Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple Choice Test
LU Decomposition Method
Simultaneous Linear Equations

1. LU decomposition method is computationally more efficient than Naïve Gauss elimination for solving

   (A) a single set of simultaneous linear equations
   (B) multiple sets of simultaneous linear equations with different coefficient matrices and same right hand side vectors.
   (C) multiple sets of simultaneous linear equations with same coefficient matrix and different right hand side vectors.
   (D) less than ten simultaneous linear equations.

2. The lower triangular matrix [L] in the [L][U] decomposition of matrix given below

$$\begin{bmatrix} 25 & 5 & 4 \\ 10 & 8 & 16 \\ 8 & 12 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.7333 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 6 & 14.400 \\ 0 & 0 & -4.2400 \end{bmatrix}$

(C) $\begin{bmatrix} 1 & 0 & 0 \\ 10 & 1 & 0 \\ 8 & 12 & 0 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.5000 & 1 \end{bmatrix}$

3. The upper triangular matrix [U] in the [L][U] decomposition of matrix given below

$$\begin{bmatrix} 25 & 5 & 4 \\ 0 & 8 & 16 \\ 0 & 12 & 22 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ \ell_{21} & 1 & 0 \\ \ell_{31} & \ell_{32} & 1 \end{bmatrix} \begin{bmatrix} u_{11} & u_{12} & u_{13} \\ 0 & u_{22} & u_{23} \\ 0 & 0 & u_{33} \end{bmatrix}$$

is

(A) $\begin{bmatrix} 1 & 0 & 0 \\ 0.40000 & 1 & 0 \\ 0.32000 & 1.7333 & 1 \end{bmatrix}$

(B) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 6 & 14.400 \\ 0 & 0 & -4.2400 \end{bmatrix}$

(C) $\begin{bmatrix} 25 & 5 & 4 \\ 0 & 8 & 16 \\ 0 & 0 & -2 \end{bmatrix}$

(D) $\begin{bmatrix} 1 & 0.2000 & 0.16000 \\ 0 & 1 & 2.4000 \\ 0 & 0 & -4.240 \end{bmatrix}$

4. For a given 2000×2000 matrix [A], assume that it takes about 15 seconds to find the inverse of [A] by the use of the [L][U] decomposition method, that is, finding the [L][U] once, and then doing forward substitution and back substitution 2000 times using the 2000 columns of the identity matrix as the right hand side vector. The approximate time, in seconds, that it will take to find the inverse if found by repeated use of Naive Gauss Elimination method, that is, doing forward elimination and back substitution 2000 times by using the 2000 columns of the identity matrix as the right hand side vector is

(A) 300
(B) 1500
(C) 7500
(D) 30000

5. The algorithm in solving the set of equations [A][X] = [C], where [A] = [L][U] involves solving [L][Z] = [C] by forward substitution. The algorithm to solve [L][Z]=[C] is given by

(A) $z_1 = c_1 / l_{11}$
for i from 2 to n do
sum = 0
for j from 1 to i do
sum = sum + $l_{ij} * z_j$
end do
$z_i = (c_i - \text{sum}) / l_{ii}$
end do

(B) $z_1 = c_1 / l_{11}$
for i from 2 to n do
sum = 0
for j from 1 to (i-1) do
sum = sum + $l_{ij} * z_j$
end do
$z_i = (c_i - \text{sum}) / l_{ii}$
end do

(C) $z_1 = c_1 / l_{11}$
for i from 2 to n do
for j from 1 to (i-1) do
sum = sum + $l_{ij} * z_j$
end do
$z_i = (c_i - \text{sum}) / l_{ii}$
end do

(D) for i from 2 to n do
sum = 0
for j from 1 to (i-1) do
sum = sum + $l_{ij} * z_j$
end do
$z_i = (c_i - \text{sum}) / l_{ii}$
end do

6. To solve boundary value problems, finite difference method is used resulting in simultaneous linear equations with tri-diagonal coefficient matrices. These are solved using the specialized [L][U] decomposition method. The set of equations in matrix form with a tri-diagonal coefficient matrix for

$$\frac{d^2y}{dx^2} = 6x - 0.5x^2,\ y(0) = 0,\ y(12) = 0,$$

using finite difference method with a second order accurate central divided difference method and a step size of $h = 4$ is

(A) $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0625 & 0.125 & 0.0625 & 0 \\ 0 & 0.0625 & 0.125 & 0.0625 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 16.0 \\ 16.0 \\ 0 \end{bmatrix}$$

(B) $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0.0625 & -0.125 & 0.0625 & 0 \\ 0 & 0.0625 & -0.125 & 0.0625 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 16.0 \\ 16.0 \\ 0 \end{bmatrix}$$

(C) $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.0625 & -0.125 & 0.0625 & 0 \\ 0 & 0.0625 & -0.125 & 0.0625 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 16.0 \\ 16.0 \end{bmatrix}$$

(D) $$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0.0625 & 0.125 & 0.0625 & 0 \\ 0 & 0.0625 & 0.125 & 0.0625 \end{bmatrix} \begin{bmatrix} y_1 \\ y_2 \\ y_3 \\ y_4 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 16.0 \\ 16.0 \end{bmatrix}$$