Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Chapter 4.10
Eigenvalues and Eigenvectors
COMPLETE SOLUTION SET

1. The eigenvalues of

$$\begin{bmatrix} 5 & 6 & 17 \\ 0 & -19 & 23 \\ 0 & 0 & 37 \end{bmatrix}$$

are

(A) $-19,5,37$
(B) $19,-5,-37$
(C) $2,-3,7$
(D) $3,-5,37$

Solution
The correct answer is (A).

The eigenvalues of an upper triangular matrix are simply the diagonal entries of the matrix. Hence $5,-19,$ and 37 are the eigenvalues of the matrix. Alternately, look at $\det([A]-\lambda[I])=0$

$$\det\left(\begin{bmatrix} 5 & 6 & 17 \\ 0 & -19 & 23 \\ 0 & 0 & 37 \end{bmatrix} - \lambda \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 5-\lambda & 6 & 17 \\ 0 & -19-\lambda & 23 \\ 0 & 0 & 37-\lambda \end{bmatrix}\right) = 0$$

$$(5-\lambda)(-19-\lambda)(37-\lambda) = 0$$

Then

$$\lambda = 5,-19,37$$

are the roots of the equation; and hence, the eigenvalues of $[A]$.

2. If $\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$ is an eigenvector of $\begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -4 \end{bmatrix}$, the eigenvalue corresponding to the eigenvector is

(A) 1
(B) 4
(C) -4.5
(D) 6

Solution

The correct answer is (B).

If $[A]$ is a $n \times n$ matrix and λ is one of the eigenvalues and $[X]$ is a $n \times 1$ corresponding eigenvector, then

$$[A][X] = \lambda[X]$$

$$\begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -4 \end{bmatrix}\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix} = \lambda\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} -18 \\ -16 \\ 4 \end{bmatrix} = \lambda\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$$

$$4\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix} = \lambda\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$$

$$\lambda = 4$$

3. The eigenvalues of the following matrix

$$\begin{bmatrix} 3 & 2 & 9 \\ 7 & 5 & 13 \\ 6 & 17 & 19 \end{bmatrix}$$

are given by solving the cubic equation

(A) $\lambda^3 - 27\lambda^2 + 167\lambda - 285$

(B) $\lambda^3 - 27\lambda^2 - 122\lambda - 313$

(C) $\lambda^3 + 27\lambda^2 + 167\lambda + 285$

(D) $\lambda^3 + 23.23\lambda^2 - 158.3\lambda + 313$

Solution

The correct answer is (B).

To find the equations of

$$[A] = \begin{bmatrix} 3 & 2 & 9 \\ 7 & 5 & 13 \\ 6 & 17 & 19 \end{bmatrix}$$

we solve $\det([A] - \lambda[I]) = 0$

$$\det\left(\begin{bmatrix} 3 & 2 & 9 \\ 7 & 5 & 13 \\ 6 & 17 & 19 \end{bmatrix} - \lambda\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\right) = 0$$

$$\det\left(\begin{bmatrix} 3-\lambda & 2 & 9 \\ 7 & 5-\lambda & 13 \\ 6 & 17 & 19-\lambda \end{bmatrix}\right) = 0$$

Using the cofactor method with $Row 1$

$$(3-\lambda)\begin{vmatrix} 5-\lambda & 13 \\ 17 & 19-\lambda \end{vmatrix} - 2\begin{vmatrix} 7 & 13 \\ 6 & 19-\lambda \end{vmatrix} + 9\begin{vmatrix} 7 & 5-\lambda \\ 6 & 17 \end{vmatrix} = 0$$

$$(3-\lambda)((5-\lambda)(19-\lambda) - 13\times 17) - 2(7(19-\lambda) - 13\times 6) + 9(7\times 17 - 6(5-\lambda)) = 0$$

$$\lambda^3 - 27\lambda^2 - 122\lambda - 313 = 0$$

4. The eigenvalues of a 4×4 matrix $[A]$ are given as $2, -3, 13$, and 7. The $|\det(A)|$ then is

(A) 546
(B) 19
(C) 25
(D) cannot be determined

Solution

The correct answer is (A).

If $\lambda_1, \lambda_2, ...\lambda_{n-1}, \lambda_n$ are the eigenvalues of a $n \times n$ matrix $[A]$, then

$$|\det(A)| = |\lambda_1 \times \lambda_2 \times \lambda_3 \times ... \times \lambda_n|$$

$$|\lambda_1 \times \lambda_2 \times \lambda_3 \times \lambda_4|$$

$$= |2 \times (-3) \times 13 \times 7|$$

$$= 546$$

5. If one of the eigenvalues of $[A]_{n\times n}$ is zero, it implies
   (A) The solution to $[A][X]=[C]$ system of equations is unique
   (B) The determinant of $[A]$ is zero
   (C) The solution to $[A][X]=[0]$ system of equations is trivial
   (D) The determinant of $[A]$ is nonzero

Solution

The correct answer is (B).

For a $n\times n$ matrix $[A]$ with $\lambda_1, \lambda_2, \ldots \lambda_{n-1}, \lambda_n$ as the eigenvectors

$$\left|\det(A)\right| = \left|\lambda_1 \times \lambda_2 \times \lambda_3 \times \ldots \times \lambda_n\right|$$

Since one of the eigenvalues is zero,

$$\left|\det(A)\right| = 0$$

$$\det(A) = 0$$

6. Given that matrix $[A]=\begin{bmatrix} 8 & -4 & 2 \\ 4 & 0 & 2 \\ 0 & -2 & -3 \end{bmatrix}$ has an eigenvalue value of 4 with the corresponding eigenvectors of $[x]=\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$, then $[A]^5[X]$ is

(A) $\begin{bmatrix} -18 \\ -16 \\ 4 \end{bmatrix}$

(B) $\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$

(C) $\begin{bmatrix} -4608 \\ -4096 \\ 1024 \end{bmatrix}$

(D) $\begin{bmatrix} -0.004395 \\ -0.003906 \\ 0.0009766 \end{bmatrix}$

Solution

The correct answer is (C).

If for a $n \times n$ matrix $[A]$, λ is an eigenvalue and $[X]$ is the corresponding eigenvector, then

$$[A]^m[x] = \lambda^m[X]$$

$$[A]^5[X] = \lambda^5[X]$$

$$= 4^5\begin{bmatrix} -4.5 \\ -4 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} -4608 \\ -4096 \\ 1024 \end{bmatrix}$$

Appendix for Question 6

For a $n \times n$ matrix $[A]$, if λ is an eigenvalue and $[X]$ is an eigenvector prove for

$$[A]^m [x] = \lambda^m [X], m = 1,2,3 \ldots$$

For a $n \times n$ matrix $[A]$, if λ is an eigenvalue and $[X]$ is an eigenvector then

$$[A][X] = \lambda[X]$$

$$[A]^2[X] = [A][A][X]$$

$$= \lambda[A][X]$$

$$= \lambda \times \lambda[X]$$

$$= \lambda^2[X]$$

If

$$[A]^{m-1}[X] = \lambda^{m-1}[X], m \geq 0, m = integer$$

Then

$$[A]^m[X] = [A][A]^{m-1}[X]$$

$$= \lambda^{m-1}[A][X]$$

$$= [A]\lambda^{m-1}[X]$$

$$= \lambda^{m-1} \times \lambda[X]$$

$$= \lambda^m[X]$$