Multiple-Choice Test
Background
Simultaneous Linear Equations
COMPLETE SOLUTION SET

1. Given \[ A = \begin{bmatrix} 6 & 2 & 3 & 9 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 6 \end{bmatrix} \] then \( A \) is a (an) ______________ matrix.

(A) diagonal  
(B) identity  
(C) lower triangular  
(D) upper triangular

Solution
The correct answer is (D).

A square matrix \( A \) is an upper triangular matrix if \( a_{ij} = 0 \) when \( i > j \), that is, all the elements below the diagonal are zero. Note that the statement \( a_{ij} = 0 \) when \( i > j \) implies that the matrix elements are zero for all elements where the row number is strictly greater than the column number.
2. A square matrix $[A]$ is lower triangular if

- (A) $a_{ij} = 0, j > i$
- (B) $a_{ij} = 0, i > j$
- (C) $a_{ij} \neq 0, i > j$
- (D) $a_{ij} \neq 0, j > i$

**Solution**

The correct answer is (A).

A $n \times n$ matrix $[A]$ is lower triangular if $a_{ij} = 0$ for $j > i$. That is, all elements above the diagonal are zero. Note that all the elements of $[A]$ for which the column number is greater than the row numbers are zero. An example of a lower triangular matrix is

$$[A] = \begin{bmatrix}
2 & 0 & 0 & 0 \\
6 & 3 & 0 & 0 \\
4 & -5 & 0 & 0 \\
2 & 9 & 3 & -3.2
\end{bmatrix}$$
3. Given
\[
A = \begin{bmatrix}
12.3 & -12.3 & 20.3 \\
11.3 & -10.3 & -11.3 \\
10.3 & -11.3 & -12.3
\end{bmatrix}, \quad
B = \begin{bmatrix}
2 & 4 \\
-5 & 6 \\
11 & -20
\end{bmatrix}
\]

then if
\[
C = A \begin{bmatrix} B \end{bmatrix},
\]

\[
c_{31} = \]

(A) \(-58.2\)
(B) \(-37.6\)
(C) 219.4
(D) 259.4

**Solution**

The correct answer is (A).

The \(i^{th}\) row and \(j^{th}\) column of the \(C\) matrix in \(C = A \begin{bmatrix} B \end{bmatrix}\) is calculated by multiplying the \(i^{th}\) row of \(A\) by the \(j^{th}\) column of \(B\), that is,

\[
c_{ij} = a_{i1}a_{i2} \ldots a_{ip} b_{1j} b_{2j} \ldots b_{pj}
\]

\[
= a_{i1} b_{1j} + a_{i2} b_{2j} + \ldots + a_{ip} b_{pj}
\]

\[
= \sum_{k=1}^{p} a_{ik} b_{kj}
\]

Therefore,

\[
c_{31} = \begin{bmatrix}
10.3 & -11.3 & -12.3
\end{bmatrix}
\begin{bmatrix}
2 \\
-5 \\
11
\end{bmatrix}
\]

\[
= (10.3 \times 2) + (-11.3 \times -5) + (-12.3 \times 11)
\]

\[
= 20.6 + 56.5 - 135.3
\]

\[
= -58.2
\]
4. The following system of equations has ____________ solution(s).

\[
\begin{align*}
  x + y &= 2 \\
  6x + 6y &= 12
\end{align*}
\]

(A) infinite  
(B) no  
(C) two  
(D) unique

**Solution**

_The correct answer is (A)._

The system of equations

\[
\begin{align*}
  x + y &= 2 \quad (1) \\
  6x + 6y &= 12 \quad (2)
\end{align*}
\]

has an infinite number of solutions because the two equations are the same. Equation (2) is a multiple of 6 of Equation (1).
5. Consider there are only two computer companies in a country. The companies are named *Dude* and *Imac*. Each year, *Dude* keeps $1/5^{th}$ of its customers, while the rest switch to *Imac*. Each year, *Imac* keeps $1/3^{rd}$ of its customers, while the rest switch to *Dude*. If in 2003, *Dude* had $1/6^{th}$ of the market and *Imac* had $5/6^{th}$ of the market, what will be the share of *Dude* computers when the market becomes stable?

(A) 37/90
(B) 5/11
(C) 6/11
(D) 53/90

**Solution**

The correct answer is (B).

If $D$ is the current market if *Dude* computers and $M$ is the current market of *Imac* computers, and if $D_n$ is the next year’s *Dude* market and $M_n$ is the next year’s *Imac* market, then since we want when the market is stable, the market share should not change from year to year.

$$
\begin{bmatrix}
D_n \\
M_n
\end{bmatrix}
=
\begin{bmatrix}
\frac{1}{5} & \frac{2}{3} \\
\frac{4}{5} & \frac{1}{3}
\end{bmatrix}
\begin{bmatrix}
D_n \\
M_n
\end{bmatrix}
$$

$$
D_n = \frac{1}{5} \times D_n + \frac{2}{3} \times M_n
$$

$$
M_n = \frac{4}{5} \times D_n + \frac{1}{3} \times M_n
$$

This has a trivial solution if $D_n = 0, M_n = 0$, but we know that $D_n + M_n = 1$. So we are looking for a non-trivial solution. Note also that the coefficient matrix is singular.

$$
0 = \frac{4}{5} \times D_n - \frac{2}{3} \times M_n
$$

$D_n + M_n = 1$

gives,

$$
D_n = \frac{5}{11}
$$

*Extra notes for the student:*

If one was going to find what the market share would be in 2004
\[
\begin{bmatrix}
D_n \\
M_n
\end{bmatrix}
= \begin{bmatrix}
\frac{1}{5} & \frac{2}{3} & \frac{1}{6} \\
\frac{4}{5} & \frac{1}{1} & \frac{5}{6}
\end{bmatrix}
= \begin{bmatrix}
\frac{53}{90} \\
\frac{37}{90}
\end{bmatrix}
\]

One would use this number to find the market share in 2005 and so on. Eventually the market share would stabilize. But that would be a lengthier way to solve the problem.
6. Three kids - Jim, Corey and David receive an inheritance of $2,253,453. The money is put in three trusts but is not divided equally to begin with. Corey's trust is three times that of David's because Corey made an A in Dr. Kaw’s class. Each trust is put in an interest generating investment. The three trusts of Jim, Corey and David pays an interest of 6%, 8%, 11%, respectively. The total interest of all the three trusts combined at the end of the first year is $190,740.57. The equations to find the trust money of Jim ($J$), Corey ($C$) and David ($D$) in a matrix form is

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 3 & -1 \\
0.06 & 0.08 & 0.11
\end{bmatrix}
\begin{bmatrix}
J \\
C \\
D
\end{bmatrix}
=
\begin{bmatrix}
2,253,453 \\
0 \\
190,740.57
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & -3 \\
0.06 & 0.08 & 0.11
\end{bmatrix}
\begin{bmatrix}
J \\
C \\
D
\end{bmatrix}
=
\begin{bmatrix}
2,253,453 \\
0 \\
190,740.57
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & -3 \\
6 & 8 & 11
\end{bmatrix}
\begin{bmatrix}
J \\
C \\
D
\end{bmatrix}
=
\begin{bmatrix}
2,253,453 \\
0 \\
190,740.57
\end{bmatrix}
\]

\[
\begin{bmatrix}
1 & 1 & 1 \\
0 & 3 & -1 \\
6 & 8 & 11
\end{bmatrix}
\begin{bmatrix}
J \\
C \\
D
\end{bmatrix}
=
\begin{bmatrix}
2,253,453 \\
0 \\
19,074,057
\end{bmatrix}
\]

**Solution**

*The correct answer is (B).*

Let $J$, $C$, and $D$ be the inheritance portions of Jim, Corey and David, respectively.

The total inheritance is $2,253,453$ gives

\[J + C + D = 2,253,453\]

Corey's trust is three times that of David's

\[C = 3D\]

gives

\[C - 3D = 0\]

The three trusts of Jim, Corey and David pay an interest of 6%, 8%, 11%, respectively. The total interest of all the three trusts combined at the end of the first year is $190,740.57.

The total interest is

\[
\frac{6}{100} J + \frac{8}{100} C + \frac{11}{100} D = 190,740.57
\]

gives
$0.06J + 0.08C + 0.11D = $190,740.57$

Three equations can be made from the information given:

$J + C + D = $2,253,453$

$C - 3D = 0$

$0.06J + 0.08C + 0.11D = $190,740.57$

Setting the three equations in matrix form is as follows:

$$
\begin{bmatrix}
1 & 1 & 1 \\
0 & 1 & -3 \\
0.06 & 0.08 & 0.11
\end{bmatrix}
\begin{bmatrix}
J \\
C \\
D
\end{bmatrix}
=
\begin{bmatrix}
2,253,453 \\
0 \\
190,740.57
\end{bmatrix}
$$