Multiple-Choice Test
Parabolic Partial Differential Equations
Partial Differential Equations
COMPLETE SOLUTION SET

1. In a general second order linear partial differential equation with two independent variables

\[ A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial x \partial y} + C \frac{\partial^2 u}{\partial y^2} + D = 0 \]

where \( A, B, C \) are functions of \( x \) and \( y \), and \( D \) is a function of \( x, y, \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \), then the partial differential equation is parabolic if

(A) \( B^2 - 4AC < 0 \)
(B) \( B^2 - 4AC > 0 \)
(C) \( B^2 - 4AC = 0 \)
(D) \( B^2 - 4AC \neq 0 \)

Solution
The correct answer is (C).

A general second order linear partial differential equation is parabolic if \( B^2 - 4AC = 0 \).
2. The region in which the following partial differential equation acts as parabolic equation is

\[ x^3 \frac{\partial^2 u}{\partial x^2} + 27 \frac{\partial^2 u}{\partial y^2} + 3 \frac{\partial^2 u}{\partial x \partial y} + 5u = 0 \]

is

(A) \( x > \left( \frac{1}{12} \right)^{1/3} \)

(B) \( x < \left( \frac{1}{12} \right)^{1/3} \)

(C) for all values of \( x \)

(D) \( x = \left( \frac{1}{12} \right)^{1/3} \)

Solution

The correct answer is (D).

A general partial differential equation with two independent variables is of the form

\[ A \frac{\partial^2 u}{\partial x^2} + B \frac{\partial^2 u}{\partial xy} + C \frac{\partial^2 u}{\partial y^2} + D = 0 \]

where \( A, B, \) and \( C \) are functions of \( x \) and \( y \) and is a function of \( x, y, u \) and \( \frac{\partial u}{\partial x}, \frac{\partial u}{\partial y} \).

For this equation to be parabolic, \( B^2 - 4AC = 0 \).

In the above question, \( A = x^3, B = 3, C = 27 \), giving

\[ B^2 - 4AC = 0 \]

\[ (3)^2 - 4(x^3)(27) = 0 \]

\[ 9 - 108x^3 = 0 \]

\[ 9 = 108x^3 \]

\[ x^3 = \frac{1}{12} \]

\[ x = \left( \frac{1}{12} \right)^{1/3} \]
3. The partial differential equation of the temperature in a long thin rod is given by

\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]

If \( \alpha = 0.8 \text{cm}^2/\text{s} \), the initial temperature of rod is \( 40^\circ C \), and the rod is divided into three equal segments, the temperature at node 1 (using \( \Delta t = 0.1 \text{s} \)) by using an explicit solution at \( t = 0.2 \text{sec} \) is

(A) 40.7134°C
(B) 40.6882°C
(C) 40.7033°C
(D) 40.6956°C

**Solution**

The correct answer is (C).

Given

\( \alpha = 0.8 \text{cm}^2/\text{s} \)
\( \Delta t = 0.1 \text{s} \)
\( t = 0.2 \text{sec} \)
\( L = 9 \text{cms} \)

Number of divisions of the rod, \( n = 3 \)

\( \Delta x = \frac{L}{n} \)
\( = \frac{9}{3} \)
\( = 3 \)

Number of time steps=\( \frac{t_{\text{final}} - t_{\text{initial}}}{\Delta t} \)
\( = \frac{0.2 - 0}{0.1} \)
\( = 2 \)

\( \lambda = \alpha \frac{\Delta t}{(\Delta x)^2} \)
The boundary conditions
\[
\begin{align*}
T_0^j &= 80^\circ C \\
T_3^j &= 20^\circ C
\end{align*}
\] for all \( j = 0,1 \) \( \text{ (E3.1) } \)

The initial temperature of the rod is \( 40^\circ C \), that is, all the temperatures of the nodes inside the rod are at \( 40^\circ C \) when time, \( t = 0 \text{ sec} \) except for the boundary nodes as given by Equation \( \text{ (E3.1) } \). This could be represented as
\[
T_i^0 = 20^\circ C , \text{ for all } i = 1,2 . \text{ (E3.2) }
\]

Initial temperature at the nodes inside the rod (when \( t = 0 \text{ sec} \))
\[
\begin{align*}
T_0^0 &= 80^\circ C \quad \text{ from Equation (E3.1)} \\
T_1^0 &= 40^\circ C \quad \text{ from Equation (E3.2)} \\
T_2^0 &= 40^\circ C \\
T_3^0 &= 20^\circ C \quad \text{ from Equation (E3.1)}
\end{align*}
\]

Temperature at the nodes inside the rod when \( t = 0.1 \text{ sec} \)
Setting \( j = 0 \) and \( i = 0,1,2,3 \) in Equation (7) (from Chapter 10.02) gives the temperature of the nodes inside the rod when time, \( t = 0.1 \text{ sec} \)
\[
T_0 = 80^\circ C \quad \text{ Boundary Condition (E3.1)}
\]
\[
T_1 = T_1^0 + \lambda \left( T_2^0 - 2T_1^0 + T_0^0 \right)
\]
\[
= 40 + 0.0089 \left( (40 - 2(40) + 80) \right)
\]
\[
= 40 + 0.0089 \times 40
\]
\[
= 40 + 0.3556
\]
\[
= 40.3556^\circ C
\]

\[
T_2 = T_2^0 + \lambda \left( T_3^0 - 2T_2^0 + T_1^0 \right)
\]
\[
= 40 + 0.0089 \left( (20 - 2(40) + 40) \right)
\]
\[
= 40 + 0.0089 \times (-20)
\]
\[
= 40 - 0.1778
\]
\[
= 39.8222^\circ C
\]

\[
T_3 = 20^\circ C \quad \text{ Boundary Condition (E3.1)}
\]
Temperature at the nodes inside the rod when $t=0.2$ sec

Setting $j=1$ and $i=0,1,2,3$ in Equation (6) (from Chapter 11.02) gives the temperature of the nodes inside the rod when time, $t=0.2$ sec

$T_0^2 = 80^\circ C$  

Boundary Condition (E3.1)

\[
T_i^2 = T_i^1 + \lambda (T_2^1 - 2T_i^1 + T_0^1)
\]

\[
= 40.3556 + 0.0089(39.8222 - 2(40.3556) + 80)
\]

\[
= 40.3556 + 0.0089(39.1110)
\]

\[
= 40.3556 + 0.3477
\]

\[
= 40.7033^\circ C
\]
4. The partial differential equation of the temperature in a long thin rod is given by
\[
\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2}
\]

If \( \alpha = 0.8 \text{cm}^2 / \text{s} \), the initial temperature of rod is \( 40^\circ \text{C} \), and the rod is divided into three equal segments, the temperature at node 1 (using \( \Delta t = 0.1 \text{s} \)) by using an implicit solution for \( t = 0.2 \text{sec} \) is

(A) 40.7134 \( ^\circ \text{C} \)
(B) 40.6882 \( ^\circ \text{C} \)
(C) 40.7033 \( ^\circ \text{C} \)
(D) 40.6956 \( ^\circ \text{C} \)

**Solution**

The correct answer is (B).

Given
\( \alpha = 0.8 \text{cm}^2 / \text{s} \)
\( \Delta t = 0.1 \text{s} \)
\( t = 0.2 \text{sec} \)
\( L = 9 \text{cms} \)

Number of divisions of the rod, \( n = 3 \)
\[
\Delta x = \frac{L}{n} = \frac{9}{3} = 3
\]

Number of time steps
\[
= \frac{t_{\text{final}} - t_{\text{initial}}}{\Delta t} = \frac{0.2 - 0}{0.1} = 2
\]

\[
\lambda = \alpha \frac{\Delta t}{(\Delta x)^2}
\]
The boundary conditions

\[
\begin{align*}
T_0^j &= 80^\circ C \quad \text{for all } j = 0,1, \quad (E4.1) \\
T_3^j &= 20^\circ C 
\end{align*}
\]

The initial temperature of the rod is 40°C, that is, all the temperatures of the nodes inside the rod are at 40°C when time, \( t = 0 \) sec except for the boundary nodes as given by Equation (E3.1). This could be represented as

\[
T_i^0 = 20^\circ C, \quad \text{for all } i = 1,2. \quad (E4.2)
\]

Initial temperature at the nodes inside the rod (when \( t=0 \) sec)

\[
\begin{align*}
T_0^0 &= 80^\circ C \quad \text{from Equation (E4.1)} \\
T_1^0 &= 40^\circ C \quad \text{from Equation (E4.2)} \\
T_2^0 &= 40^\circ C \\
T_3^0 &= 20^\circ C \quad \text{from Equation (E4.1)}
\end{align*}
\]

Temperature at the nodes inside the rod when \( t=0.1 \) sec

\[
\begin{align*}
T_0^1 &= 80^\circ C \quad \text{Boundary Condition (E4.1)} \\
T_3^1 &= 20^\circ C
\end{align*}
\]

For all the interior nodes, putting \( j = 0 \) and \( i = 1,2 \) in Equation (11) (from Chapter 10.02) gives the following equations

\[i=1\]

\[
\begin{align*}
-\lambda T_0^1 + (1 + 2\lambda)T_1^1 - \lambda T_2^1 &= T_0^0 \\
-0.0089 \times 80 + (1 + 2 \times 0.0089)T_1^1 - (0.0089 T_2^1) &= 40 \\
-0.7111 + 1.0178 T_1^1 - 0.0089 T_2^1 &= 40 \\
1.0178 T_1^1 - 0.0089 T_2^1 &= 40.7111 
\end{align*}
\]

\[i=2\]

\[
\begin{align*}
-\lambda T_1^1 + (1 + 2\lambda)T_2^1 - \lambda T_3^1 &= T_2^0 \\
-0.0089 T_1^1 + 1.0178 T_2^1 - (0.0089 \times 20) &= 40 \\
-0.0089 T_1^1 + 1.0178 T_2^1 - 0.1778 &= 40 \\
-0.0089 T_1^1 + 1.0178 T_2^1 &= 40.1778 
\end{align*}
\]

The simultaneous linear equations (E4.3) – (E4.4) can be written in matrix form as

\[
\begin{bmatrix}
1.0178 & -0.0089 \\
-0.0089 & 1.0178
\end{bmatrix}
\begin{bmatrix}
T_1^1 \\
T_2^1
\end{bmatrix}
= 
\begin{bmatrix}
40.7111 \\
40.1778
\end{bmatrix}
\]

The above coefficient matrix is tri-diagonal. Special algorithms such as Thomas’ algorithm can be used to solve simultaneous linear equation with tri-diagonal coefficient matrices. The solution is given by
Temperature at the nodes inside the rod when $t=0.2$ sec

\[
\begin{align*}
T_0^2 &= 80\,^\circ C, \\
T_3^2 &= 20\,^\circ C
\end{align*}
\]

Boundary Condition (E4.1)

For all the interior nodes, putting $j = 1$ and $i = 1, 2$ in Equation (11) (from Chapter 10.02) gives the following equations

\begin{align*}
\text{(i)} & = 1 \\
-\lambda T_0^2 + (1 + 2\lambda)T_1^2 - \lambda T_2^2 &= T_1^1 \\
(-0.0089 \times 80) + (1 + 2 \times 0.0089)T_1^2 - 0.0089T_2^2 &= 40.3478 \\
-0.7111 + 1.0178T_1^2 - 0.0089T_2^2 &= 40.3478 \\
1.0178T_1^2 - 0.0089T_2^2 &= 41.0590 \\
\text{(E4.5)} \\

\text{(i)} & = 2 \\
-\lambda T_1^2 + (1 + 2\lambda)T_2^2 - \lambda T_3^2 &= T_2^1 \\
-0.0089T_1^2 + 1.0178T_2^2 - (0.0089 \times 20) &= 39.8284 \\
-0.0089T_1^2 + 1.0178T_2^2 - 0.1778 &= 39.8284 \\
-0.0089T_1^2 + 1.0178T_2^2 &= 40.0061 \\
\text{(E4.6)}
\end{align*}

The simultaneous linear equations (E4.5) – (E4.6) can be written in matrix form as

\[
\begin{bmatrix}
1.0178 & -0.0089 \\
-0.0089 & 1.0178
\end{bmatrix}
\begin{bmatrix}
T_1^2 \\
T_2^2
\end{bmatrix}
= 
\begin{bmatrix}
41.0590 \\
40.0061
\end{bmatrix}
\]

The above coefficient matrix is tri-diagonal. Special algorithms such as Thomas’ algorithm can be used to solve simultaneous linear equation with tri-diagonal coefficient matrices. The solution is given by

\[
\begin{bmatrix}
T_1^2 \\
T_2^2
\end{bmatrix}
= 
\begin{bmatrix}
40.6882 \\
39.6627
\end{bmatrix}
\]
5. The partial differential equation of the temperature in a long thin rod is given by
\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]

If \( \alpha = 0.8 \text{ cm}^2 / \text{s} \), the initial temperature of rod is 40°C, and the rod is divided into three equal segments, the temperature at node 1 (using \( \Delta t = 0.1 \text{s} \)) by using a Crank-Nicolson solution for \( t = 0.2 \text{ sec} \) is

(A) 40.7134°C
(B) 40.6882°C
(C) 40.7033°C
(D) 40.6956°C

Solution
The correct answer is (D).

Given
\( \alpha = 0.8 \text{ cm}^2 / \text{s} \)
\( \Delta t = 0.1 \text{s} \)
\( t = 0.2 \text{ sec} \)
\( L = 9 \text{ cm} \)

Number of divisions of the rod, \( n = 3 \)
\[ \Delta x = \frac{L}{n} = \frac{9}{3} = 3 \]

Number of time steps
\[ \lambda = \alpha \frac{\Delta t}{(\Delta x)^2} \]
\[
\frac{0.8}{(3)^2} = 0.0089
\]

The boundary conditions
\[
\begin{align*}
T_0^j &= 80^\circ\text{C} \\
T_3^j &= 20^\circ\text{C}
\end{align*}
\]
\quad \text{for all } j = 0,1
\quad \text{(E5.1)}

The initial temperature of the rod is 40\(^\circ\text{C}\), that is, all the temperatures of the nodes inside the rod are at 40\(^\circ\text{C}\) when time, \(t = 0\) sec except for the boundary nodes as given by Equation (E5.1). This could be represented as
\[
T_i^0 = 20^\circ\text{C}, \text{ for all } i = 1,2. \quad \text{(E5.2)}
\]

Initial temperature at the nodes inside the rod (when \(t=0\) sec)
\[
\begin{align*}
T_0^0 &= 80^\circ\text{C} & \text{from Equation (E4.1)} \\
T_1^0 &= 40^\circ\text{C} & \text{from Equation (E4.2)} \\
T_2^0 &= 40^\circ\text{C} & \text{from Equation (E4.1)} \\
T_3^0 &= 20^\circ\text{C} & \text{from Equation (E4.1)}
\end{align*}
\]

Temperature at the nodes inside the rod when \(t=0.1\) sec
\[
\begin{align*}
T_0^1 &= 80^\circ\text{C} & \text{Boundary Condition (E4.1)} \\
T_3^1 &= 20^\circ\text{C}
\end{align*}
\]
For all the interior nodes, putting \(j = 0\) and \(i = 1,2\) in Equation (15) (from Chapter 10.02) gives the following equations
\[
i = 1
\begin{align*}
-\lambda T_0^1 + 2(1+\lambda)T_1^1 - \lambda T_2^1 &= \lambda T_0^0 + 2(1-\lambda)T_1^0 + \lambda T_2^0 \\
(-0.0089 \times 80) + 2(1+0.0089)T_1^1 - 0.0089T_2^1 &= (0.0089)80 + 2(1-0.0089)40 + (0.0089)40 \\
-0.7111 + 2.0178T_1^1 - 0.0089T_2^1 &= 0.7111 + 79.2889 + 0.3556 \\
2.0178T_1^1 - 0.0089T_2^1 &= 81.0667 \quad \text{(E5.3)}
\end{align*}
\]
\[
i = 2
\begin{align*}
-\lambda T_1^1 + 2(1+\lambda)T_2^1 - \lambda T_3^1 &= \lambda T_1^0 + 2(1-\lambda)T_2^0 + \lambda T_3^0 \\
-0.0089T_1^1 + 2(1+0.0089)T_2^1 - (0.0089 \times 20) &= (0.0089)40 + 2(1-0.0089)40 + (0.0089)20 \\
-0.0089T_1^1 + 2.0178T_2^1 - 0.1778 &= 0.3556 + 79.2889 + 0.1778 \\
-0.0089T_1^1 + 2.0178T_2^1 &= 80.0000 \quad \text{(E5.4)}
\end{align*}
\]

The simultaneous linear equations (E5.3) – (E4.4) can be written in matrix form as
\[
\begin{bmatrix}
2.0178 & -0.0089 \\
-0.0089 & 2.0178
\end{bmatrix}
\begin{bmatrix}
T_1^1 \\
T_2^1
\end{bmatrix} =
\begin{bmatrix}
81.0667 \\
80.0000
\end{bmatrix}
\]

The above coefficient matrix is tri-diagonal. Special algorithms such as Thomas’ algorithm can be used to solve simultaneous linear equation with tri-diagonal coefficient matrices. The solution is given by
\[
\begin{bmatrix}
T_1^1 \\
T_2^1
\end{bmatrix} = \begin{bmatrix}
40.3517 \\
39.8253
\end{bmatrix}
\]

Temperature at the nodes inside the rod when \(t=0.2\) sec

\[
\begin{align*}
T_0^2 &= 80^\circ C \\
T_3^2 &= 20^\circ C
\end{align*}
\]

Boundary Condition (E4.1)

For all the interior nodes, putting \(j=1\) and \(i=1,2\) in Equation (11) (from Chapter 10.02) gives the following equations

\(i=1\)

\[-\lambda T_0^2 + 2(1 + \lambda)T_1^2 - \lambda T_2^2 = \lambda T_0^1 + 2(1 - \lambda)T_1^1 + \lambda T_2^1\]

\((-0.0089 \times 80) + 2(1 + 0.0089)T_1^2 - 0.0089T_2^2 =
\]

\[(0.0089)80 + 2(1 - 0.0089)40.3517 + (0.0089)39.8253\]

\[-0.7111 + 2.0178T_1^2 - 0.0089T_2^2 = 0.7111 + 79.9860 + 0.3540\]

\[2.0178T_1^2 - 0.0089T_2^2 = 81.7622\]  \hspace{1cm} (E5.5)

\(i=2\)

\[-\lambda T_1^2 + 2(1 + \lambda)T_2^2 - \lambda T_3^2 = \lambda T_1^1 + 2(1 - \lambda)T_2^1 + \lambda T_3^1\]

\[-0.0089T_1^2 + 2(1 + 0.0089)T_2^2 - (0.0089 \times 20) =
\]

\[(0.0089)40.3517 + 2(1 - 0.0089)39.8253 + (0.0089)20\]

\[-0.0089T_1^2 + 2.0178T_2^2 - 0.1780 = 0.3587 + 78.9426 + 0.1780\]

\[-0.0089T_1^2 + 2.0178T_2^2 = 79.6569\]  \hspace{1cm} (E5.6)

The simultaneous linear equations (E5.5) – (E5.6) can be written in matrix form as

\[
\begin{bmatrix}
2.0178 & -0.0089 \\
-0.0089 & 2.0178
\end{bmatrix}
\begin{bmatrix}
T_1^2 \\
T_2^2
\end{bmatrix} =
\begin{bmatrix}
81.7622 \\
79.6569
\end{bmatrix}
\]

The above coefficient matrix is tri-diagonal. Special algorithms such as Thomas’ algorithm can be used to solve simultaneous linear equation with tri-diagonal coefficient matrices. The solution is given by

\[
\begin{bmatrix}
T_1^2 \\
T_2^2
\end{bmatrix} =
\begin{bmatrix}
40.6956 \\
39.6568
\end{bmatrix}
\]
6. The partial differential equation of the temperature in a long thin rod is given by

\[ \frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \]

If \( \alpha = 0.8 \text{cm}^2 / \text{s} \), the initial temperature of rod is 40°C, and the rod is divided into three equal segments, the temperature at node 1 (using \( \Delta t = 0.1 \text{s} \)) by using an explicit solution at \( t = 0.2 \text{sec} \) is. (For node 0, \( -k \frac{\partial T}{\partial x} = h(T_a - T_0) \)), where

\[ k = 9 \text{W/(m°C)}, h = 20 \text{W/m}^2, T_a = 25°C, T_0 = \text{the temperature of rod at node 0} \]

(A) 41.6478°C  
(B) 38.4356°C  
(C) 39.9983°C  
(D) 37.5798°C  

**Solution**  
*The correct answer is (C).*

Given  
\( \alpha = 0.8 \text{cm}^2 / \text{s} \)  
\( \Delta t = 0.1 \text{s} \)  
\( t = 0.2 \text{sec} \)  
\( L = 9 \text{cms} \)  
\( T_a = 25°C \)  
\( h = 20 \text{W/m}^2 \)  
\( k = 9 \text{W/(m°C)} \)

Number of divisions of the rod, \( n = 3 \)

\[ \Delta x = \frac{L}{n} = \frac{9}{3} = 3\text{cm} \]
Number of time steps = \( \frac{t_{\text{final}} - t_{\text{initial}}}{\Delta t} \)
\[ = \frac{0.2 - 0}{0.1} \]
\[ = 2 \]

\[ \lambda = \alpha \frac{\Delta t}{(\Delta x)^2} \]
\[ = 0.8 \frac{0.1}{(3)^2} \]
\[ = 0.0089 \]

The boundary conditions
\[ T_j^0 = 20^\circ C \]
\[ -k \frac{\partial T}{\partial x}_{i,j} = h(T_a - T_{i,j}), \quad i = 0 \]

(E6.1)

Approximating the derivative by the central divided difference,
\[ \frac{\partial T}{\partial x}_{i,j} \cong \frac{T_{i+1,j} - T_{i-1,j}}{2(\Delta x)} \]  \( (E6.2) \)

Substituting Equation (E6.2) in the derivative boundary condition of Equation (E6.3) we have
\[ -k \left[ \frac{T_{i+1,j} - T_{i-1,j}}{2(\Delta x)} \right] = h(T_a - T_{i,j}), \quad i = 0 \]

Rewriting the above equation we have
\[ T_{i,j} = 2 \frac{h}{k} \Delta x(T_a - T_{i,j}^0) + T_{i+1,j}, \quad i = 0 \]  \( (E6.4) \)

From the explicit method (Equation 7 from Chapter 10.02) we have
\[ T_i^{j+1} = T_i^{j} + \lambda \left( T_{i+1,j}^{j} - 2T_i^{j} + T_{i-1,j}^{j} \right), i \neq 0 \]  \( (E6.5) \)

Substituting Equation (E6.4) in Equation (E6.5) we get
\[ T_i^{j+1} = T_i^{j} + \lambda \left( T_{i+1,j}^{j} - 2T_i^{j} + 2 \frac{h}{k} \Delta x(T_a - T_{i,j}^0) + T_{i+1,j}^{j} \right) \]
\[ = T_i^{j} + 2\lambda \left( T_{i+1,j}^{j} - T_i^{j} + \frac{h}{k} \Delta x(T_a - T_{i,j}^0) \right), i = 0 \]  \( (E6.8) \)

Equation (E6.8) is used for the node 0 (insulated edge)
The initial temperature of the rod is 40°C, that is, all the temperatures of the nodes inside the rod are at 40°C when time, \( t = 0 \) sec except for the boundary nodes as given by Equation (E6.1). This could be represented as
\[ T_i^0 = 20^\circ C, \quad \text{for all } i = 1, 2, 3 \]  \( (E6.9) \)

Initial temperature at the nodes inside the rod (when \( t = 0 \) sec)
\[ T_0^0 = 40^\circ C \]
\[ T_1^0 = 40^\circ C \]  \( \text{from Equation (E6.9)} \)
\[ T_2^0 = 40^\circ C \]
\[ T_2^0 = 20^\circ C \quad \text{from Equation (E6.1)} \]

Temperature at the nodes inside the rod when \( t=0.1 \) sec

Setting \( j = 0 \) and \( i = 0,1,2,3 \) in Equation (7) (from Chapter 10.02) gives the temperature of the nodes inside the rod when time, \( t = 0.1 \) sec.

\( i=0 \)

\[
T_0^1 = T_0^0 + 2\lambda \left( T_1^0 - T_0^0 + \frac{h}{k} \Delta x(T_a - T_0^0) \right) \quad \text{(From Equation E6.8)}
\]

\[
= 40 + 2 \times 0.0089 \left( 40 - 40 + \frac{20}{9} \times 0.03 \times (25 - 40) \right) \quad \text{note: } \Delta x = 0.03 m
\]

\[
= 40 + 0.017778(-1)
\]

\[
= 40 - 0.017778
\]

\[
= 39.9822 \, ^\circ C
\]

\( i=1 \)

\[
T_1^1 = T_1^0 + \lambda \left( T_2^0 - 2T_1^0 + T_0^0 \right)
\]

\[
= 40 + 0.0089(40 - 2(40) + 40)
\]

\[
= 40 + 0.0089(0)
\]

\[
= 40 + 0
\]

\[
= 40.0000 \, ^\circ C
\]

\( i=2 \)

\[
T_2^1 = T_2^0 + \lambda \left( T_3^0 - 2T_2^0 + T_1^0 \right)
\]

\[
= 40 + 0.0089(20 - 2(40) + 40)
\]

\[
= 40 + 0.0089(-20)
\]

\[
= 40 - 0.1778
\]

\[
= 39.8222 \, ^\circ C
\]

\( i=3 \)

\[
T_3^1 = 20^\circ C \quad \text{Boundary Condition (E3.1)}
\]
Temperature at the nodes inside the rod when $t = 0.2 \text{ sec}$

Setting $j = 1$ and $i = 0, 1, 2, 3$ in Equation (7) (from Chapter 10.02) gives the temperature of the nodes inside the rod when time, $t = 0.2 \text{ sec}$

$i = 0$

\[
T_0^2 = T_0^1 + 2\lambda \left( T_1^i - T_0^i + \frac{h}{k} \Delta x (T_0^i - T_0^i) \right) \quad \text{(From Equation E6.8)}
\]

\[
= 39.9822 + 2 \times 0.0089 \left( 40 - 39.9822 + \frac{20}{9} \times 0.03(25 - 39.9822) \right) \quad \text{note: } \Delta x = 0.03m
\]

\[
= 39.9822 + 0.17778(-0.98101)
\]

\[
= 39.9822 - 0.01744
\]

\[
= 39.9648 \degree C
\]

$i = 1$

\[
T_1^2 = T_1^1 + \lambda \left( T_2^1 - 2T_1^1 + T_0^1 \right)
\]

\[
= 40.0000 + 0.0089(39.8222 - 2(40.0000) + 39.9822)
\]

\[
= 40.0000 + 0.0089(-0.1956)
\]

\[
= 40.0000 - 0.001741
\]

\[
= 39.9983 \degree C
\]