Multiple-Choice Test

Chapter 08.06
Shooting Method

1. The exact solution to the boundary value problem
   \[ \frac{d^2y}{dx^2} = 6x - 0.5x^2, \quad y(0) = 0, \quad y(12) = 0 \]
   for \( y(4) \) is
   (A) -234.66
   (B) 0.00
   (C) 16.000
   (D) 106.66

2. Given
   \[ \frac{d^2y}{dx^2} = 6x - 0.5x^2, \quad y(0) = 0, \quad y(12) = 0, \]
   the exact value of \( \frac{dy}{dx}(0) \) is
   (A) -72.0
   (B) 0.00
   (C) 36.0
   (D) 72.0

3. Given
   \[ \frac{d^2y}{dx^2} = 6x - 0.5x^2, \quad y(0) = 0, \quad y(12) = 0, \]
   If one was using shooting method with Euler’s method with a step size of \( h = 4 \), and an assumed value of \( \frac{dy}{dx}(0) = 20 \), then the estimated value of \( y(12) \) in the first iteration most nearly is
   (A) 60.00
   (B) 496.0
   (C) 1088
   (D) 1102
4. The transverse deflection, \( u \) of a cable of length, \( L \), fixed at both ends, is given as a solution to
\[
\frac{d^2u}{dx^2} = \frac{T}{R} + \frac{q x (x - L)}{2R}
\]
where
- \( T = \) tension in cable
- \( R = \) flexural stiffness
- \( q = \) distributed transverse load

Given are \( L = 50'' \), \( T = 2000 \) lbs, \( q = 75 \) lbs/in, \( R = 75 \times 10^6 \) lbs\cdot\text{in}^2.\) The shooting method is used with Euler’s method assuming a step size of \( h = 12.5'' \). Initial slope guesses at \( x = 0 \) of \( \frac{du}{dx} = 0.003 \) and \( \frac{du}{dx} = 0.004 \) are used in order, and then refined for the next iteration using linear interpolation after the value of \( u(L) \) is found. The deflection in inches at the center of the cable found during the second iteration is most nearly
- (A) 0.075000
- (B) 0.10000
- (C) -0.061291
- (D) 0.00048828
5. The radial displacement, \( u \) is a pressurized hollow thick cylinder (inner radius=5\( " \), outer radius=8\( " \)) is given at different radial locations.

<table>
<thead>
<tr>
<th>Radius (in)</th>
<th>Radial Displacement (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.0</td>
<td>0.0038731</td>
</tr>
<tr>
<td>5.6</td>
<td>0.0036165</td>
</tr>
<tr>
<td>6.2</td>
<td>0.0034222</td>
</tr>
<tr>
<td>6.8</td>
<td>0.0032743</td>
</tr>
<tr>
<td>7.4</td>
<td>0.0031618</td>
</tr>
<tr>
<td>8.0</td>
<td>0.0030769</td>
</tr>
</tbody>
</table>

The maximum normal stress, in psi, on the cylinder is given by

\[
\sigma_{\text{max}} = 3.2967 \times 10^6 \left( \frac{u(5)}{5} + 0.3 \frac{du}{dr}(5) \right)
\]

The maximum stress, in psi, with second order accuracy for the approximation of the first derivative most nearly is

(A) 2079.3
(B) 2104.5
(C) 2130.7
(D) 2182.0
6. For a simply supported beam (at \( x = 0 \) and \( x = L \)) with a uniform load \( q \), the vertical deflection \( v(x) \) is described by the boundary value ordinary differential equation as

\[
\frac{d^2v}{dx^2} = \frac{qx(x-L)}{2EI}, \quad 0 \leq x \leq L
\]

where

\[
E = \text{Young’s modulus of elasticity of beam},
\]

\[
I = \text{second moment of area}.
\]

This ordinary differential equations is based on assuming that \( \frac{dv}{dx} \) is small. If \( \frac{dv}{dx} \) is not small, then the ordinary differential equation is

(A) \[
\frac{d^2v}{dx^2} = \frac{qx(x-L)}{2EI} \sqrt{1 + \left(\frac{dv}{dx}\right)^2}
\]

(B) \[
\frac{d^2v}{dx^2} = \frac{qx(x-L)}{2EI} \left(1 + \left(\frac{dv}{dx}\right)^2\right)^{\frac{3}{2}}
\]

(C) \[
\frac{d^2v}{dx^2} = \frac{qx(x-L)}{2EI} \sqrt{1 + \left(\frac{dv}{dx}\right)^2}
\]

(D) \[
\frac{d^2v}{dx^2} = \frac{qx(x-L)}{2EI} \left(1 + \frac{dv}{dx}\right)^2
\]

For a complete solution, refer to the links at the end of the book.