Multiple-Choice Test
Nonlinear Regression
Regression

1. When using the linearized data model to find the constants of the regression model \( y = ae^{bx} \) to best fit \((x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)\), the sum is the square of the residuals that is minimized is

(A) \( \sum_{i=1}^{n} (y_i - ae^{bx_i})^2 \)

(B) \( \sum_{i=1}^{n} (\ln(y_i) - \ln a - bx_i)^2 \)

(C) \( \sum_{i=1}^{n} (y_i - \ln a - bx_i)^2 \)

(D) \( \sum_{i=1}^{n} (\ln(y_i) - \ln a - b \ln(x_i))^2 \)

2. It is suspected from theoretical considerations that the rate of flow from a firehouse is proportional to some power of the nozzle pressure. Assume pressure data is more accurate. You are linearizing the data.

<table>
<thead>
<tr>
<th>Flow rate, F (gallons/min)</th>
<th>96</th>
<th>129</th>
<th>135</th>
<th>145</th>
<th>168</th>
<th>235</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure, p (psi)</td>
<td>11</td>
<td>17</td>
<td>20</td>
<td>25</td>
<td>40</td>
<td>55</td>
</tr>
</tbody>
</table>

The exponent of the power of the nozzle pressure in the regression model, \( F = ap^b \) most nearly is

(A) 0.497
(B) 0.556
(C) 0.578
(D) 0.678
3. The linearized data model for the stress-strain curve $\sigma = K_1 \varepsilon e^{-K_2 \varepsilon}$ for concrete in compression, where $\sigma$ is the stress and $\varepsilon$ is the strain is

(A) $\ln \sigma = \ln k_1 + \ln \varepsilon - k_2 \varepsilon$

(B) $\ln \frac{\sigma}{\varepsilon} = \ln k_1 - k_2 \varepsilon$

(C) $\ln \frac{\sigma}{\varepsilon} = \ln k_1 + k_2 \varepsilon$

(D) $\ln \sigma = \ln(k_1 \varepsilon) - k_2 \varepsilon$

4. In nonlinear regression, finding the constants of the model requires solution of simultaneous nonlinear equations. However in the exponential model, $y = ae^{bx}$ that is best fit to $(x_1, y_1), (x_2, y_2), \ldots, (x_n, y_n)$, the value of $b$ can be found as a solution of a sample nonlinear equation. That equation is given by

(A) $\sum_{i=1}^{n} y_i x_i e^{bx_i} - \sum_{i=1}^{n} y_i e^{bx_i} \sum_{i=1}^{n} x_i = 0$

(B) $\sum_{i=1}^{n} y_i x_i e^{bx_i} = \frac{\sum_{i=1}^{n} y_i e^{bx_i}}{\sum_{i=1}^{n} e^{2bx_i}} \sum_{i=1}^{n} x_i e^{2bx_i} = 0$

(C) $\sum_{i=1}^{n} y_i x_i e^{bx_i} = \frac{\sum_{i=1}^{n} y_i e^{bx_i}}{\sum_{i=1}^{n} e^{2bx_i}} \sum_{i=1}^{n} e^{bx_i} = 0$

(D) $\sum_{i=1}^{n} y_i e^{bx_i} = \frac{\sum_{i=1}^{n} y_i e^{bx_i}}{\sum_{i=1}^{n} e^{2bx_i}} \sum_{i=1}^{n} x_i e^{2bx_i} = 0$
5. There is a functional relationship between the mass density $\rho$ of air and altitude $h$ above the sea level.

<table>
<thead>
<tr>
<th>Altitude above sea level, $h$ (km)</th>
<th>0.32</th>
<th>0.64</th>
<th>1.28</th>
<th>1.60</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mass Density, $\rho$ (kg/m$^3$)</td>
<td>1.15</td>
<td>1.10</td>
<td>1.05</td>
<td>0.95</td>
</tr>
</tbody>
</table>

In the regression model $\rho = k_1e^{k_2h}$, the constant $k_2$ is found as $k_2 = 0.1315$. Assuming the mass density of air at the top of the atmosphere is $\frac{1}{1000}$ of the mass density of air at sea level. The altitude in km of the top of the atmosphere most nearly is

(A) 46.2  
(B) 46.6  
(C) 49.7  
(D) 52.5

6. A steel cylinder at 80°F of length 12" is placed in a liquid nitrogen bath (−315°F). If thermal expansion coefficient of steel behaves as a second order polynomial of temperature and the polynomial is found by regressing the data below,

<table>
<thead>
<tr>
<th>Temperature (°F)</th>
<th>Thermal expansion coefficient (μ in/in/°F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-320</td>
<td>2.76</td>
</tr>
<tr>
<td>-240</td>
<td>3.83</td>
</tr>
<tr>
<td>-160</td>
<td>4.72</td>
</tr>
<tr>
<td>-80</td>
<td>5.43</td>
</tr>
<tr>
<td>0</td>
<td>6.00</td>
</tr>
<tr>
<td>80</td>
<td>6.47</td>
</tr>
</tbody>
</table>

the reduction in the length of cylinder most nearly is

(A) 0.0219"  
(B) 0.0231"  
(C) 0.0235"  
(D) 0.0307"