is (do not use the exact value of $\sin(0.1)$ or $\cos(0.1)$ to find the answer, but the knowledge that $|\sin(x)| \leq 1$ and $|\cos(x)| \leq 1$).

(A) 3
(B) 5
(C) 7
(D) 9

<u>For a complete solution, refer to the links at the end of the book.</u>