Multiple-Choice Test

Chapter 02.03
Differentiation of Discrete Functions

1. The definition of the first derivative of a function \( f(x) \) is
   \[
   (A) \quad f'(x) = \frac{f(x + \Delta x) + f(x)}{\Delta x} \\
   (B) \quad f'(x) = \frac{f(x + \Delta x) - f(x)}{\Delta x} \\
   (C) \quad f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) + f(x)}{\Delta x} \\
   (D) \quad f'(x) = \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}
   \]

2. Using the forward divided difference approximation with a step size of 0.2, the derivative of the function at \( x = 2 \) is given as
   \[
   \begin{array}{|c|c|c|c|c|c|}
   \hline
   x & 1.8 & 2.0 & 2.2 & 2.4 & 2.6 \\
   \hline
   \end{array}
   
   (A) 6.697 \\
   (B) 7.389 \\
   (C) 7.438 \\
   (D) 8.180
   
3. A student finds the numerical value of \( f'(x) = 20.220 \) at \( x = 3 \) using a step size of 0.2. Which of the following methods did the student use to conduct the differentiation if \( f(x) \) is given in the table below?
   \[
   \begin{array}{|c|c|c|c|c|c|}
   \hline
   x & 2.6 & 2.8 & 3.0 & 3.2 & 3.4 & 3.6 \\
   f(x) & e^{2.6} & e^{2.8} & e^{3.0} & e^{3.2} & e^{3.4} & e^{3.6} \\
   \hline
   \end{array}
   
   (A) Backward divided difference \\
   (B) Calculus, that is, exact \\
   (C) Central divided difference \\
   (D) Forward divided difference
4. The upward velocity of a body is given as a function of time as

<table>
<thead>
<tr>
<th>t, s</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>22</th>
</tr>
</thead>
<tbody>
<tr>
<td>v, m/s</td>
<td>22</td>
<td>36</td>
<td>57</td>
<td>10</td>
</tr>
</tbody>
</table>

To find the acceleration at \( t = 17 \) s, a scientist finds a second order polynomial approximation for the velocity, and then differentiates it to find the acceleration. The estimate of the acceleration in \( \text{m/s}^2 \) at \( t = 17 \) s is most nearly

(A) 4.060  
(B) 4.200  
(C) 8.157  
(D) 8.498

5. The velocity of a rocket is given as a function of time as

<table>
<thead>
<tr>
<th>t, s</th>
<th>0</th>
<th>0.5</th>
<th>1.2</th>
<th>1.5</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>v, m/s</td>
<td>0</td>
<td>213</td>
<td>223</td>
<td>275</td>
<td>300</td>
</tr>
</tbody>
</table>

Allowed to use the forward divided difference, backward divided difference or central divided difference approximation of the first derivative, your best estimate for the acceleration \( a = \frac{dv}{dt} \) of the rocket in \( \text{m/s}^2 \) at \( t = 1.5 \) seconds is

(A) 83.33  
(B) 128.33  
(C) 173.33  
(D) 183.33

6. In a circuit with an inductor of inductance \( L \), a resistor with resistance \( R \), and a variable voltage source \( E(t) \),

\[
E(t) = L \frac{di}{dt} + Ri
\]

The current, \( i \), is measured at several values of time as

<table>
<thead>
<tr>
<th>Time, ( t ) (secs)</th>
<th>1.00</th>
<th>1.01</th>
<th>1.03</th>
<th>1.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Current, ( i ) (amperes)</td>
<td>3.10</td>
<td>3.12</td>
<td>3.18</td>
<td>3.24</td>
</tr>
</tbody>
</table>

If \( L = 0.98 \) henries and \( R = 0.142 \) ohms, the most accurate expression for \( E(1.00) \) is

(A) \( 0.98 \left( \frac{3.24 - 3.10}{0.1} \right) + (0.142)(3.10) \)  
(B) \( 0.142 \times 3.10 \)  
(C) \( 0.98 \left( \frac{3.12 - 3.10}{0.01} \right) + (0.142)(3.10) \)  
(D) \( 0.98 \left( \frac{3.12 - 3.10}{0.01} \right) \)

For a complete solution, refer to the links at the end of the book.