Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Chapter 09.01 Golden Section Search Method
Optimization

COMPLETE SOLUTION SET

1. Which of the following statements is incorrect regarding the Equal Interval Search and Golden Section Search methods?
   (A) Both methods require an initial boundary region to start the search
   (B) The number of iterations in both methods are affected by the size of ε
   (C) Everything else being equal, the Golden Section Search method should find an optimal solution faster.
   (D) Everything else being equal, the Equal Interval Search method should find an optimal solution faster.

Solution
The correct answer is (D).

Due to the manner in which the intermediate points in the Golden Section Search method are determined, the initial search region size is reduced much quicker than the Equal Interval Search method and hence converges to an optimal solution faster.

2. Which of the following parameters is <u>not</u> required to use the Golden Section Search method for optimization?
   (A) The lower bound for the search region
   (B) The upper bound for the search region
   (C) The golden ratio
   (D) The function to be optimized

Solution
The correct answer is (C).

The Golden Section Search method is an optimization algorithm that requires search boundaries (lower and upper) and a one-dimensional function to be optimized. The Golden Ratio is simply the ratio of the distance between the intermediary points to the search boundary.

3. When applying the Golden Section Search method to a function $f(x)$ to find its maximum, the $f(x_1) > f(x_2)$ condition holds true for the intermediate points x_1 and x_2. Which of the following statements is <u>incorrect</u>?

(A) The new search region is determined by $[x_2, x_u]$

(B) The Intermediate point x_1 stays as one of the intermediate points

(C) The upper bound x_u stays the same

(D) The new search region is determined by $[x_l, x_1]$

Solution

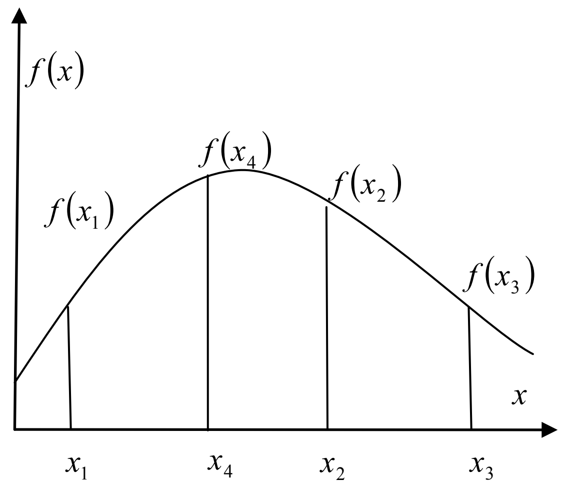
The correct answer is (D).

If $f(x_1) > f(x_2)$, then the new x_l, x_1, x_2 and x_u are determined as follows:

$$x_l = x_2$$
$$x_2 = x_1$$
$$x_u = x_u$$
$$x_1 = x_l + \frac{\sqrt{5}-1}{2}(x_u - x_l)$$

Therefore, the statement "The new search region is determined by $[x_l, x_1]$" is incorrect.

4. In the graph below, the lower and upper boundary of the search is given by x_1 and x_3 respectively. If x_4 and x_2 are the initial intermediary points, which of the following statement is false?



(A) The distance between x_2 and x_1 is equal to the distance between x_4 and x_3

(B) The distance between x_4 and x_2 is approximately 0.618 times the distance between x_2 and x_1

(C) The distance between x_4 and x_1 is approximately 0.618 times the distance between x_4 and x_3

(D) The distance between x_4 and x_1 is equal to the distance between x_2 and x_3

Solution

The correct answer is (B).

Referring to Figure 6 in the chapter (also shown below) , we can see that choices A, C and D are true based on the Golden Ratio, however no such assertion can be made about choice B.

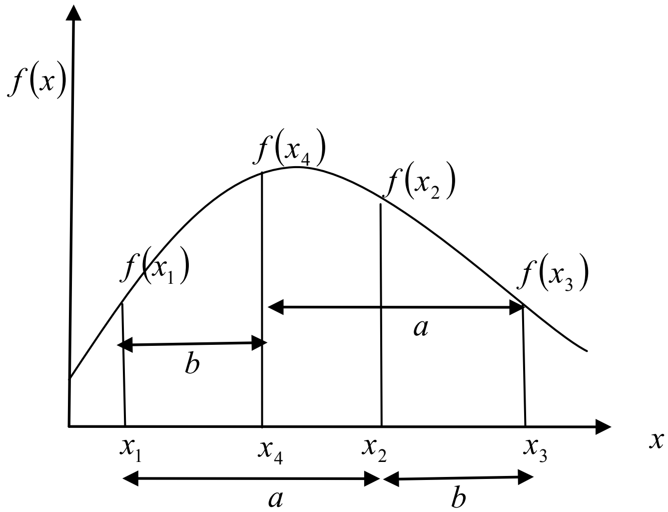


Figure 6.

5. Using the Golden Section Search method, find two numbers whose sum is 90 and their product is as large as possible. Conduct two iterations on the interval [0,90].

   (A) 30 and 60
   (B) 45 and 45
   (C) 38 and 52
   (D) 20 and 70

Solution
The correct answer is (C).

To model this problem we must recognize that the two numbers x and y are related to each other as

$$x + y = 90$$

and the function to be maximized is

$$f(x, y) = xy .$$

We can model the problem as a one-dimensional optimization problem by substituting the value of y in terms of x as

$$f(x) = x(90 - x) = 90x - x^2$$

Iteration 1: Using [0, 90] as the search boundaries

$$\begin{aligned} x_1 &= x_l + \frac{\sqrt{5}-1}{2}(x_u - x_l) \\ &= 0 + \frac{\sqrt{5}-1}{2}(90) \\ &= 55.6231 \end{aligned}$$

$$\begin{aligned} x_2 &= x_u - \frac{\sqrt{5}-1}{2}(x_u - x_l) \\ &= 90 - \frac{\sqrt{5}-1}{2}(90) \\ &= 34.3769 \end{aligned}$$

The function is evaluated at the intermediate points as $f(55.6231) = 1912.1506$ and $f(34.3769) = 1912.1506$. This is an interesting case where $f(x_1) = f(x_2)$, therefore we can proceed either way. Assume we eliminate the region to the right of x_1 and update the upper boundary point as $x_u = x_1$. The lower boundary point x_l remains unchanged. The first intermediate point x_1 is updated to assume the value of x_2 and finally the second intermediate point x_2 is re-calculated as follows:

$$
\begin{aligned}
x_2 &= x_u - \frac{\sqrt{5}-1}{2}(x_u - x_l) \\
&= 55.623 - \frac{\sqrt{5}-1}{2}(55.623 - 0) \\
&= 21.246
\end{aligned}
$$

Iteration 2: The process is repeated in the second iteration with the new values for the boundary and intermediate points calculated in the previous iteration as shown below.

$$
\begin{aligned}
x_l &= 0.00000 \\
x_u &= 55.623 \\
x_1 &= 34.377 \\
x_2 &= 21.246
\end{aligned}
$$

Again the function is evaluated at the intermediate points as $f(34.377) = 1912.2$ and $f(21.246) = 1460.8$. Since $f(x_1) > f(x_2)$, we eliminate the region to the left of x_2 and update the lower boundary point as $x_l = x_2$. The upper boundary point x_u remains unchanged.

At the end of the second iteration the solution is for x is

$$
\begin{aligned}
\frac{x_u + x_l}{2} &= \frac{21.246 + 55.623}{2} \\
&= 38.435
\end{aligned}
$$

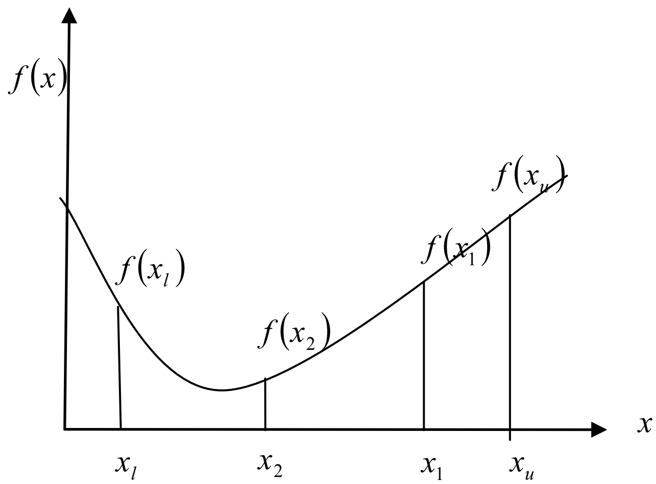
The value for y is calculated by subtracting it from 90 as

$$
y = 90 - 38.435 = 51.565
$$

The iterations will continue until the stopping criterion is met when $x = 44.99$. Summary results of all the iterations are shown below assuming $\varepsilon<0.05$. The theoretical optimal solution for this problem is when the numbers are equal to each other at 45.

Iteration	xl	xu	x1	x2	f(x1)	f(x2)	ε
1	0.00000	90.000	55.623	34.377	1912.2	1912.2	90.000
2	0.00000	55.623	34.377	21.246	1912.2	1460.8	55.623
3	21.246	55.623	42.492	34.377	2018.7	1912.2	34.377
4	34.377	55.623	47.508	42.492	2018.7	2018.7	21.246
5	34.377	47.508	42.492	39.392	2018.7	1993.6	13.131
6	39.392	47.508	44.408	42.492	2024.6	2018.7	8.1153
7	42.492	47.508	45.592	44.408	2024.6	2025.0	5.0155
8	42.492	45.592	44.408	43.676	2024.6	2025.0	3.0998
9	43.676	45.592	44.860	44.408	2025.0	2025.0	1.9158
10	44.408	45.592	45.140	44.860	2025.0	2025.0	1.1840
11	44.408	45.140	44.860	44.688	2025.0	2024.9	0.73176
12	44.688	45.140	44.967	44.860	2025.0	2025.0	0.45225
13	44.860	45.140	45.033	44.967	2025.0	2025.0	0.27951
14	44.860	45.033	44.967	44.926	2025.0	2025.0	0.17274
15	44.926	45.033	44.992	44.967	2025.0	2025.0	0.10676
16	44.967	45.033	45.008	44.992	2025.0	2025.0	0.065982
17	44.967	45.008	44.992	44.983	2025.0	2025.0	0.040779

6. Consider the problem of finding the minimum of the function shown below. Given the intermediate points in the drawing, what would be the search region in the next iteration?



(A) $[x_2, x_u]$
(B) $[x_1, x_u]$
(C) $[x_l, x_1]$
(D) $[x_l, x_2]$

Solution
The correct answer is (C).

As seen in the drawing the optimal solution is between $[x_l, x_2]$. Either the region to the right of x_1 or the region to the left of x_2 will be eliminated. Due to location of the optimal solution we eliminate $[x_1, x_u]$ leaving the new search region as $[x_l, x_1]$.