1. Physically, integrating $\int_a^b f(x)dx$ means finding the

(A) area under the curve from $a$ to $b$
(B) area to the left of point $a$
(C) area to the right of point $b$
(D) area above the curve from $a$ to $b$

Solution
The correct answer is (A).

Integrating $\int_a^b f(x)dx$ means finding the area under the curve of the function $f(x)$ from $a$ to $b$. 
2. The mean value of a function \( f(x) \) from \( a \) to \( b \) is given by

(A) \( \frac{f(a) + f(b)}{2} \)

(B) \( \frac{f(a) + 2f\left(\frac{a+b}{2}\right) + f(b)}{4} \)

(C) \( \int_{a}^{b} f(x) \, dx \)

(D) \( \frac{\int_{a}^{b} f(x) \, dx}{b-a} \)

Solution

The correct answer is (D).

The mean value of a function \( f(x) \) from \( a \) to \( b \) is given by

\[
\bar{f} = \frac{\text{Area under the curve from } a \text{ to } b}{\text{Width of the interval from } a \text{ to } b} = \frac{\int_{a}^{b} f(x) \, dx}{b-a}
\]
3. The exact value of \( \int_{0.2}^{2.2} xe^x \, dx \) is most nearly

(A) 7.8036
(B) 11.807
(C) 14.034
(D) 19.611

**Solution**

The correct answer is (B).

To solve this integral we must integrate by parts.

\[
\int u \, dv = uv - \int v \, du
\]

where

\[
u = x
\]
\[du = dx
\]

and

\[
v = e^x
\]

\[
\int_{0.2}^{2.2} xe^x \, dx = \int_{0.2}^{2.2} x \, (e^x) \, dx
\]

\[
= \left[ xe^x \right]_{0.2}^{2.2} - \int_{0.2}^{2.2} e^x \, dx
\]

\[
= \left[ xe^x - e^x \right]_{0.2}^{2.2}
\]

\[
= (2.2e^{2.2} - e^{2.2}) - (0.2e^{0.2} - e^{0.2})
\]

\[
= 10.83 - (-0.9771)
\]

\[
= 11.807
\]
4. \[ \int_{0.2}^{2} f(x)dx \] for

\[ f(x) = x, \quad 0 \leq x \leq 1.2 \]
\[ = x^2, \quad 1.2 < x \leq 2.4 \]

is most nearly

(A) 1.9800
(B) 2.6640
(C) 2.7907
(D) 4.7520

**Solution**

*The correct answer is (C).*

\[ \int_{0.2}^{2} f(x)dx = \int_{0.2}^{1.2} x dx + \int_{1.2}^{2} x^2 dx \]

\[ = \left[ \frac{1}{2} x^2 \right]_{0.2}^{1.2} + \left[ \frac{1}{3} x^3 \right]_{1.2}^{2} \]

\[ = \left( \frac{1.2^2 - 0.2^2}{2} \right) + \left( \frac{2^3 - 1.2^3}{3} \right) \]

\[ = 0.7 + 2.0907 \]

\[ = 2.7907 \]
5. The area of a circle of radius $a$ can be found by the following integral

(A) $\int_{0}^{a} (a^2 - x^2) \, dx$

(B) $\int_{0}^{2\pi} \sqrt{a^2 - x^2} \, dx$

(C) $4\int_{0}^{a} \sqrt{a^2 - x^2} \, dx$

(D) $\int_{0}^{a} \sqrt{a^2 - x^2} \, dx$

Solution
The correct answer is (C).

The equation for a circle of radius $a$ is

\[ y^2 + x^2 = a^2 \]

\[ y = \sqrt{a^2 - x^2} \]

To find the area of the shaded quarter circle shown, we can find the area under the curve of the integral of $y = \sqrt{a^2 - x^2}$ from 0 to $a$.

\[ A = \int_{0}^{a} \sqrt{a^2 - x^2} \, dx \]

Thus, the area of the full circle is four times the quarter circle

\[ A = 4\int_{0}^{a} \sqrt{a^2 - x^2} \, dx \]
6. Velocity distribution of a fluid flow through a pipe varies along the radius and is given by \( v(r) \). The flow rate through the pipe of radius \( a \) is given by

(A) \( \pi v(a)a^2 \)

(B) \( \frac{\pi v(0) + v(a)}{2}a^2 \)

(C) \( \int_{0}^{a} v(r)dr \)

(D) \( 2\pi \int_{0}^{a} v(r)rdr \)

Solution

The correct answer is (D).

The differential area is the area bound by the concentric circles and is given by

\[
dA = \pi (r + dr)^2 - \pi r^2 = \pi r^2 + \pi (dr)^2 + 2\pi r dr - \pi r^2 = 2\pi r dr + \pi (dr)^2
\]

Since \((dr)^2 \ll dr\)

we have \(dA \approx 2\pi r \, dr\)

The flow rate \(dQ\) through the differential element of the pipe is
\[ d\dot{Q} = (velocity) \times (differential\ area) \]
\[ = (v(r)) \times (2\pi dr) \]
\[ \dot{Q} = \int_0^a v(r) 2\pi r \, dr \]
\[ = 2\pi \int_0^a v(r) r \, dr \]