Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Parabolic Partial Differential Equations
Partial Differential Equations
COMPLETE SOLUTION SET

1. In a general second order linear partial differential equation with two independent variables

$$A\frac{\partial^2 u}{\partial x^2}+B\frac{\partial^2 u}{\partial x\partial y}+C\frac{\partial^2 u}{\partial y^2}+D=0$$

where A, B, C are functions of x and y, and D is a function of x, y, $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$, then the partial differential equation is parabolic if

(A) $B^2-4AC<0$
(B) $B^2-4AC>0$
(C) $B^2-4AC=0$
(D) $B^2-4AC\neq 0$

Solution
The correct answer is (C).

A general second order linear partial differential equation is parabolic if $B^2-4AC=0$.

2. The region in which the following partial differential equation

$$x^3 \frac{\partial^2 u}{\partial x^2} + 27\frac{\partial^2 u}{\partial y^2} + 3\frac{\partial^2 u}{\partial x \partial y} + 5u = 0$$

acts as parabolic equation is

(A) $x > \left(\frac{1}{12}\right)^{1/3}$

(B) $x < \left(\frac{1}{12}\right)^{1/3}$

(C) for all values of x

(D) $x = \left(\frac{1}{12}\right)^{1/3}$

Solution

The correct answer is (D).

A general partial differential equation with two independent variables is of the form

$$A\frac{\partial^2 u}{\partial x^2} + B\frac{\partial^2 u}{\partial xy} + C\frac{\partial^2 u}{\partial y^2} + D = 0$$

where A, B, and C are functions of x and y and is a function of x, y, u and $\frac{\partial u}{\partial x}, \frac{\partial u}{\partial y}$.

For this equation to be parabolic, $B^2 - 4AC = 0$.

In the above question, $A = x^3, B = 3, C = 27$, giving

$$B^2 - 4AC = 0$$

$$(3)^2 - 4(x^3)(27) = 0$$

$$9 - 108x^3 = 0$$

$$108x^3 = 9$$

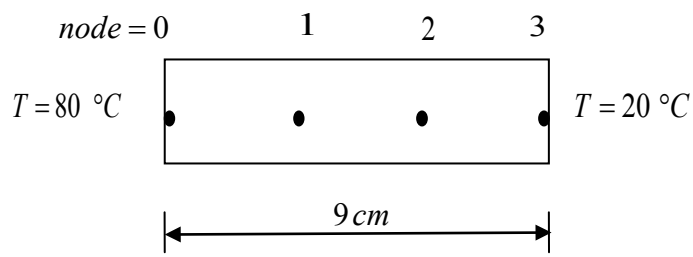
$$x^3 = \frac{9}{108}$$

$$x^3 = \frac{1}{12}$$

$$x = \left(\frac{1}{12}\right)^{1/3}$$

3. The partial differential equation of the temperature in a long thin rod is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



If $\alpha = 0.8\,cm^2/s$, the initial temperature of rod is $40°C$, and the rod is divided into three equal segments, the temperature at node 1 (using $\Delta t = 0.1\,s$) by using an <u>explicit</u> solution at $t = 0.2\,\text{sec}$ is

(A) $40.7134\,^0C$
(B) $40.6882\,^0C$
(C) $40.7033\,^0C$
(D) $40.6956\,^0C$

Solution

The correct answer is (C).

Given

$$\alpha = 0.8\,cm^2/s$$
$$\Delta t = 0.1\,s$$
$$t = 0.2\,\text{sec}\,s$$
$$L = 9\,\text{cms}$$

Number of divisions of the rod, $n = 3$

$$\Delta x = \frac{L}{n}$$
$$= \frac{9}{3}$$
$$= 3$$

Number of time steps $= \dfrac{t_{final} - t_{initial}}{\Delta t}$

$$= \frac{0.2 - 0}{0.1}$$
$$= 2$$

$$\lambda = \alpha \frac{\Delta t}{(\Delta x)^2}$$

$$= 0.8\frac{0.1}{(3)^2}$$
$$= 0.0089$$

The boundary conditions

$$\left.\begin{aligned} T_0^j &= 80°C \\ T_3^j &= 20°C \end{aligned}\right\} \text{ for all } j = 0,1 \qquad \text{(E3.1)}$$

The initial temperature of the rod is $40°C$, that is, all the temperatures of the nodes inside the rod are at $40°C$ when time, $t = 0\,\text{sec}$ except for the boundary nodes as given by Equation (E3.1). This could be represented as

$$T_i^0 = 20°C, \text{ for all } i = 1,2. \qquad \text{(E3.2)}$$

<u>Initial temperature at the nodes inside the rod (when *t*=0 sec)</u>

$$T_0^0 = 80°C \quad \text{from Equation (E3.1)}$$

$$\left.\begin{aligned} T_1^0 &= 40°C \\ T_2^0 &= 40°C \end{aligned}\right\} \quad \text{from Equation (E3.2)}$$

$$T_3^0 = 20°C \quad \text{from Equation (E3.1)}$$

<u>Temperature at the nodes inside the rod when *t*=0.1 sec</u>

Setting $j = 0$ and $i = 0,1,2,3$ in Equation (7) (from Chapter 10.02) gives the temperature of the nodes inside the rod when time, $t = 0.1\,\text{sec}$.

$$T_0^1 = 80°C \quad \text{Boundary Condition (E3.1)}$$

$$\begin{aligned} T_1^1 &= T_1^0 + \lambda\left(T_2^0 - 2T_1^0 + T_0^0\right) \\ &= 40 + 0.0089(40 - 2(40) + 80) \\ &= 40 + 0.0089(40) \\ &= 40 + 0.3556 \\ &= 40.3556°C \end{aligned}$$

$$\begin{aligned} T_2^1 &= T_2^0 + \lambda\left(T_3^0 - 2T_2^0 + T_1^0\right) \\ &= 40 + 0.0089(20 - 2(40) + 40) \\ &= 40 + 0.0089(-20) \\ &= 40 - 0.1778 \\ &= 39.8222°C \end{aligned}$$

$$T_3^1 = 20°C \quad \text{Boundary Condition (E3.1)}$$

Temperature at the nodes inside the rod when t=0.2 sec

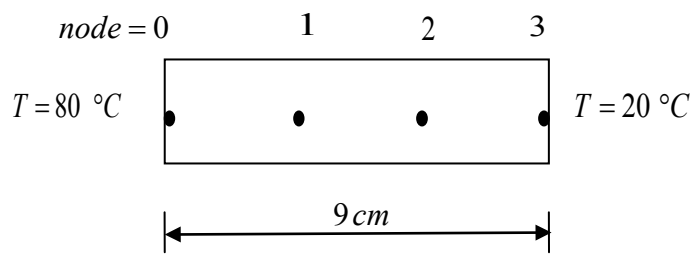
Setting $j = 1$ and $i = 0,1,2,3$ in Equation (6) (from Chapter 11.02) gives the temperature of the nodes inside the rod when time, $t = 0.2\,\text{sec}$

$$T_0^2 = 80°C \quad \text{Boundary Condition (E3.1)}$$

$$\begin{aligned}
T_1^2 &= T_1^1 + \lambda\left(T_2^1 - 2T_1^1 + T_0^1\right) \\
&= 40.3556 + 0.0089\left(39.8222 - 2(40.3556) + 80\right) \\
&= 40.3556 + 0.0089\left(39.1110\right) \\
&= 40.3556 + 0.3477 \\
&= 40.7033°C
\end{aligned}$$

4. The partial differential equation of the temperature in a long thin rod is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



If $\alpha = 0.8\,cm^2/s$, the initial temperature of rod is $40°C$, and the rod is divided into three equal segments, the temperature at node 1 (using $\Delta t = 0.1\,s$) by using an <u>implicit</u> solution for $t = 0.2\,\text{sec}$ is

(A) $40.7134\,^0C$
(B) $40.6882\,^0C$
(C) $40.7033\,^0C$
(D) $40.6956\,^0C$

Solution

The correct answer is (B).

Given

$$\alpha = 0.8\,cm^2/s$$
$$\Delta t = 0.1\,s$$
$$t = 0.2\,\text{sec}\,s$$
$$L = 9\,\text{cms}$$

Number of divisions of the rod, $n = 3$

$$\Delta x = \frac{L}{n}$$
$$= \frac{9}{3}$$
$$= 3$$

Number of time steps $= \dfrac{t_{final} - t_{initial}}{\Delta t}$

$$= \frac{0.2 - 0}{0.1}$$
$$= 2$$

$$\lambda = \alpha \frac{\Delta t}{(\Delta x)^2}$$

$$= 0.8\frac{0.1}{(3)^2}$$
$$= 0.0089$$

The boundary conditions

$$\left.\begin{aligned} T_0^j &= 80°C \\ T_3^j &= 20°C \end{aligned}\right\} \text{ for all } j = 0,1 \qquad \text{(E4.1)}$$

The initial temperature of the rod is $40°C$, that is, all the temperatures of the nodes inside the rod are at $40°C$ when time, $t = 0\,\text{sec}$ except for the boundary nodes as given by Equation (E3.1). This could be represented as

$$T_i^0 = 20°C, \text{ for all } i = 1,2. \qquad \text{(E4.2)}$$

<u>Initial temperature at the nodes inside the rod (when *t*=0 sec)</u>

$$T_0^0 = 80°C \quad \text{from Equation (E4.1)}$$

$$\left.\begin{aligned} T_1^0 &= 40°C \\ T_2^0 &= 40°C \end{aligned}\right\} \quad \text{from Equation (E4.2)}$$

$$T_3^0 = 20°C \quad \text{from Equation (E4.1)}$$

<u>Temperature at the nodes inside the rod when *t*=0.1 sec</u>

$$\left.\begin{aligned} T_0^1 &= 80°C \\ T_3^1 &= 20°C \end{aligned}\right\} \text{Boundary Condition (E4.1)}$$

For all the interior nodes, putting $j = 0$ and $i = 1,2$ in Equation (11) (from Chapter 10.02) gives the following equations

<u>*i*=1</u>

$$-\lambda T_0^1 + (1+2\lambda)T_1^1 - \lambda T_2^1 = T_1^0$$
$$(-0.0089 \times 80) + (1 + 2 \times 0.0089)T_1^1 - (0.0089T_2^1) = 40$$
$$-0.7111 + 1.0178T_1^1 - 0.0089T_2^1 = 40$$
$$1.0178T_1^1 - 0.0089T_2^1 = 40.7111 \qquad \text{(E4.3)}$$

<u>*i*=2</u>

$$-\lambda T_1^1 + (1+2\lambda)T_2^1 - \lambda T_3^1 = T_2^0$$
$$-0.0089T_1^1 + 1.0178T_2^1 - (0.0089 \times 20) = 40$$
$$-0.0089T_1^1 + 1.0178T_2^1 - 0.1778 = 40$$
$$-0.0089T_1^1 + 1.0178T_2^1 = 40.1778 \qquad \text{(E4.4)}$$

The simultaneous linear equations (E4.3) – (E4.4) can be written in matrix form as

$$\begin{bmatrix} 1.0178 & -0.0089 \\ -0.0089 & 1.0178 \end{bmatrix} \begin{bmatrix} T_1^1 \\ T_2^1 \end{bmatrix} = \begin{bmatrix} 40.7111 \\ 40.1778 \end{bmatrix}$$

The above coefficient matrix is tri-diagonal. Special algorithms such as Thomas' algorithm can be used to solve simultaneous linear equation with tri-diagonal coefficient matrices. The solution is given by

$$\begin{bmatrix} T_1^1 \\ T_2^1 \end{bmatrix} = \begin{bmatrix} 40.3478 \\ 39.8284 \end{bmatrix}$$

<u>Temperature at the nodes inside the rod when *t*=0.2 sec</u>

$$\left.\begin{aligned} T_0^2 &= 80°C \\ T_3^2 &= 20°C \end{aligned}\right\} \text{Boundary Condition (E4.1)}$$

For all the interior nodes, putting $j = 1$ and $i = 1, 2$ in Equation (11) (from Chapter 10.02) gives the following equations

<u>*i*=1</u>

$$-\lambda T_0^2 + (1+2\lambda)T_1^2 - \lambda T_2^2 = T_1^1$$

$$(-0.0089 \times 80) + (1 + 2 \times 0.0089)T_1^2 - 0.0089T_2^2 = 40.3478$$

$$-0.7111 + 1.0178T_1^2 - 0.0089T_2^2 = 40.3478$$

$$1.0178T_1^2 - 0.0089T_2^2 = 41.0590 \qquad \text{(E4.5)}$$

<u>*i*=2</u>

$$-\lambda T_1^2 + (1+2\lambda)T_2^2 - \lambda T_3^2 = T_2^1$$

$$-0.0089T_1^2 + 1.0178T_2^2 - (0.0089 \times 20) = 39.8284$$

$$-0.0089T_1^2 + 1.0178T_2^2 - 0.1778 = 39.8284$$

$$-0.0089T_1^2 + 1.0178T_2^2 = 40.0061 \qquad \text{(E4.6)}$$

The simultaneous linear equations (E4.5) – (E4.6) can be written in matrix form as

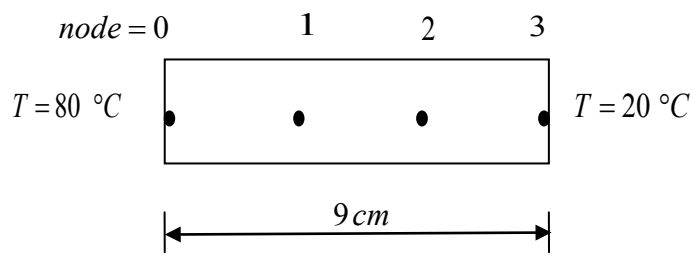
$$\begin{bmatrix} 1.0178 & -0.0089 \\ -0.0089 & 1.0178 \end{bmatrix} \begin{bmatrix} T_1^2 \\ T_2^2 \end{bmatrix} = \begin{bmatrix} 41.0590 \\ 40.0061 \end{bmatrix}$$

The above coefficient matrix is tri-diagonal. Special algorithms such as Thomas' algorithm can be used to solve simultaneous linear equation with tri-diagonal coefficient matrices. The solution is given by

$$\begin{bmatrix} T_1^2 \\ T_2^2 \end{bmatrix} = \begin{bmatrix} 40.6882 \\ 39.6627 \end{bmatrix}$$

5. The partial differential equation of the temperature in a long thin rod is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



If $\alpha = 0.8\,cm^2/s$, the initial temperature of rod is $40°C$, and the rod is divided into three equal segments, the temperature at node 1 (using $\Delta t = 0.1\,s$) by using a <u>Crank-Nicolson</u> solution for $t = 0.2\,\text{sec}$ is

(A) $40.7134\,^0C$
(B) $40.6882\,^0C$
(C) $40.7033\,^0C$
(D) $40.6956\,^0C$

Solution

The correct answer is (D).

Given

$$\alpha = 0.8\,cm^2/s$$
$$\Delta t = 0.1\,s$$
$$t = 0.2\,\text{sec}\,s$$
$$L = 9\,\text{cms}$$

Number of divisions of the rod, $n = 3$

$$\Delta x = \frac{L}{n}$$
$$= \frac{9}{3}$$
$$= 3$$

Number of time steps $= \dfrac{t_{final} - t_{initial}}{\Delta t}$

$$= \frac{0.2 - 0}{0.1}$$
$$= 2$$

$$\lambda = \alpha \frac{\Delta t}{(\Delta x)^2}$$

$$= 0.8\frac{0.1}{(3)^2}$$
$$= 0.0089$$

The boundary conditions

$$\left.\begin{aligned} T_0^j &= 80°C \\ T_3^j &= 20°C \end{aligned}\right\} \text{ for all } j = 0,1 \qquad \text{(E5.1)}$$

The initial temperature of the rod is $40°C$, that is, all the temperatures of the nodes inside the rod are at $40°C$ when time, $t = 0\,\text{sec}$ except for the boundary nodes as given by Equation (E5.1). This could be represented as

$$T_i^0 = 20°C, \text{ for all } i = 1,2. \qquad \text{(E5.2)}$$

<u>Initial temperature at the nodes inside the rod (when *t*=0 sec)</u>

$$T_0^0 = 80°C \quad \text{from Equation (E4.1)}$$

$$\left.\begin{aligned} T_1^0 &= 40°C \\ T_2^0 &= 40°C \end{aligned}\right\} \quad \text{from Equation (E4.2)}$$

$$T_3^0 = 20°C \quad \text{from Equation (E4.1)}$$

<u>Temperature at the nodes inside the rod when *t*=0.1 sec</u>

$$\left.\begin{aligned} T_0^1 &= 80°C \\ T_3^1 &= 20°C \end{aligned}\right\} \text{Boundary Condition (E4.1)}$$

For all the interior nodes, putting $j = 0$ and $i = 1,2$ in Equation (15) (from Chapter 10.02) gives the following equations

<u>*i*=1</u>

$$-\lambda T_0^1 + 2(1+\lambda)T_1^1 - \lambda T_2^1 = \lambda T_0^0 + 2(1-\lambda)T_1^0 + \lambda T_2^0$$
$$(-0.0089 \times 80) + 2(1+0.0089)T_1^1 - 0.0089T_2^1 = (0.0089)80 + 2(1-0.0089)40 + (0.0089)40$$
$$-0.7111 + 2.0178T_1^1 - 0.0089T_2^1 = 0.7111 + 79.2889 + 0.3556$$
$$2.0178T_1^1 - 0.0089T_2^1 = 81.0667 \qquad \text{(E5.3)}$$

<u>*i*=2</u>

$$-\lambda T_1^1 + 2(1+\lambda)T_2^1 - \lambda T_3^1 = \lambda T_1^0 + 2(1-\lambda)T_2^0 + \lambda T_3^0$$
$$-0.0089T_1^1 + 2(1+0.0089)T_2^1 - (0.0089 \times 20) = (0.0089)40 + 2(1-0.0089)40 + (0.0089)20$$
$$-0.0089T_1^1 + 2.0178T_2^1 - 0.1778 = 0.3556 + 79.2889 + 0.1778$$
$$-0.0089T_1^1 + 2.0178T_2^1 = 80.0000 \qquad \text{(E5.4)}$$

The simultaneous linear equations (E5.3) – (E4.4) can be written in matrix form as

$$\begin{bmatrix} 2.0178 & -0.0089 \\ -0.0089 & 2.0178 \end{bmatrix} \begin{bmatrix} T_1^1 \\ T_2^1 \end{bmatrix} = \begin{bmatrix} 81.0667 \\ 80.0000 \end{bmatrix}$$

The above coefficient matrix is tri-diagonal. Special algorithms such as Thomas' algorithm can be used to solve simultaneous linear equation with tri-diagonal coefficient matrices. The solution is given by

$$\begin{bmatrix} T_1^1 \\ T_2^1 \end{bmatrix} = \begin{bmatrix} 40.3517 \\ 39.8253 \end{bmatrix}$$

<u>Temperature at the nodes inside the rod when *t*=0.2 sec</u>

$$\left.\begin{aligned} T_0^2 &= 80°C \\ T_3^2 &= 20°C \end{aligned}\right\} \text{Boundary Condition (E4.1)}$$

For all the interior nodes, putting $j = 1$ and $i = 1, 2$ in Equation (11) (from Chapter 10.02) gives the following equations

<u>*i*=1</u>

$$-\lambda T_0^2 + 2(1+\lambda)T_1^2 - \lambda T_2^2 = \lambda T_0^1 + 2(1-\lambda)T_1^1 + \lambda T_2^1$$

$$(-0.0089 \times 80) + 2(1+0.0089)T_1^2 - 0.0089T_2^2 =$$

$$(0.0089)80 + 2(1-0.00899)40.3517 + (0.0089)39.8253$$

$$-0.7111 + 2.0178T_1^2 - 0.0089T_2^2 = 0.7111 + 79.9860 + 0.3540$$

$$2.0178T_1^2 - 0.0089T_2^2 = 81.7622 \qquad \text{(E5.5)}$$

<u>*i*=2</u>

$$-\lambda T_1^2 + 2(1+\lambda)T_2^2 - \lambda T_3^2 = \lambda T_1^1 + 2(1-\lambda)T_2^1 + \lambda T_3^1$$

$$-0.0089T_1^2 + 2(1+0.0089)T_2^2 - (0.0089 \times 20) =$$

$$(0.0089)40.3517 + 2(1-0.0089)39.8253 + (0.0089)20$$

$$-0.0089T_1^2 + 2.0178T_2^2 - 0.1780 = 0.3587 + 78.9426 + 0.1780$$

$$-0.0089T_1^2 + 2.0178T_2^2 = 79.6569 \qquad \text{(E5.6)}$$

The simultaneous linear equations (E5.5) – (E5.6) can be written in matrix form as

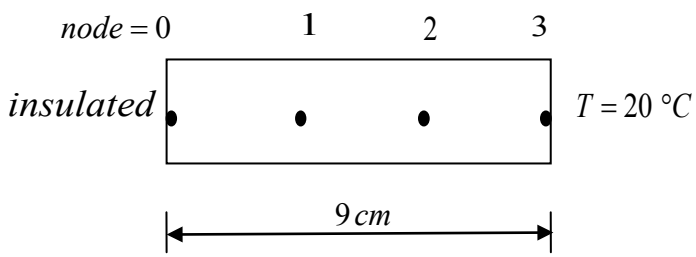
$$\begin{bmatrix} 2.0178 & -0.0089 \\ -0.0089 & 2.0178 \end{bmatrix} \begin{bmatrix} T_1^2 \\ T_2^2 \end{bmatrix} = \begin{bmatrix} 81.7622 \\ 79.6569 \end{bmatrix}$$

The above coefficient matrix is tri-diagonal. Special algorithms such as Thomas' algorithm can be used to solve simultaneous linear equation with tri-diagonal coefficient matrices. The solution is given by

$$\begin{bmatrix} T_1^2 \\ T_2^2 \end{bmatrix} = \begin{bmatrix} 40.6956 \\ 39.6568 \end{bmatrix}$$

6. The partial differential equation of the temperature in a long thin rod is given by

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}$$



If $\alpha = 0.8\,cm^2/s$, the initial temperature of rod is $40°C$, and the rod is divided into three equal segments, the temperature at node 1 (using $\Delta t = 0.1\,s$) by using an <u>explicit</u> solution at $t = 0.2\,\text{sec}$ is. (For node 0, $-k\frac{\partial T}{\partial x} = h(T_a - T_0)$), where $k = 9\,W/(m°C)$, $h = 20\,W/m^2$, $T_a = 25°C$, $T_0 =$ the temperature of rod at node 0

(A) $41.6478\,^0C$
(B) $38.4356\,^0C$
(C) $39.9983\,^0C$
(D) $37.5798\,^0C$

Solution

The correct answer is (C).

Given

$$\alpha = 0.8\,cm^2/s$$
$$\Delta t = 0.1\,s$$
$$t = 0.2\,\text{sec}\,s$$
$$L = 9\,\text{cms}$$
$$T_a = 25°C$$
$$h = 20\,W/m^2$$
$$k = 9\,W/(m°C)$$

Number of divisions of the rod, $n = 3$

$$\Delta x = \frac{L}{n}$$
$$= \frac{9}{3}$$
$$= 3cm$$

Number of time steps= $\dfrac{t_{final} - t_{initial}}{\Delta t}$

$$= \frac{0.2 - 0}{0.1}$$

$$= 2$$

$$\lambda = \alpha \frac{\Delta t}{(\Delta x)^2}$$

$$= 0.8 \frac{0.1}{(3)^2}$$

$$= 0.0089$$

The boundary conditions

$$\left.\begin{aligned} &T_3^j = 20°C \\ &-k \left.\frac{\partial T}{\partial x}\right|_{i,j} = h(T_a - T_{i,j}), \quad i = 0 \end{aligned}\right\} \text{ for all } j = 0,1 \tag{E6.1}$$

Approximating the derivative by the central divided difference,

$$\left.\frac{\partial T}{\partial x}\right|_{i,j} \cong \frac{T_{i+1}^j - T_{i-1}^j}{2(\Delta x)} \tag{E6.2}$$

Substituting Equation (E6.2) in the derivative boundary condition of Equation (E6.3) we have

$$-k \left[\frac{T_{i+1}^j - T_{i-1}^j}{2(\Delta x)} \right] = h(T_a - T_i^j), \; i = 0$$

Rewriting the above equation we have

$$T_{i-1}^j = 2 \frac{h}{k} \Delta x (T_a - T_i^j) + T_{i+1}^j, \quad i = 0 \tag{E6.4}$$

From the explicit method (Equation 7 from Chapter 10.02) we have

$$T_i^{j+1} = T_i^j + \lambda \left(T_{i+1}^j - 2T_i^j + T_{i-1}^j \right), i \neq 0 \tag{E6.5}$$

Substituting Equation (E6.4) in Equation (E6.5) we get

$$T_i^{j+1} = T_i^j + \lambda \left(T_{i+1}^j - 2T_i^j + 2 \frac{h}{k} \Delta x (T_a - T_i^j) + T_{i+1}^j \right)$$

$$= T_i^j + 2\lambda \left(T_{i+1}^j - T_i^j + \frac{h}{k} \Delta x (T_a - T_i^j) \right), \; i = 0 \tag{E6.8}$$

Equation (E6.8) is used for the node 0 (insulated edge)

The initial temperature of the rod is $40°C$, that is, all the temperatures of the nodes inside the rod are at $40°C$ when time, $t = 0\,\text{sec}$ except for the boundary nodes as given by Equation (E6.1). This could be represented as

$$T_i^0 = 20°C, \text{ for all } i = 1,2,3\,. \tag{E6.9}$$

<u>Initial temperature at the nodes inside the rod (when *t*=0 sec)</u>

$$\left.\begin{aligned} T_0^0 &= 40°C \\ T_1^0 &= 40°C \\ T_2^0 &= 40°C \end{aligned}\right\} \quad \text{from Equation (E6.9)}$$

$T_3^0 = 20°C$ from Equation (E6.1)

<u>Temperature at the nodes inside the rod when *t*=0.1 sec</u>

Setting $j = 0$ and $i = 0,1,2,3$ in Equation (7) (from Chapter 10.02) gives the temperature of the nodes inside the rod when time, $t = 0.1\text{sec}$.

<u>*i*=0</u>

$$T_0^1 = T_0^0 + 2\lambda\left(T_1^0 - T_0^0 + \frac{h}{k}\Delta x(T_a - T_0^0)\right) \qquad \text{(From Equation E6.8)}$$

$$= 40 + 2\times 0.0089\left(40 - 40 + \frac{20}{9}0.03(25-40)\right) \quad \text{note: } \Delta x = 0.03m$$

$$= 40 + 0.017778(-1)$$

$$= 40 - 0.017778$$

$$= 39.9822\ °C$$

<u>*i*=1</u>

$$T_1^1 = T_1^0 + \lambda\left(T_2^0 - 2T_1^0 + T_0^0\right)$$

$$= 40 + 0.0089(40 - 2(40) + 40)$$

$$= 40 + 0.0089(0)$$

$$= 40 + 0$$

$$= 40.0000°C$$

<u>*i*=2</u>

$$T_2^1 = T_2^0 + \lambda\left(T_3^0 - 2T_2^0 + T_1^0\right)$$

$$= 40 + 0.0089(20 - 2(40) + 40)$$

$$= 40 + 0.0089(-20)$$

$$= 40 - 0.1778$$

$$= 39.8222°C$$

<u>*i*=3</u>

$T_3^1 = 20°C$ Boundary Condition (E3.1)

Temperature at the nodes inside the rod when t=0.2 sec

Setting $j = 1$ and $i = 0,1,2,3$ in Equation (7) (from Chapter 10.02) gives the temperature of the nodes inside the rod when time, $t = 0.2\,\text{sec}$

i=0

$$T_0^2 = T_0^1 + 2\lambda\left(T_1^1 - T_0^1 + \frac{h}{k}\Delta x(T_a - T_0^1)\right) \qquad \text{(From Equation E6.8)}$$

$$= 39.9822 + 2\times 0.0089\left(40 - 39.9822 + \frac{20}{9}0.03(25 - 39.9822)\right) \text{ note}: \Delta x = 0.03m$$

$$= 39.9822 + 0.017778(-0.98101)$$

$$= 39.9822 - 0.01744$$

$$= 39.9648\ °C$$

i=1

$$T_1^2 = T_1^1 + \lambda\left(T_2^1 - 2T_1^1 + T_0^1\right)$$

$$= 40.0000 + 0.0089(39.8222 - 2(40.0000) + 39.9822)$$

$$= 40.0000 + 0.0089(-0.1956)$$

$$= 40.0000 - 0.001741$$

$$= 39.9983\ °C$$