Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Differentiation of Discrete Functions
DIfferentiation
COMPLETE SOLUTION SET

1. The definition of the first derivative of a function $f(x)$ is

(A) $f'(x)=\dfrac{f(x+\Delta x)+f(x)}{\Delta x}$

(B) $f'(x)=\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$

(C) $f'(x)=\lim\limits_{\Delta x\to 0}\dfrac{f(x+\Delta x)+f(x)}{\Delta x}$

(D) $f'(x)=\lim\limits_{\Delta x\to 0}\dfrac{f(x+\Delta x)-f(x)}{\Delta x}$

Solution

The correct answer is (D).

$$f'(x)=\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

Choice (B) is incorrect as it is an approximate method to calculate the first derivative of a function $f(x)$. In fact, choice (B) is the forward divided difference method of approximately calculating the first derivative of a function.

2. Using the forward divided difference approximation with a step size of 0.2, the derivative of the function at $x = 2$ is given as

x	1.8	2.0	2.2	2.4	2.6
$f(x)$	6.0496	7.3890	9.0250	11.023	13.464

(A) 6.697
(B) 7.389
(C) 7.438
(D) 8.180

Solution
The correct answer is (D).

The forward divided difference approximation is

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_i)}{\Delta x}$$

where

$$x_i = 2.0$$
$$x_{i+1} = 2.2$$
$$\Delta x = 0.2$$

Thus,

$$\begin{aligned} f'(2.0) &= \frac{f(2.2) - f(2.0)}{0.2} \\ &= \frac{9.0250 - 7.3890}{0.2} \\ &= 8.180 \end{aligned}$$

3. A student finds the numerical value of $f'(x) = 20.220$ at $x = 3$ using a step size of 0.2. Which of the following methods did the student use to conduct the differentiation if $f(x)$ is given in the table below?

x	2.6	2.8	3.0	3.2	3.4	3.6
$f(x)$	$e^{2.6}$	$e^{2.8}$	e^{3}	$e^{3.2}$	$e^{3.4}$	$e^{3.6}$

(A) Backward divided difference
(B) Calculus, that is, exact
(C) Central divided difference
(D) Forward divided difference

Solution
The correct answer is (C).

Choice (A)
The backward divided difference approximation is

$$f'(x_i) \approx \frac{f(x_i) - f(x_{i-1})}{\Delta x}$$

where

$$x_i = 3$$
$$x_{i-1} = 2.8$$
$$\Delta x = 0.2$$

Thus,

$$\begin{aligned} f'(3) &\approx \frac{f(3) - f(2.8)}{(0.2)} \\ &= \frac{f(3) - f(2.8)}{(0.2)} \\ &= \frac{e^3 - e^{2.8}}{0.2} \\ &= \frac{20.086 - 16.445}{0.2} \\ &= 18.205 \end{aligned}$$

Choice (B)
Differential calculus cannot be used as the function is not a continuous function.

Choice (C)
The central divided difference approximation is

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_{i-1})}{2\Delta x}$$

where

$$x_i = 3$$

$$x_{i+1} = 3.2$$

$$x_{i-1} = 2.8$$

$$\Delta x = 0.2$$

Thus,

$$f'(3) = \frac{f(3.2) - f(2.8)}{2(0.2)}$$

$$= \frac{e^{3.2} - e^{2.8}}{0.4}$$

$$= \frac{24.533 - 16.445}{0.4}$$

$$= 20.219$$

Choice (D)

The forward divided difference approximation is

$$f'(x_i) \approx \frac{f(x_{i+1}) - f(x_i)}{\Delta x}$$

where

$$x_i = 3$$

$$x_{i+1} = 3.2$$

$$\Delta x = 0.2$$

Thus,

$$f'(3) = \frac{f(3.2) - f(3)}{(0.2)}$$

$$= \frac{e^{3.2} - e^{3}}{0.2}$$

$$= \frac{24.533 - 20.086}{0.2}$$

$$= 22.235$$

4. The upward velocity of a body is given as a function of time as

t, s	10	15	20	22
v, m/s	22	36	57	10

To find the acceleration at $t=17\text{ s}$, a scientist finds a second order polynomial approximation for the velocity, and then differentiates it to find the acceleration. The estimate of the acceleration in m/s^2 at $t=17\text{ s}$ is most nearly

(A) 4.060
(B) 4.200
(C) 8.157
(D) 8.498

Solution

The correct answer is (C).

Find a 2nd order polynomial

$$v(t)=a_0+a_1t+a_2t^2$$

at $t=17\text{ s}$

First choose the three points closest to $t=17\text{ s}$ that also bracket it.

$$t_0=15\text{ s}, v(t_0)=36\text{ m/s}$$

$$t_1=20\text{ s}, v(t_1)=57\text{ m/s}$$

$$t_2=22\text{ s}, v(t_2)=10\text{ m/s}$$

Such that

$$v(15)=36=a_0+a_1(15)+a_2(15)^2$$

$$v(20)=57=a_0+a_1(20)+a_2(20)^2$$

$$v(22)=10=a_0+a_1(22)+a_2(22)^2$$

Rewrite in matrix form as

$$\begin{bmatrix}1 & 15 & 225\\ 1 & 20 & 400\\ 1 & 22 & 484\end{bmatrix}\begin{bmatrix}a_0\\ a_1\\ a_2\end{bmatrix}=\begin{bmatrix}36\\ 57\\ 10\end{bmatrix}$$

Solving the above set of 3 simultaneous linear equations gives

$$a_0=-1214.1429$$

$$a_1=142.7$$

$$a_2=-3.9571$$

Thus,

$$v(t)=-1214.14+142.7(t)-3.9571(t)^2, 15\le t\le 22$$

$$a(t)=\frac{d}{dt}(-1214.14)+\frac{d}{dt}(142.7(t))+\frac{d}{dt}(-3.9571(t)^2)$$

$$
\begin{aligned}
&= 142.7 - 7.9143(t), 15 \le t \le 22 \\
a(17) &= 142.7 - 7.9143(17) \\
&= 8.1571 \text{ m/s}^2
\end{aligned}
$$

5. The velocity of a rocket is given as a function of time as

t, s	0	0.5	1.2	1.5	1.8
v, m/s	0	213	223	275	300

Allowed to use the forward divided difference, backward divided difference or central divided difference approximation of the first derivative, the best estimate for the acceleration $\left(a = \frac{dv}{dt}\right)$ in $\mathrm{m/s^2}$ of the rocket at $t = 1.5$ seconds is

(A) 83.33

(B) 128.33

(C) 173.33

(D) 183.33

Solution

The correct answer is (B).

Using the central divided difference will give the best estimate since the error is of the order of $0(\Delta x)^2$, as opposed to forward divided difference and backward divided difference approximation where the error is of the order of $0(\Delta x)$.

$$a(t) = \frac{v(t_{i+1}) - v(t_{i-1})}{2(\Delta x)}$$

where

$t_i = 1.5\,\mathrm{s}$

$t_{i+1} = 1.8\,\mathrm{s}$

$t_{i-1} = 1.2\,\mathrm{s}$

$\Delta x = 0.3\,\mathrm{s}$

Thus,

$$\begin{aligned} a(1.5) &= \frac{v(1.8) - v(1.2)}{2(0.3)} \\ &= \frac{300 - 223}{0.6} \\ &= 128.33\,\mathrm{m/s^2} \end{aligned}$$

6. In a circuit with an inductor of inductance L, a resistor with resistance R, and a variable voltage source $E(t)$,

$$E(t) = L\frac{di}{dt} + Ri$$

The current, i, is measured at several values of time as

Time, t (secs)	1.00	1.01	1.03	1.1
Current, i (amperes)	3.10	3.12	3.18	3.24

If $L = 0.98$ henries and $R = 0.142$ ohms, the most accurate expression for $E(1.00)$ is

(A) $0.98\left(\frac{3.24-3.10}{0.1}\right)+(0.142)(3.10)$

(B) 0.142×3.10

(C) $0.98\left(\frac{3.12-3.10}{0.01}\right)+(0.142)(3.10)$

(D) $0.98\left(\frac{3.12-3.10}{0.01}\right)$

Solution

The correct answer is (C).

The central difference and backward difference approximation cannot be used because there are no values for t_{i-1}. Hence, the forward divided difference will give the best approximation.

$$\frac{di}{dt} \approx \frac{i(t_{i+1}) - i(t_i)}{t_{i+1} - t_i}$$

where

$$t_i = 1.00$$

$$t_{i+1} = 1.01$$

$$\frac{di}{dt} \approx \frac{3.12-3.10}{0.01}$$

$$E(t) = L\frac{di}{dt} + Ri$$

$$E(1) \approx 0.98\left(\frac{3.12-3.10}{0.01}\right)+(0.142)(3.10)$$