Multiple-Choice Test

Chapter 08.04
Runge-Kutta 4th Order Method

1. To solve the ordinary differential equation

$$3\frac{dy}{dx} + xy^2 = \sin x, y(0) = 5,$$

by Runge-Kutta 4th order method, you need to rewrite the equation as

(A) $\frac{dy}{dx} = \sin x - xy^2, y(0) = 5$

(B) $\frac{dy}{dx} = \frac{1}{3}\left(\sin x - xy^2\right), y(0) = 5$

(C) $\frac{dy}{dx} = \frac{1}{3}\left(-\cos x - \frac{xy^3}{3}\right), y(0) = 5$

(D) $\frac{dy}{dx} = \frac{1}{3}\sin x, y(0) = 5$

2. Given $3\frac{dy}{dx} + 5y^2 = \sin x, y(0.3) = 5$ and using a step size of $h = 0.3$, the value of $y(0.9)$ using Runge-Kutta 4th order method is most nearly

(A) -0.25011×10^{40}
(B) –4297.4
(C) –1261.5
(D) 0.88498

3. Given $3\frac{dy}{dx} + y^2 = e^x, y(0.3) = 5$, and using a step size of $h = 0.3$, the best estimate of $\frac{dy}{dx}(0.9)$ Runge-Kutta 4th order method is most nearly

(A) -1.6604
(B) -1.1785
(C) -0.45831
(D) 2.7270

4. The velocity (m/s) of a parachutist is given as a function of time (seconds) by

$$v(t) = 55.8\tanh(0.17t),\ \ t \geq 0$$

Using Runge-Kutta 4th order method with a step size of 5 seconds, the distance in meters traveled by the body from $t = 2$ to $t = 12$ seconds is estimated most nearly as

(A) 341.43
(B) 428.97
(C) 429.05
(D) 703.50

5. Runge-Kutta method can be derived from using first three terms of Taylor series of writing the value of y_{i+1}, that is the value of y at x_{i+1}, in terms of y_i and all the derivatives of y at x_i. If $h = x_{i+1} - x_i$, the explicit expression for y_{i+1} if the first five terms of the Taylor series are chosen for the ordinary differential equation

$$\frac{dy}{dx} + 5y = 3e^{-2x}, y(0) = 7,$$

would be

(A) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \frac{5h^2}{2}$

(B) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-21e^{-2x_i} + 25y_i\right)\frac{h^2}{2} + \left(-483e^{-2x_i} + 625y_i\right)\frac{h^3}{6} + \left(-300909e^{-2x_i} + 390625y_i\right)\frac{h^4}{24}$

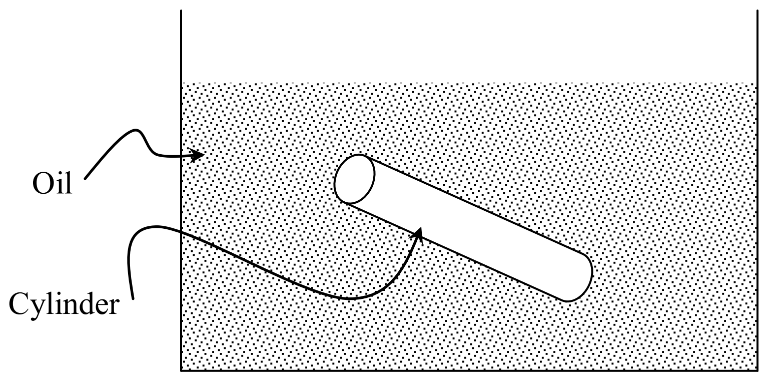
(C) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-6e^{-2x_i}\right)\frac{h^2}{2} + \left(12e^{-2x_i}\right)\frac{h^3}{6} + \left(-24e^{-2x_i}\right)\frac{h^4}{24}$

(D) $y_{i+1} = y_i + \left(3e^{-2x_i} - 5y_i\right)h + \left(-6e^{-2x_i} + 5\right)\frac{h^2}{2} + \left(12e^{-2x_i}\right)\frac{h^3}{6} + \left(-24e^{-2x_i}\right)\frac{h^4}{24}$

6. A hot solid cylinder is immersed in a cool oil bath as part of a quenching process. This process makes the temperature of the cylinder, θ_c, and the bath, θ_b, change with time. If the initial temperature of the bar and the oil bath is given as 600° C and 27°C, respectively, and

Length of cylinder = 30 cm
Radius of cylinder = 3 cm
Density of cylinder = 2700 kg/m^3
Specific heat of cylinder = 895 J/kg · K
Convection heat transfer coefficient = 100 $\text{W/m}^2 \cdot \text{K}$
Specific heat of oil = 1910 J/kg · K
Mass of oil = 2 kg

the coupled ordinary differential equation giving the heat transfer are given by



(A)
$$362.4\frac{d\theta_c}{dt} + \theta_c = \theta_b$$
$$675.5\frac{d\theta_b}{dt} + \theta_b = \theta$$

(B)
$$362.4\frac{d\theta_c}{dt} - \theta_c = \theta_b$$
$$675.5\frac{d\theta_b}{dt} - \theta_b = \theta_c$$

(C)
$$675.5\frac{d\theta_c}{dt} + \theta_c = \theta_b$$
$$362.4\frac{d\theta_b}{dt} + \theta_b = \theta_c$$

(D) $$675.5\frac{d\theta_c}{dt} - \theta_c = \theta_b$$

<u>For a complete solution, refer to the links at the end of the book.</u>