

Multiple-Choice Test
Background
Nonlinear Equations
COMPLETE SOLUTION SET

1. The value of x that satisfies $f(x) = 0$ is called the
 - (A) root of an equation $f(x) = 0$
 - (B) root of a function $f(x)$
 - (C) zero of an equation $f(x) = 0$
 - (D) none of the above

Solution

The correct answer is (A).

Only an equation has roots, while functions have zeros. A root of an equation $f(x) = 0$ is defined as the point where $f(x)$ is zero.

2. A quadratic equation has _____ root(s).

- (A) one
- (B) two
- (C) three
- (D) four

Solution

The correct answer is (B).

The quadratic equation

$$ax^2 + bx + c = 0$$

has two roots

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

These two roots may be real or complex. If $b^2 - 4ac = 0$, the roots are repeated.

3. For a certain cubic equation, at least one of the roots is known to be a complex root. How many total complex roots does the cubic equation have?

- (A) one
- (B) two
- (C) three
- (D) cannot be determined

Solution

The correct answer is (B).

The equation would have two complex roots. There are two possible cases for the roots of a cubic equation:

1. Three real roots
2. Two complex roots and one real root

Complex roots always come in conjugate pairs $(a+ib, a-ib)$.

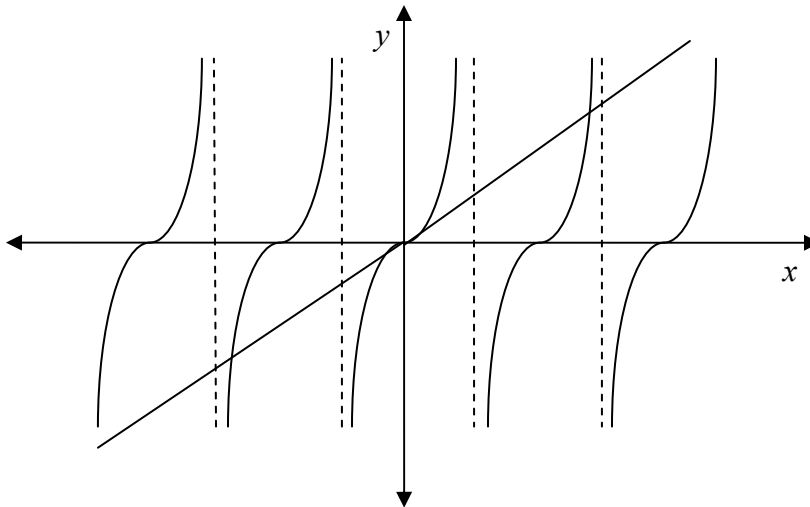
4. An equation such as $\tan x = x$ has _____ root(s).

- (A) zero
- (B) one
- (C) two
- (D) infinite

Solution

The correct answer is (D).

The equation,
 $\tan x = x$
has infinite roots.



For example, the roots of

$$\tan x = x$$

include

$$x_1 = 0$$

$$x_2 = 4.49341$$

$$x_3 = 7.72525$$

$$x_4 = 54.9597$$

5. A polynomial of order n has _____ zeros.

- (A) $n - 1$
- (B) n
- (C) $n + 1$
- (D) $n + 2$

Solution

The correct answer is (B).

A polynomial of order n has n zeros.

For example a second order polynomial

$$ax^2 + bx + c$$

has two zeros

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Similarly a third order polynomial has three zeros.

6. The velocity of a body is given by $v(t) = 5e^{-t} + 4$, where t is in seconds and v is in m/s. The velocity of the body is 6 m/s at $t =$ _____ seconds.

- (A) 0.1823
- (B) 0.3979
- (C) 0.9163
- (D) 1.609

Solution

The correct answer is (C).

$$v(t) = 5e^{-t} + 4$$

where

$$v(t) = 6 \text{ m/s}$$

Thus,

$$5e^{-t} + 4 = 6$$

$$5e^{-t} = 6 - 4 = 2$$

$$e^{-t} = \frac{2}{5}$$

If we take the natural log of both sides

$$\ln(e^{-t}) = \ln\left(\frac{2}{5}\right)$$

$$-t = -0.9162$$

$$t = 0.9163 \text{ s}$$

Or if we take the \log_{10} of both sides

$$\log_{10}(e^{-t}) = \log_{10}\left(\frac{2}{5}\right)$$

$$-t \times \log_{10}(e) = -0.3979$$

$$t = \frac{-0.3979}{-0.4343}$$

$$t = 0.9163 \text{ s}$$