Multiple-Choice Test

Chapter 10.02
Parabolic Partial Differential Equations

1. In a general second order linear partial differential equation with two independent variables

\[ A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D = 0 \]

where \( A, B, C \) are functions of \( x \) and \( y \), and \( D \) is a function of \( x, y, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \),
then the partial differential equation is parabolic if

(A) \( B^2 - 4AC < 0 \)
(B) \( B^2 - 4AC > 0 \)
(C) \( B^2 - 4AC = 0 \)
(D) \( B^2 - 4AC \neq 0 \)

2. The region in which the following partial differential equation

\[ x^3 \frac{\partial^2 u}{\partial x^2} + 27 \frac{\partial^2 u}{\partial y^2} + 3 \frac{\partial^2 u}{\partial x \partial y} + 5u = 0 \]

acts as parabolic equation is

(A) \( x > \left( \frac{1}{12} \right)^{1/3} \)
(B) \( x < \left( \frac{1}{12} \right)^{1/3} \)
(C) for all values of \( x \)
(D) \( x = \left( \frac{1}{12} \right)^{1/3} \)

3. The partial differential equation of the temperature in a long thin rod is given by

\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]
If \( \alpha = 0.8 \text{cm}^2/\text{s} \), the initial temperature of rod is 40° C, and the rod is divided into three equal segments, the temperature at node 1 (using \( \Delta t = 0.1 \text{s} \)) by using an explicit solution at \( t = 0.2 \text{sec} \) is

(A) 40.7134°C
(B) 40.6882°C
(C) 40.7033°C
(D) 40.6956°C

4. The partial differential equation of the temperature in a long thin rod is given by

\[
\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}
\]

If \( \alpha = 0.8 \text{cm}^2/\text{s} \), the initial temperature of rod is 40° C, and the rod is divided into three equal segments, the temperature at node 1 (using \( \Delta t = 0.1 \text{s} \)) by using an implicit solution for \( t = 0.2 \text{sec} \) is

(A) 40.7134°C
(B) 40.6882°C
(C) 40.7033°C
(D) 40.6956°C
5. The partial differential equation of the temperature in a long thin rod is given by
\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]

If \( \alpha = 0.8 \text{ cm}^2 / \text{s} \), the initial temperature of rod is \( 40^\circ C \), and the rod is divided into three equal segments, the temperature at node 1 (using \( \Delta t = 0.1 \text{s} \)) by using a Crank-Nicolson solution for \( t = 0.2 \text{ sec} \) is
(A) 40.7134 \(^\circ\)C  
(B) 40.6882 \(^\circ\)C  
(C) 40.7033 \(^\circ\)C  
(D) 40.6956 \(^\circ\)C

6. The partial differential equation of the temperature in a long thin rod is given by
\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]

If \( \alpha = 0.8 \text{ cm}^2 / \text{s} \), the initial temperature of rod is \( 40^\circ C \), and the rod is divided into three equal segments, the temperature at node 1 (using \( \Delta t = 0.1 \text{s} \)) by using an explicit solution at \( t = 0.2 \text{ sec} \) is
(For node 0, \( k \frac{\partial T}{\partial x} = h(T_a - T_0) \)), where \( k = 9 W/(m \cdot ^\circ C) \), \( h = 20 W/m^2 \), \( T_a = 25^\circ C \), and \( T_0 = \) (the temperature of rod at node 0)

(A) \( 41.6478^\circ C \)
(B) \( 38.4356^\circ C \)
(C) \( 39.9983^\circ C \)
(D) \( 37.5798^\circ C \)