Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Chapter 04.02
Vectors
COMPLETE SOLUTION SET

1. A set of equations

$$4x_1 + 7x_2 + 11x_3 = 13$$
$$17x_1 + 39x_2 + 23x_3 = 31$$
$$13x_1 + 67x_2 + 59x_3 = 37$$

can also be written as

(A) $x_1\begin{bmatrix}4\\17\\13\end{bmatrix} + x_2\begin{bmatrix}7\\39\\23\end{bmatrix} + x_3\begin{bmatrix}11\\23\\59\end{bmatrix} = \begin{bmatrix}13\\31\\37\end{bmatrix}$

(B) $4\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + 39\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} + 59\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix} = \begin{bmatrix}13\\31\\37\end{bmatrix}$

(C) $x_1\begin{bmatrix}4\\7\\11\end{bmatrix} + x_2\begin{bmatrix}17\\39\\23\end{bmatrix} + x_3\begin{bmatrix}13\\67\\59\end{bmatrix} = \begin{bmatrix}13\\31\\37\end{bmatrix}$

(D) $x_1\begin{bmatrix}13\\17\\4\end{bmatrix} + x_2\begin{bmatrix}67\\39\\7\end{bmatrix} + x_3\begin{bmatrix}59\\23\\11\end{bmatrix} = \begin{bmatrix}57\\13\\31\end{bmatrix}$

Solution

The correct answer is (*C*).

The set of equations

$$4x_1 + 7x_2 + 11x_3 = 13$$
$$17x_1 + 39x_2 + 23x_3 = 31$$
$$13x_1 + 67x_2 + 59x_3 = 37$$

can be rewritten as two matrices being equal to each other as

$$\begin{bmatrix} 4x_1 + 7x_2 + 11x_3 \\ 17x_1 + 39x_2 + 23x_3 \\ 13x_1 + 67x_2 + 59x_3 \end{bmatrix} = \begin{bmatrix} 13 \\ 31 \\ 37 \end{bmatrix}$$

and then as a linear combination as

$$x_1 \begin{bmatrix} 4 \\ 7 \\ 11 \end{bmatrix} + x_2 \begin{bmatrix} 17 \\ 39 \\ 23 \end{bmatrix} + x_3 \begin{bmatrix} 13 \\ 67 \\ 59 \end{bmatrix} = \begin{bmatrix} 13 \\ 31 \\ 37 \end{bmatrix}$$

2. The magnitude of the vector, $V = (5, -3, 2)$ is

(A) 4
(B) 10
(C) $\sqrt{38}$
(D) $\sqrt{20}$

Solution
The correct answer is (C).

The magnitude of the vector, $\vec{v} = (5, -3, 2)$ is

$$\left|\vec{v}\right| = \sqrt{(5)^2 + (-3)^2 + (2)^2}$$

$$= \sqrt{38}$$

3. The rank of the vector

$$\vec{A}\begin{bmatrix}2\\3\\7\end{bmatrix},\begin{bmatrix}6\\9\\21\end{bmatrix},\begin{bmatrix}3\\2\\7\end{bmatrix}$$

is

(A) 1
(B) 2
(C) 3
(D) 4

Solution

The correct answer is (*B*).

$$\vec{A}_1 = \begin{bmatrix}2\\3\\7\end{bmatrix}$$

$$\vec{A}_2 = \begin{bmatrix}6\\9\\21\end{bmatrix}$$

$$\vec{A}_3 = \begin{bmatrix}3\\2\\7\end{bmatrix}$$

From inspection

$$\vec{A}_1 = \frac{1}{3}\vec{A}_2$$

which implies

$$1\vec{A}_1 - \frac{1}{3}\vec{A}_2 + 0\vec{A}_3 = \vec{0}$$

Hence

$$k_1\vec{A}_1 + k_2\vec{A}_2 + k_3\vec{A}_3 = \vec{0}$$

has a non-trivial solution of $k_1 = 1, k_2 = -\frac{1}{3}, k_3 = 0$. $\vec{A}_1, \vec{A}_2, \vec{A}_3$ are hence, linearly dependent. The rank of vectors hence is not 3, but less than 3.

However,

$$k_2\vec{A}_2 + k_3\vec{A}_3 = \vec{0}$$

has only one solution and that is the trivial solution of $k_1 = 0, k_2 = 0$. So $\vec{A}_2$ and $\vec{A}_3$ are linearly independent. The rank of the vectors hence is 2.

4. If $\vec{A} = (5,2,3)$ and $\vec{B} = (6,-7,3)$, then $4\vec{A} + 5\vec{B}$ is

(A) $(50,-5,6)$

(B) $(50,-27,27)$

(C) $(11,-5,6)$

(D) $(20,8,12)$

Solution

The correct answer is (*B*).

$$\vec{A} = (5,2,3)$$

$$\vec{B} = (6,-7,3)$$

$$\begin{aligned} 4\vec{A} + 5\vec{B} &= 4(5,2,3) + 5(6,-7,3) \\ &= (20,8,12) + (30,-35,15) \\ &= (50,-27,27) \end{aligned}$$

5. The dot product of two vectors $\vec{A}$ and $\vec{B}$

$$\vec{A} = 3i + 5j + 7k$$

$$\vec{B} = 11i + 13j + 17k$$

most nearly is

(A) 14.80
(B) 33.00
(C) 56.00
(D) 217.0

Solution

The correct answer is (D).

The dot product of two vectors

$$\vec{u} = (u_x, u_y, u_z)$$

and

$$\vec{v} = (v_x, v_y, v_z)$$

is

$$\begin{aligned}\vec{u} \cdot \vec{v} &= u_x v_x + u_y v_y + u_z v_z \\ &= 3 \times 11 + 5 \times 13 + 7 \times 17 \\ &= 217\end{aligned}$$

6. The angle in degrees between two vectors $\vec{u}$ and $\vec{v}$

$$\vec{u} = 3i + 5j + 7k$$

$$\vec{v} = 11i + 13j + 17k$$

most nearly is

(A) 8.124
(B) 11.47
(C) 78.52
(D) 81.88

Solution

The correct answer is (*A*).

The dot product of two vectors $\vec{u} = (u_x, u_y, u_z)$ and $\vec{v} = (v_x, v_y, v_z)$ is

$$\vec{u} \cdot \vec{v} = u_x v_x + u_y v_y + u_z v_z$$
$$= 3 \times 11 + 5 \times 13 + 7 \times 17$$
$$= 217$$

The dot product is also defined as

$$\vec{u} . \vec{v} = |u||v| \cos\theta$$

where

$$u = \sqrt{u_x^2 + u_y^2 + u_t^2}$$
$$= \sqrt{3^2 + 5^2 + 7^2}$$
$$= 9.110$$

and

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$
$$= \sqrt{11^2 + 13^2 + 17^2}$$
$$= 24.06$$

then

$$\vec{u} \cdot \vec{v} = |u||v| \cos\theta$$
$$= 9.110 \times 24.06 \cos\theta$$
$$= 219.2 \cos\theta$$

Hence

$$219.2 \cos\theta = 217$$
$$\cos\theta = \frac{217}{219.2}$$
$$= 0.9900$$

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$$\theta = 8.124^\circ$$