1. The two-segment trapezoidal rule of integration is exact for integrating at most _______ order polynomials.
   (A) first
   (B) second
   (C) third
   (D) fourth

Solution
The correct answer is (A).

The single segment trapezoidal rule is exact for at most a first order polynomial. The two segment trapezoidal rule is also exact only for the same order of polynomial, that is, a first order polynomial.
2. The value of $\int_{0.2}^{2.2} xe^x \, dx$ by the using one-segment trapezoidal rule is most nearly

(A) 11.672
(B) 11.807
(C) 20.099
(D) 24.119

Solution

The correct answer is (C).

$$\int_{a}^{b} f(x) \, dx \approx (b - a) \left[ \frac{f(a) + f(b)}{2} \right]$$

where

- $a = 0.2$
- $b = 0.2$
- $f(x) = xe^x$
- $f(0.2) = 0.2e^{0.2}$
  $$= 0.24428$$
- $f(2.2) = 2.2e^{2.2}$
  $$= 19.855$$

$$\int_{0.2}^{2.2} xe^x \, dx \approx (2.2 - 0.2) \left[ \frac{0.24428 + 19.855}{2} \right]$$

$$= 2 \times 10.050$$
$$= 20.099$$
3. The value of \( \int_{0.2}^{2.2} xe^x \, dx \) by using the three-segment trapezoidal rule is most nearly

(A) 11.672  
(B) 11.807  
(C) 12.811  
(D) 14.633

**Solution**

The correct answer is (C).

\[
\int_{a}^{b} f(x) \, dx \approx \frac{b-a}{2n} \left[ f(a) + 2 \left( \sum_{i=1}^{n-1} f(a + ih) \right) + f(b) \right]
\]

where

\[
a = 0.2 \\
b = 2.2 \\
n = 3 \\
h = \frac{b-a}{n} = \frac{2.2 - 0.2}{3} = 0.66667 \\
f(x) = xe^x
\]

Thus

\[
\int_{0.2}^{2.2} xe^x \, dx \approx \frac{2.2 - 0.2}{2 \times 3} \left[ f(0.2) + 2 \left( \sum_{i=1}^{3-1} f(0.2 + i \times 0.66667) \right) + f(2.2) \right]
\]

\[
= \frac{2}{6} \left[ f(0.2) + 2 \left( \sum_{i=1}^{2} f(0.2 + i \times 0.66667) \right) + f(2.2) \right]
\]

\[
= \frac{1}{3} \left[ f(0.2) + 2 f(0.86667) + 2 f(1.5333) + f(2.2) \right]
\]

where
\[ f(0.2) = 0.2e^{0.2} \]
\[ = 0.24428 \]
\[ f(0.86667) = 0.86667e^{0.86667} \]
\[ = 2.0618 \]
\[ f(1.5333) = 1.5333e^{1.5333} \]
\[ = 7.1048 \]
\[ f(2.2) = 2.2e^{2.2} \]
\[ = 19.855 \]

Hence
\[ \int_{0.2}^{2.2} xe^x \, dx \approx 0.33333 \left[ 0.24428 + 2 \times 2.0618 + 2 \times 7.1048 + 19.855 \right] \]
\[ = 0.33333 \left[ 38.433 \right] \]
\[ = 12.811 \]
4. The velocity of a body is given by

\[ v(t) = 2t, \quad 1 \leq t \leq 5 \]
\[ = 5t^2 + 3, \quad 5 < t \leq 14 \]

where \( t \) is given in seconds, and \( v \) is given in m/s. Use the two-segment trapezoidal rule to find the distance covered by the body from \( t = 2 \) to \( t = 9 \) seconds.

(A) 935.0 m
(B) 1039.7 m
(C) 1260.9 m
(D) 5048.9 m

**Solution**

The correct answer is (C).

\[
\int_a^b v(t) \, dt \approx \frac{b-a}{2n} \left[ v(a) + 2 \sum_{i=1}^{n-1} v(a + ih) \right] + v(b)
\]

where

\[ a = 2 \]
\[ b = 9 \]
\[ n = 2 \]
\[ h = \frac{b-a}{n} \]
\[ = \frac{9-2}{2} \]
\[ = 3.5 \]

\[ v(t) = 2t, \quad 1 \leq t \leq 5 \]
\[ = 5t^2 + 3, \quad 5 < t \leq 14 \]

Thus

\[
\int_2^9 v(t) \, dt \approx \frac{9-2}{2 \times 2} \left[ v(2) + 2 \left\{ \sum_{i=1}^{2-1} v(2 + i \times 3.5) \right\} + v(9) \right]
\]
\[ = \frac{7}{4} [v(2) + 2v(5.5) + v(9)] \]
\[ v(2) = 2 \times 2 \]
\[ = 4 \text{ m/s} \]
\[ v(5.5) = 5 \times 5.5^2 + 3 \]
\[ = 154.25 \text{ m/s} \]
\[ v(9) = 5 \times 9^2 + 3 \]
\[ = 408 \text{ m/s} \]
\[ \int_{t_1}^{t_2} v(t) dt \approx 1.75 \left[ 4 + 2 \times 154.25 + 408 \right] \\
= 1.75[720.5] \\
= 1260.9 \text{ m} \]
5. The shaded area shows a plot of land available for sale. Your best estimate of the area of the land is most nearly
   (A) 2500 m\(^2\)
   (B) 4775 m\(^2\)
   (C) 5250 m\(^2\)
   (D) 6000 m\(^2\)

Solution

The correct answer is (B).
\[ A = 60 \times 60 \]
\[ = 3600 \, \text{m}^2 \]
\[ B = 45 \times 15 \]
\[ = 675 \, \text{m}^2 \]
\[ C = 20 \times 25 \]
\[ = 500 \, \text{m}^2 \]
\[ \text{Area} \approx A + B + C \]
\[ = 3600 + 675 + 500 \]
\[ = 4775 \, \text{m}^2 \]
6. The following data of the velocity of a body is given as a function of time.

<table>
<thead>
<tr>
<th>Time (s)</th>
<th>0</th>
<th>15</th>
<th>18</th>
<th>22</th>
<th>24</th>
</tr>
</thead>
<tbody>
<tr>
<td>Velocity (m/s)</td>
<td>22</td>
<td>24</td>
<td>37</td>
<td>25</td>
<td>123</td>
</tr>
</tbody>
</table>

The distance in meters covered by the body from $t = 12$ s to $t = 18$ s calculated using the trapezoidal rule with unequal segments is

(A) 162.90  
(B) 166.00  
(C) 181.70  
(D) 436.50

**Solution**

The correct answer is (A).

Use the trapezoidal rule with unequal segments.

$$\int_{12}^{18} v(t) \, dt = \int_{12}^{15} v(t) \, dt + \int_{15}^{18} v(t) \, dt$$

$v(15) = 24 \text{ m/s}$  
$v(18) = 37 \text{ m/s}$

To find the value of the velocity at 12 s, we will use linear interpolation.

$v(t) = a_0 + a_1 t, \ 0 \leq t \leq 15$

At $t = 0$ s  
$22 = a_0 + a_1 0$

At $t = 15$ s  
$24 = a_0 + a_1 15$

which gives  
$a_0 = 22$  
$a_1 = 0.13333$

Hence,

$v(t) = 22 + 0.13333t, \ 0 \leq t \leq 15$

$v(12) = 22 + 0.13333 \times 12 = 23.600 \text{ m/s}$

$$\int_{12}^{18} v(t) \, dt \approx (15 - 12) \left[ \frac{v(12) + v(15)}{2} \right] + (18 - 15) \left[ \frac{v(15) + v(18)}{2} \right]$$

$$= (15 - 12) \left[ \frac{23.6 + 24}{2} \right] + (18 - 15) \left[ \frac{24 + 37}{2} \right]$$

$$= 162.90 \text{ m}$$