Multiple-Choice Test

Chapter 10.03
Elliptic Partial Differential Equations

1. In a general second order linear partial differential equation with two independent variables,

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D=0$$

where A, B, C are functions of x and y, and D is a function of x, y, $\frac{\partial u}{\partial x},\frac{\partial u}{\partial y}$,

then the PDE is elliptic if

(A) $B^2-4AC<0$
(B) $B^2-4AC>0$
(C) $B^2-4AC=0$
(D) $B^2-4AC\neq 0$

2. The region in which the following equation

$$x^3\frac{\partial^2 u}{\partial x^2}+27\frac{\partial^2 u}{\partial y^2}+3\frac{\partial^2 u}{\partial x\partial y}+5u=0$$

acts as an elliptic equation is

(A) $x>\left(\frac{1}{12}\right)^{1/3}$
(B) $x<\left(\frac{1}{12}\right)^{1/3}$
(C) for all values of x
(D) $x=\left(\frac{1}{12}\right)^{1/3}$

3. The finite difference approximation of $\frac{\partial^2 u}{\partial x^2}$ in the elliptic equation

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

at (x, y) can be approximated as

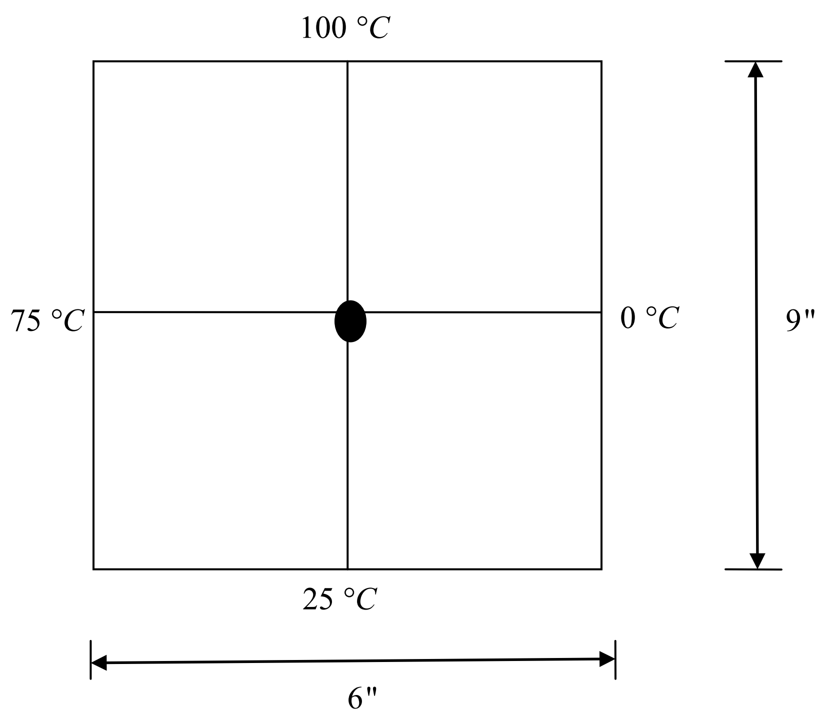
(A) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x+\Delta x, y) - 2u(x, y) + u(x-\Delta x, y)}{(\Delta x)^2}$

(B) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x+\Delta x, y) - u(x, y) + u(x-\Delta x, y)}{(\Delta x)^2}$

(C) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x, y+\Delta y) - 2u(x, y) + u(x, y-\Delta y)}{(\Delta x)^2}$

(D) $\frac{\partial^2 u}{\partial x^2} \cong \frac{u(x+\Delta x, y) - u(x-\Delta x, y)}{2\Delta x}$

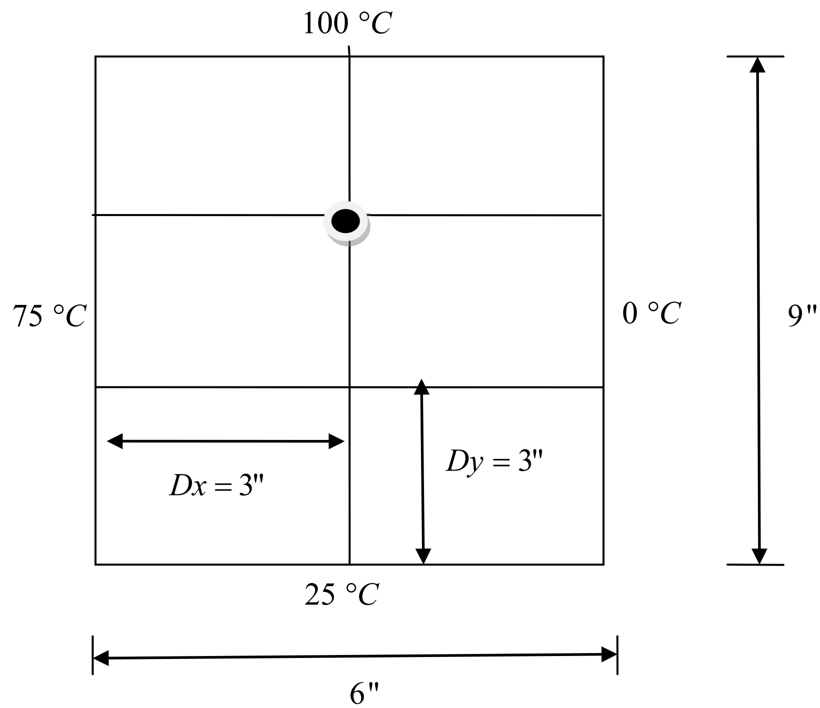
4. Find the temperature at the interior node given in the following figure using the direct method



(A) 45.19 °C
(B) 48.64 °C
(C) 50.00 °C

(D) 56.79 °*C*

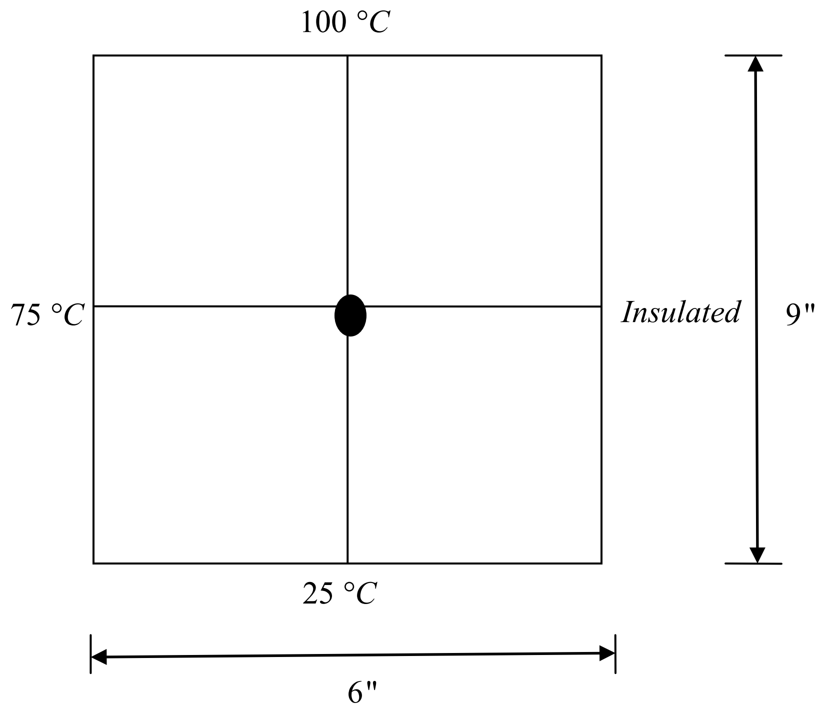
5. Find the temperature at the interior node given in the following figure



Using the Lieberman method and relaxation factor of 1.2, the temperature at $x = 3, y = 6$ estimated after 2 iterations is (use the temperature of interior nodes as $50\ °C$ for the initial guess)

(A) 52.36 °*C*
(B) 53.57 °*C*
(C) 56.20 °*C*
(D) 58.64 °*C*

6. Find the steady-state temperature at the interior node as given in the following figure



(A) 53.57 °C
(B) 66.40 °C
(C) 68.20 °C
(D) 69.59 °C