Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

# Multiple-Choice Test
# Gauss-Seidel Method of Solving Simultaneous Linear Equations

1. A square matrix $[A]_{nxn}$ is diagonally dominant if

(A) $\left|a_{ii}\right| \geq \sum_{\substack{j=1 \\ i \neq j}}^{n}\left|a_{ij}\right|, \quad i = 1, 2, \ldots, n$

*(B)* $\left|a_{ii}\right| \geq \sum_{\substack{j=1 \\ i \neq j}}^{n}\left|a_{ij}\right|, \ i = 1, 2, \ldots, n$ and $\left|a_{ii}\right| > \sum_{\substack{j=1 \\ i \neq j}}^{n}\left|a_{ij}\right|,$ for any $i = 1, 2, \ldots, n$

(C) $\left|a_{ii}\right| \geq \sum_{j=1}^{n}\left|a_{ij}\right|, \ i = 1, 2, \ldots, n$ and $\left|a_{ii}\right| > \sum_{j=1}^{n}\left|a_{ij}\right|,$ for any $i = 1, 2, \ldots, n$

(D) $\left|a_{ii}\right| \geq \sum_{j=1}^{n}\left|a_{ij}\right|, \ i = 1, 2, \ldots, n$

2. Using $[x_1 \quad x_2 \quad x_3] = [1 \quad 3 \quad 5]$ as the initial guess, the value of $[x_1 \quad x_2 \quad x_3]$ after three iterations in Gauss-Seidel method for

$$\begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -5 \\ 6 \end{bmatrix}$$

is

(A) [-2.8333 -1.4333 -1.9727]
(B) [1.4959 -0.90464 -0.84914]
(C) [0.90666 -1.0115 -1.0242]
(D) [1.2148 -0.72060 -0.82451]

3. To ensure that the following system of equations,

$$\begin{aligned} 2x_1 + 7x_2 - 11x_3 &= 6 \\ x_1 + 2x_2 + x_3 &= -5 \\ 7x_1 + 5x_2 + 2x_3 &= 17 \end{aligned}$$

converges using Gauss-Seidel Method, one can rewrite the above equations as follows:

(A) $\begin{bmatrix} 2 & 7 & -11 \\ 1 & 2 & 1 \\ 7 & 5 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 17 \end{bmatrix}$

(B) $\begin{bmatrix} 7 & 5 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 17 \\ -5 \\ 6 \end{bmatrix}$

(C) $\begin{bmatrix} 7 & 5 & 2 \\ 1 & 2 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 6 \\ -5 \\ 17 \end{bmatrix}$

(D) The equations cannot be rewritten in a form to ensure convergence.

4. For $\begin{bmatrix} 12 & 7 & 3 \\ 1 & 5 & 1 \\ 2 & 7 & -11 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 22 \\ 7 \\ -2 \end{bmatrix}$ and using $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix} = \begin{bmatrix} 1 & 2 & 1 \end{bmatrix}$ as the initial guess, the values of $\begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}$ are found at the end of each iteration as

| Iteration # | $x_1$ | $x_2$ | $x_3$ |
|---|---|---|---|
| 1 | 0.41666 | 1.1166 | 0.96818 |
| 2 | 0.93989 | 1.0183 | 1.0007 |
| 3 | 0.98908 | 1.0020 | 0.99930 |
| 4 | 0.99898 | 1.0003 | 1.0000 |

At what first iteration number would you trust at least 1 significant digit in your solution?

(A) 1
(B) 2
(C) 3
(D) 4

5. The algorithm for the Gauss-Seidel Method to solve [A] [X] = [C] is given as follows for using nmax iterations. The initial value of [X] is stored in [X].

(A)
```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = (rhs(i) - Sum) / a(i, i)
Next i
Next k
End Sub
```

(B)
```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
Sum = 0
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = (rhs(i) - Sum) / a(i, i)
Next i
Next k
End Sub
```

(C)
```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
Sum = 0
For j = 1 To n
Sum = Sum + a(i, j) * x(j)
Next j
x(i) = (rhs(i) - Sum) / a(i, i)
Next i
Next k
End Sub
```

(D)
```
Sub Seidel(n, a, x, rhs, nmax)
For k = 1 To nmax
For i = 1 To n
Sum = 0
For j = 1 To n
If (i <> j) Then
Sum = Sum + a(i, j) * x(j)
endif
Next j
x(i) = rhs(i)  / a(i, i)
Next i
Next k
End Sub
```

6. Thermistors measure temperature, have a nonlinear output and are valued for a limited range. So when a thermistor is manufactured, the manufacturer supplies a resistance vs. temperature curve. An accurate representation of the curve is generally given by

$$\frac{1}{T} = a_0 + a_1 \ln(R) + a_2 \{\ln(R)\}^2 + a_3 \{\ln(R)\}^3$$

where T is temperature in Kelvin, R is resistance in ohms, and $a_0, a_1, a_2, a_3$ are constants of the calibration curve.

Given the following for a thermistor

| **R** | **T** |
|---|---|
| **ohm** | $°C$ |
| 1101.0 | 25.113 |
| 911.3 | 30.131 |
| 636.0 | 40.120 |
| 451.1 | 50.128 |

the value of temperature in $°C$ for a measured resistance of 900 ohms most nearly is

(A) 30.002
(B) 30.472
(C) 31.272
(D) 31.445