Multiple-Choice Test
Background
Nonlinear Equations
COMPLETE SOLUTION SET

1. The value of $x$ that satisfies $f(x) = 0$ is called the
   (A) root of an equation $f(x) = 0$
   (B) root of a function $f(x)$
   (C) zero of an equation $f(x) = 0$
   (D) none of the above

Solution
*The correct answer is (A).*

Only an equation has roots, while functions have zeros. A root of an equation $f(x) = 0$ is defined as the point where $f(x)$ is zero.
2. A quadratic equation has ______ root(s).
   
   (A) one  
   (B) two  
   (C) three  
   (D) four

**Solution**

*The correct answer is (B).*

The quadratic equation

\[ ax^2 + bx + c = 0 \]

has two roots

\[ x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \]
\[ x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \]

These two roots may be real or complex. If \( b^2 - 4ac = 0 \), the roots are repeated.
3. For a certain cubic equation, at least one of the roots is known to be a complex root. How many total complex roots does the cubic equation have?

   (A) one
   (B) two
   (C) three
   (D) cannot be determined

**Solution**

*The correct answer is (B).*

The equation would have two complex roots. There are two possible cases for the roots of a cubic equation:

1. Three real roots
2. Two complex roots and one real root

Complex roots always come in conjugate pairs \((a+ib,a-ib)\).
4. An equation such as $\tan x = x$ has _____ root(s).

(A) zero  
(B) one  
(C) two  
(D) infinite

**Solution**

*The correct answer is (D).*

The equation,  

$$\tan x = x$$

has infinite roots.

For example, the roots of  

$$\tan x = x$$

include  

$$x_1 = 0$$  
$$x_2 = 4.49341$$  
$$x_3 = 7.72525$$  
$$x_4 = 54.9597$$
5. A polynomial of order $n$ has _________ zeros.

(A) $n - 1$
(B) $n$
(C) $n + 1$
(D) $n + 2$

Solution
The correct answer is (B).

A polynomial of order $n$ has $n$ zeros.
For example a second order polynomial
\[ ax^2 + bx + c \]
has two zeros
\[
\begin{align*}
x_1 &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \\
x_2 &= \frac{-b - \sqrt{b^2 - 4ac}}{2a}
\end{align*}
\]
Similarly a third order polynomial has three zeros.
6. The velocity of a body is given by \( v(t) = 5e^{-t} + 4 \), where \( t \) is in seconds and \( v \) is in m/s. The velocity of the body is 6 m/s at \( t = \) ____________ seconds.

(A) 0.1823  
(B) 0.3979  
(C) 0.9163  
(D) 1.609  

Solution  
*The correct answer is (C).*

\[ v(t) = 5e^{-t} + 4 \]

where  
\[ v(t) = 6 \text{ m/s} \]

Thus,  
\[ 5e^{-t} + 4 = 6 \]
\[ 5e^{-t} = 6 - 4 = 2 \]
\[ e^{-t} = \frac{2}{5} \]

If we take the natural log of both sides  
\[ \ln(e^{-t}) = \ln\left(\frac{2}{5}\right) \]
\[ -t = -0.9162 \]
\[ t = 0.9163 \text{ s} \]

Or if we take the \( \log_{10} \) of both sides  
\[ \log_{10}(e^{-t}) = \log_{10}\left(\frac{2}{5}\right) \]
\[ -t \times \log_{10}(e) = -0.3979 \]
\[ t = \frac{-0.3979}{-0.4343} \]
\[ t = 0.9163 \text{ s} \]