Multiple-Choice Test

Chapter 08.03
Runge-Kutta 2nd Order Method

1. To solve the ordinary differential equation

$$3\frac{dy}{dx} + xy^2 = \sin x,\ y(0) = 5$$

by the Runge-Kutta 2nd order method, you need to rewrite the equation as

(A) $\frac{dy}{dx} = \sin x - xy^2,\ y(0) = 5$

(B) $\frac{dy}{dx} = \frac{1}{3}\left(\sin x - xy^2\right),\ y(0) = 5$

(C) $\frac{dy}{dx} = \frac{1}{3}\left(-\cos x - \frac{xy^3}{3}\right),\ y(0) = 5$

(D) $\frac{dy}{dx} = \frac{1}{3}\sin x,\ y(0) = 5$

2. Given

$$3\frac{dy}{dx} + 5y^2 = \sin x,\ y(0.3) = 5$$

and using a step size of $h = 0.3$, the value of $y(0.9)$ using the Runge-Kutta 2nd order Heun method is most nearly

(A) –4297.4

(B) –4936.7

(C) -0.21336×10^{14}

(D) -0.24489×10^{14}

3. Given

$$3\frac{dy}{dx} + 5\sqrt{y} = e^{0.1x},\ y(0.3) = 5$$

and using a step size of $h = 0.3$, the best estimate of $\frac{dy}{dx}(0.9)$ using the Runge-Kutta 2nd order midpoint method most nearly is

(A) –2.2473

(B) –2.2543

(C) –2.6188

(D) –3.2045

4. The velocity (m/s) of a body is given as a function of time (seconds) by

$$v(t) = 200\ln(1+t) - t,\ t \geq 0$$

Using the Runge-Kutta 2nd order Ralston method with a step size of 5 seconds, the distance in meters traveled by the body from $t = 2$ to $t = 12$ seconds is estimated most nearly as

(A) 3904.9
(B) 3939.7
(C) 6556.3
(D) 39397

5. The Runge-Kutta 2nd order method can be derived by using the first three terms of the Taylor series of writing the value of y_{i+1} (that is the value of y at x_{i+1}) in terms of y_i (that is the value of y at x_i) and all the derivatives of y at x_i. If $h = x_{i+1} - x_i$, the explicit expression for y_{i+1} if the first three terms of the Taylor series are chosen for solving the ordinary differential equation

$$\frac{dy}{dx} + 5y = 3e^{-2x}, y(0) = 7$$

would be

(A) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + 5\frac{h^2}{2}$

(B) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-21e^{-2x_i} + 25y_i\right)\frac{h^2}{2}$

(C) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-6e^{-2x_i}\right)\frac{h^2}{2}$

(D) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-6e^{-2x_i} + 5\right)\frac{h^2}{2}$

6. A spherical ball is taken out of a furnace at 1200 K and is allowed to cool in air. You are given the following

radius of ball = 2 cm
specific heat of ball $= 420\ \text{J/kg} \cdot \text{K}$
density of ball $= 7800\ \text{kg/m}^3$
convection coefficient $= 350\ \text{J/s} \cdot \text{m}^2 \cdot \text{K}$
ambient temperature $= 300\ \text{K}$

The ordinary differential equation that is given for the temperature θ of the ball is

$$\frac{d\theta}{dt} = -2.20673 \times 10^{-13}\left(\theta^4 - 81 \times 10^8\right)$$

if only radiation is accounted for. The ordinary differential equation if convection is accounted for in addition to radiation is

(A) $\frac{d\theta}{dt} = -2.20673 \times 10^{-13}\left(\theta^4 - 81 \times 10^8\right) - 1.6026 \times 10^{-2}(\theta - 300)$

(B) $\frac{d\theta}{dt} = -2.20673 \times 10^{-13}\left(\theta^4 - 81 \times 10^8\right) - 4.3982 \times 10^{-2}(\theta - 300)$

(C) $\frac{d\theta}{dt} = -1.6026 \times 10^{-2}(\theta - 300)$

(D) $\frac{d\theta}{dt} = -4.3982 \times 10^{-2}(\theta - 300)$

For a complete solution, refer to the links at the end of the book.