The definition of the first derivative of a function $f(x)$ is

(A) $f'(x) = \frac{f(x + \Delta x) + f(x)}{\Delta x}$

(B) $f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x}$

(C) $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) + f(x)}{\Delta x}$

(D) $f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$

Solution

The correct answer is (D).

The definition of the first derivative of the function $f(x)$ is

$$f''(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Choice (B) is incorrect as it is an approximate method to calculate the first derivative of a function $f(x)$. In fact, choice (B) is the forward divided difference method of approximately calculating the first derivative of a function.
2. Given \( y = 5e^{3x} + \sin x \), \( \frac{dy}{dx} \) is

(A) \( 5e^{3x} + \cos x \)

(B) \( 15e^{3x} + \cos x \)

(C) \( 15e^{3x} - \cos x \)

(D) \( 2.666e^{3x} - \cos x \)

Solution

The correct answer is (B)

Use the sum rule of differentiation

\[
\frac{d}{dx} (u + v) = \frac{du}{dx} + \frac{dv}{dx}
\]

Re-write the function as

\[ y = u + v \]

where

\[ u = 5e^{3x} \]

\[ v = \sin x \]

Find \( \frac{du}{dx} \) and \( \frac{dv}{dx} \)

\[
\frac{d}{dx} (5e^{3x}) = 15e^{3x}
\]

\[
\frac{d}{dx} (\sin x) = \cos x
\]

\[
\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}
\]

\[
= 15e^{3x} + \cos x
\]
3. Given $y = \sin 2x$, $\frac{dy}{dx}$ at $x = 3$ is most nearly

(A) 0.9600  
(B) 0.9945  
(C) 1.920  
(D) 1.989

**Solution**

The correct answer is (C).

Using the chain rule

Let $u = 2x$  
$y = \sin u$

$\frac{du}{dx} = 2$  
$\frac{dy}{du} = \cos u$

$\frac{dy}{dx} = \cos 2x$

Since,

$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$  
$\frac{dy}{dx} = (\cos 2x) \times 2$

At $x = 3$

$\frac{dy}{dx} = (\cos 6) \times 2$  
(Don’t forget to use radians)

$= 0.9602 \times 2$

$= 1.920$
4. Given \( y = x^3 \ln x \), \( \frac{dy}{dx} \) is

(A) \( 3x^2 \ln x \)
(B) \( 3x^2 \ln x + x^2 \)
(C) \( x^2 \)
(D) \( 3x \)

Solution
The correct answer is (B).

Using the product rule,

\[
\frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx}
\]

(1)

\( u = x^3 \)
\( v = \ln x \)

\( \frac{du}{dx} = 3x^2 \)
\( \frac{dv}{dx} = \frac{1}{x} \)

Substituting into Equation (1),

\[
\frac{dy}{dx} = x^3 \times \frac{1}{x} + \ln x \times 3x^2
\]

\[
= x^2 + 3x^2 \ln x
\]
5. The velocity of a body as a function of time is given as \( v(t) = 5e^{-2t} + 4 \), where \( t \) is in seconds, and \( v \) is in m/s. The acceleration in m/s\(^2\) at \( t = 0.6 \) s is

(A) -3.012
(B) 5.506
(C) 4.147
(D) -10.00

**Solution**

*The correct answer is (A)*

\[
a(t) = \frac{dv}{dt} = \frac{d}{dt} \left(5e^{-2t} + 4\right) = -10e^{-2t}
\]

\[
a(0.6) = -10e^{-2(0.6)} = -3.012 \text{ m/s}^2
\]
6. If \( x^2 + 2xy = y^2 \), then \( \frac{dy}{dx} \) is

(A) \( \frac{x + y}{y - x} \)
(B) \( 2x + 2y \)
(C) \( \frac{x + 1}{y} \)
(D) \( -x \)

Solution

The correct answer is (A).

\[
\begin{align*}
\frac{d}{dx}[x^2] + \frac{d}{dx}[2xy] &= \frac{d}{dx}[y^2] \\
2x + 2x \frac{dy}{dx} + 2y &= 2y \frac{dy}{dx} \\
\quad &\quad \left( \frac{d}{dx}(uv) = u \frac{dv}{dx} + v \frac{du}{dx} \right) \\
x + y &= (y - x) \frac{dy}{dx} \\
\frac{dy}{dx} &= \frac{x + y}{y - x}
\end{align*}
\]