Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Chapter 4.01
Introduction
COMPLETE SOLUTION SET

1. For an $n \times n$ upper triangular matrix $[A]$,
   - (A) $a_{ij} = 0, i > j$
   - (B) $a_{ij} = 0, j > i$
   - (C) $a_{ij} \neq 0, i > j$
   - (D) $a_{ij} \neq 0, j > i$

Solution
The correct answer is (*A*).

A $n \times n$ matrix $[A]$ is upper triangular if $a_{ij} = 0, i > j$ for all i, j. A matrix being upper triangular has nothing to do with whether any of the elements $a_{ij}, j > i$ are non-zero or not.
A matrix like

$$\begin{bmatrix} 6 & 7 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

is upper triangular because $a_{ij} = 0, i > j$ for all i, j.

A matrix like

$$\begin{bmatrix} 6 & 7 & 0 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}$$

is upper triangular because $a_{ij} = 0, i > j$ for all i, j. Note that $a_{13} = 0$ does not affect the upper triangular nature of the matrix.

2. Which one of these square matrices is strictly diagonally dominant?

(A) $\begin{bmatrix} 5 & 7 & 0 \\ 3 & -6 & 2 \\ 2 & 2 & 9 \end{bmatrix}$

(B) $\begin{bmatrix} 7 & -5 & -2 \\ 6 & -13 & -7 \\ 6 & -7 & -13 \end{bmatrix}$

(C) $\begin{bmatrix} 8 & -5 & -2 \\ 6 & -14 & -7 \\ 6 & -7 & -13 \end{bmatrix}$

(D) $\begin{bmatrix} 8 & 5 & 2 \\ 6 & 14 & 7 \\ 6 & 7.5 & 14 \end{bmatrix}$

Solution

The correct answer is (D).

For a square matrix $[A]_{n\times n}$ to be strictly diagonally dominant, then

$$|a_{ii}| > \sum_{\substack{i=1 \\ i\neq j}}^{n} |a_{ij}|$$

must be true for all i.

The matrix in choice (A),

$$[A] = \begin{bmatrix} 5 & 7 & 0 \\ 3 & -6 & 2 \\ 2 & 2 & 9 \end{bmatrix}$$

is not strictly diagonally dominant as it does not meet the requirement for Row 1.

$$|a_{11}| > |a_{12}| + |a_{13}|$$

$$|5| > |7| + |0|$$

$5 > 7$ is not true.

The matrix in choice (B),

$$[B]=\begin{bmatrix} 7 & -5 & -2 \\ 6 & -13 & -7 \\ 6 & -7 & -13 \end{bmatrix}$$

is not strictly diagonally dominant as it does not meet the requirement for Row 1.

$$|b_{11}| > |b_{12}| + |b_{13}|$$

$$|7| > |-5| + |-2|$$

$7 > 7$ is not true.

The matrix in choice (C),

$$[C]=\begin{bmatrix} 8 & -5 & -2 \\ 6 & -14 & -7 \\ 6 & -7 & -13 \end{bmatrix}$$

is not strictly diagonally dominant as it meets all the requirements for Rows 1 and 2 but not for Row 3.

For Row 1

$$|c_{11}| > |c_{12}| + |c_{13}|$$

$$|8| > |-5| + |-2|$$

$8 > 7$ is true.

For Row 2

$$|c_{22}| > |c_{21}| + |c_{23}|$$

$$|-14| > |6| + |-7|$$

$14 > 13$ is true.

For Row 3

$$|c_{33}| > |c_{31}| + |c_{32}|$$

$$|-13| > |6| + |-7|$$

$13 > 13$ is not true.

Hence $[C]$ is not strictly diagonally dominant.

The matrix in choice (D)

$$[D]=\begin{bmatrix} 8 & 5 & 2 \\ 6 & 14 & 7 \\ 6 & 7.5 & 14 \end{bmatrix}$$

is strictly diagonally dominant as it meets the requirements for all rows – 1, 2, 3.

For Row 1

$$|d_{11}| > |d_{12}| + |d_{13}|$$

$$|8| > |5| + |2|$$

$8 > 7$ is true.

For Row 2

$$|d_{22}| > |d_{21}| + |d_{23}|$$

$$|14| > |6| + |7|$$

$14 > 13$ is true.

For Row 3

$$|d_{33}| > |d_{31}| + |d_{32}|$$

$$|14| > |7.5| + |6|$$

$14 > 13.5$ is true.

Hence, $[D]$ is strictly diagonally dominant.

3. The order of the following matrix is

$$\begin{bmatrix} 4 & -6 & -7 & 2 \\ 3 & 2 & -5 & 6 \end{bmatrix}$$

(A) 4×2
(B) 2×4
(C) 8×1
(D) not defined

Solution
The correct answer is (*B*).

The order of the matrix is determined by the number of rows and columns.
The matrix

$$\begin{bmatrix} 4 & -6 & -7 & 2 \\ 3 & 2 & -5 & 6 \end{bmatrix}$$

has 2 rows and 4 columns. The order of the matrix then is 2×4.

4. To make the following two matrices equal

$$[A]=\begin{bmatrix}5 & -6 & 7\\ 3 & 2 & 5\end{bmatrix}$$

$$[B]=\begin{bmatrix}5 & p & 7\\ 3 & 2 & 5\end{bmatrix}$$

the value of p is

(A) -6
(B) 6
(C) 0
(D) 7

Solution

The correct answer is (*A*).

Two matrices $[A]$ and $[B]$ are equal if they are of the same order and $a_{ij}=b_{ij}$ for all i, j. For $[A]$ and $[B]$ to be equal, we find that if $b_{12}=-6=p$, then the two matrices are equal. The matrices $[A]$ and $[B]$ are both of the same order 2×3 and $a_{ij}=b_{ij}$ for all i, j.

5. For a square $n \times n$ matrix $[A]$ to be an identity matrix,

(A) $a_{ij} \neq 0, i = j; a_{ij} = 0, i = j$

(B) $a_{ij} = 0, i \neq j; a_{ij} = 1, i = j$

(C) $a_{ij} = 0, i \neq j; a_{ij} = i, i = j$

(D) $a_{ij} = 0, i \neq j; a_{ij} > 0, i = j$

Solution

The correct answer is (B).

A square $n \times n$ matrix $[A]$ is an identity matrix if all the off-diagonal elements are zero and the diagonal elements are one. Hence

$a_{ij} = 0$ for $i \neq j$ for all i, j

$a_{ij} = 1$ for $i = j$ for all i, j.

An example of an identity matrix is

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

6. To make the following square matrix to be diagonally dominant, the value of p needs to be

$$\begin{bmatrix} 6 & -2 & -4 \\ 7 & 9 & 1 \\ 8 & -5 & p \end{bmatrix}$$

(A) greater than or equal to 13
(B) greater than 3
(C) greater than or equal to 3
(D) greater than 13

Solution

The correct answer is (*A*).

A square $n \times n$ matrix $[A]$ is considered to be diagonally dominant if

$$|a_{ii}| \geq \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|$$

for all i. and

$$|a_{ii}| > \sum_{\substack{j=1 \\ i \neq j}}^{n} |a_{ij}|$$

for at least one *i*.

For Row 1

$$|a_{11}| \geq |a_{12}| + |a_{13}|$$

$$|6| \geq |-2| + |-4|$$

$6 \geq 6$ is true.

For Row 2

$$|a_{22}| \geq |a_{21}| + |a_{23}|$$

$$|9| \geq |7| + |1|$$

$9 \geq 8$ is true.

For Row 3

$$|a_{33}| \geq |a_{31}| + |a_{32}|$$

$$|p| \geq |8| + |-5|$$

$p \geq 13$ needs to be true.

So, *p* greater than or equal to 13 is the right choice, along with the fact that the inequality is strictly met for Row 2 (that is, $9 > 8$).