Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Secant Method
Nonlinear Equations
COMPLETE SOLUTION SET

1. The secant method of finding roots of nonlinear equations falls under the category of _____________ methods.
   (A) bracketing
   (B) graphical
   (C) open
   (D) random

Solution
The correct answer is (C).

The secant method of finding roots of nonlinear equations falls under the category of open methods. The secant method uses two initial guesses of the root but unlike the bisection method, they do not have to bracket the root.

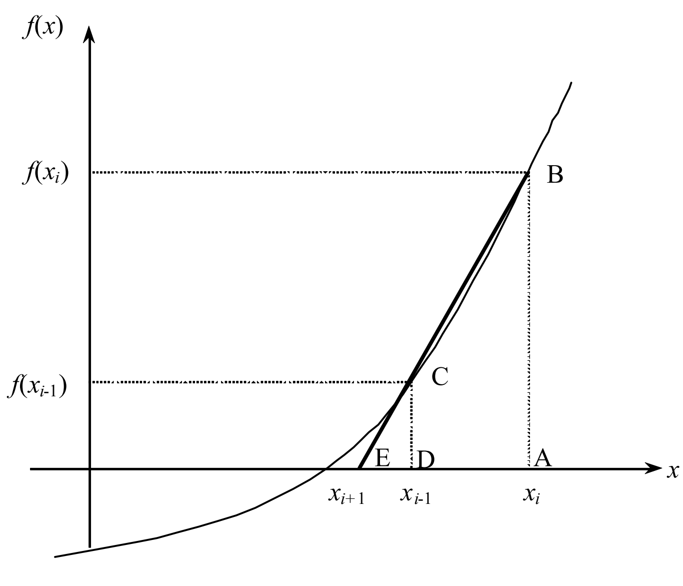


Figure 1 Geometrical representation of the secant method.

2. The secant method formula for finding the square root of a real number R from the equation $x^2 - R = 0$ is

(A) $\dfrac{x_i x_{i-1} + R}{x_i + x_{i-1}}$

(B) $\dfrac{x_i x_{i-1}}{x_i + x_{i-1}}$

(C) $\dfrac{1}{2}\left(x_i + \dfrac{R}{x_i}\right)$

(D) $\dfrac{2x_i^2 + x_i x_{i-1} - R}{x_i + x_{i-1}}$

Solution
The correct solution is (A).

The secant method formula for finding the root of $f(x) = 0$ is

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

where

$$f(x) = x^2 - R$$

Thus

$$\begin{aligned}
x_{i+1} &= x_i - \frac{(x_i^2 - R)(x_i - x_{i-1})}{(x_i^2 - R) - (x_{i-1}^2 - R)} \\
&= x_i - \frac{(x_i^2 - R)(x_i - x_{i-1})}{x_i^2 - x_{i-1}^2} \\
&= x_i - \frac{(x_i^2 - R)(x_i - x_{i-1})}{(x_i - x_{i-1})(x_i + x_{i-1})} \\
&= x_i - \frac{x_i^2 - R}{x_i + x_{i-1}} \\
&= \frac{x_i\left(x_i + x_{i-1}\right) - \left(x_i^2 - R\right)}{x_i + x_{i-1}} \\
&= \frac{x_i^2 + x_i x_{i-1} - x_i^2 + R}{x_i + x_{i-1}} \\
&= \frac{x_i x_{i-1} + R}{x_i + x_{i-1}}
\end{aligned}$$

3. The next iterative value of the root of $x^2 - 4 = 0$ using secant method, if the initial guesses are 3 and 4, is

(A) 2.2857
(B) 2.5000
(C) 5.5000
(D) 5.7143

Solution
The correct solution is (A).

The first iteration is

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

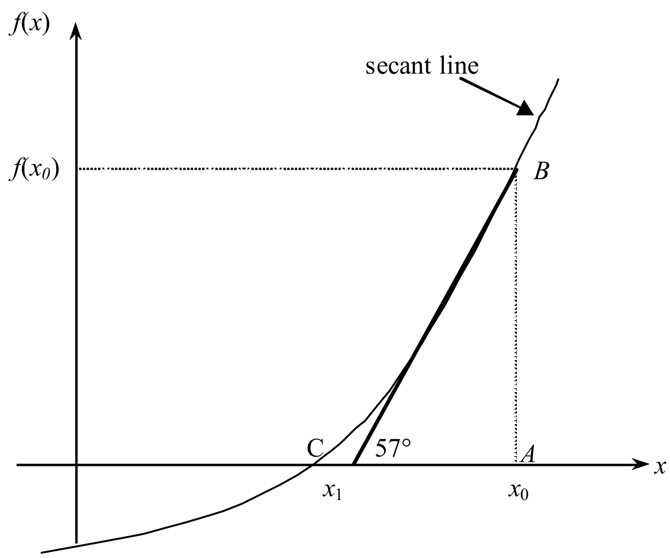
for $i = 0,\ x_0 = 4,\ x_{-1} = 3$

$$\begin{aligned}
x_1 &= x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})} \\
&= x_0 - \frac{(x_0^2 - 4)(x_0 - x_{-1})}{(x_0^2 - 4) - (x_{-1}^2 - 4)} \\
&= 4 - \frac{(4^2 - 4)(4 - 3)}{(4^2 - 4) - (3^2 - 4)} \\
&= 4 - \frac{(12)(1)}{(12) - (5)} \\
&= 4 - \frac{12}{7} \\
&= 2.2857
\end{aligned}$$

4. The root of the equation $f(x)=0$ is found by using the secant method. Given one of the initial estimates is $x_0 = 3$, $f(3)=5$, and the angle the secant line makes with the x-axis is $57°$, the next estimate of the root, x_1, is

(A) –3.2470
(B) –0.24704
(C) 3.247
(D) 6.2470

Solution
The correct answer is (B).



$$\tan(\theta) = \frac{f(x_0)}{x_0 - x_1}$$

$$\tan(57) = \frac{f(3)}{3 - x_1}$$

$$3 - x_1 = \frac{f(3)}{\tan(57)}$$

$$\begin{aligned} x_1 &= -\frac{5}{1.5399} + 3 \\ &= -3.2470 + 3 \\ &= -0.24704 \end{aligned}$$

5. For finding the root of $\sin x = 0$ by the secant method, the following choice of initial guesses would not be appropriate.

(A) $\frac{\pi}{4}$ and $\frac{\pi}{2}$

(B) $\frac{\pi}{4}$ and $\frac{3\pi}{4}$

(C) $-\frac{\pi}{2}$ and $\frac{\pi}{2}$

(D) $\frac{\pi}{3}$ and $\frac{\pi}{2}$

Solution

The correct answer is (B).

$\frac{\pi}{4}$ and $\frac{3\pi}{4}$ are not good initial guesses because $\sin\frac{\pi}{4} = \sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}}$. This would result in division by zero in the formula

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

6. When drugs are given orally to a patient, the drug concentration c in the blood stream at time t is given by a formula

$$c = Kte^{-at}$$

where K is dependent on parameters such as the dose administered while a is dependent on the absorption and elimination rates of the drug. If $K = 2$ and $a = 0.25$, and t is in seconds and c is in mg/ml, the time at which the maximum concentration is reached is given by the solution of the equation,

(A) $2te^{-0.25t} = 0$

(B) $2e^{-0.25t} - 2te^{-0.25t} = 0$

(C) $2e^{-0.25t} - 0.5te^{-0.25t} = 0$

(D) $2te^{-0.25t} = 2$

Solution

The correct answer is (C).

$$c = Kte^{-at}$$

Given

$$K = 2,\ a = 0.25$$

$$c = 2te^{-0.25t}$$

To find the time of maximum concentration, we set

$$\frac{dc}{dt} = 0$$

$$\frac{d}{dt}\left(2te^{-0.25t}\right) = 0$$

$$2e^{-0.25t} - 0.5te^{-0.25t} = 0$$

Note: The solution of the above equation will only give a local maximum or minimum. We need to do the second derivative test after finding the root of the equation.