Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Continuous Fourier Series
Chapter 11.02
COMPLETE SOLUTION SET

1. Which of the following is an "even" function of t?

   (A) t^2
   (B) t^2-4t
   (C) $\sin(2t)+3t$
   (D) t^3+6

Solution
The correct answer is (A).

Since if we replace $"t"\ by\ "-t"$, then the function value remains the same !

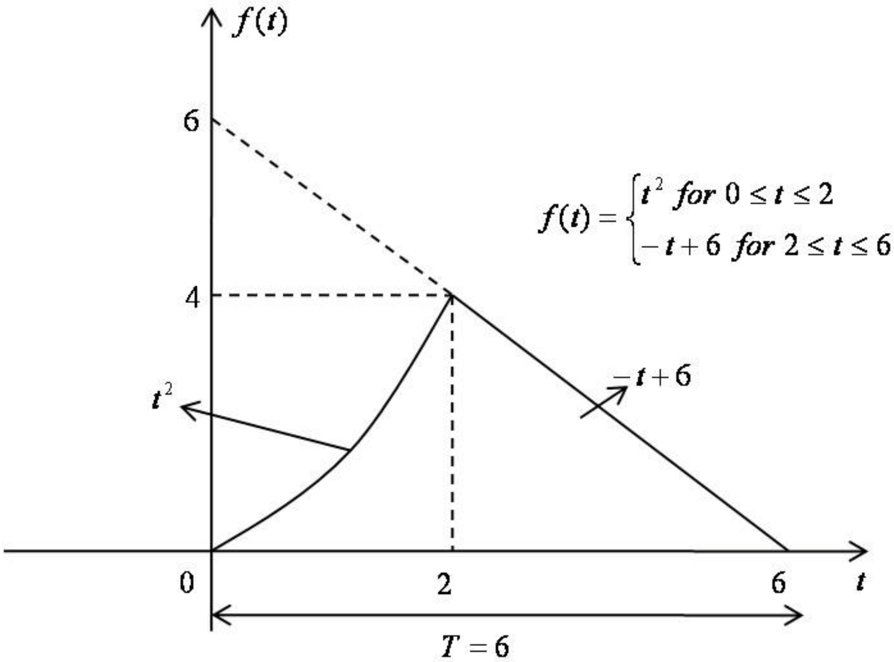
2. A “periodic function” is given by a function which

(A) has a period $T = 2\pi$
(B) satisfies $f(t+T) = f(t)$
(C) satisfies $f(t+T) = -f(t)$
(D) has a period $T = \pi$

Solution
The correct answer is (B).

Since the function's value remains the same value after a period (or multiple periods) has passed!

3. Given the following periodic function, $f(t)$.



The coefficient a_0 of the continuous Fourier series associated with the above given function $f(t)$ can be computed as

(A) $\frac{8}{9}$

(B) $\frac{16}{9}$

(C) $\frac{24}{9}$

(D) $\frac{32}{9}$

Solution

The correct answer is (B).

The coefficient a_0 of the continuous Fourier series associated with the given function $f(t)$ can be computed as (see Eq. 2, in Chapter 11.02)

$$a_0 = \left(\frac{1}{T}\right)\int \{f(t)\, dt\}$$

$$a_0 = \left(\frac{1}{6}\right)\left(\int_0^2 t^2\,dt + \int_2^6 (-t+6)\,dt\right)$$

$$a_0 = \left(\frac{1}{6}\right)\left(\left[\frac{t^3}{3}\right]_0^2 + \left[-\frac{t^2}{2} + 6\times t\right]_2^6\right)$$

$$a_0 = 1.78$$

4. For the given periodic function $f(t) = \begin{cases} 2t \text{ for } 0 \le t \le 2 \\ 4 \text{ for } 2 \le t \le 6\ (= T) \end{cases}$. The coefficient b_1 of the continuous Fourier series associated with the given function $f(t)$ can be computed as

(A) -75.6800
(B) -7.5680
(C) -6.8968
(D) -0.7468

Solution
The correct answer is (D).

The coefficient b_1 of the continuous Fourier series associated with the above given function $f(t)$ can be computed as (see Eq. 4, in Chapter 11.02)

$$b_1 = \left(\frac{2}{T}\right)\int \{f(t) \times \sin(w_0 t) dt\}$$

Since

$$w_0 = \left(\frac{2\pi}{T}\right) = \left(\frac{2 \times 3.1416}{6}\right) = 1.0472$$

and

$$\left(\frac{2}{T}\right) = \left(\frac{2}{6}\right) = 0.3333,$$

Hence

$$b_1 = (0.3333)\int_0^2 \{2t \sin(1.0472t)\}\, dt + (0.3333) \times \int_2^6 \{4 \sin(1.0472t)\}\, dt$$

$$b_1 = -0.7468$$

5. For the given periodic function $f(t)=\begin{cases} 2t \text{ for } 0 \le t \le 2 \\ 4 \text{ for } 2 \le t \le 6 \end{cases}$ with a period $T=6$. The Fourier coefficient a_1 can be computed as

(A) -9.2642
(B) -8.1275
(C) -0.9119
(D) -0.5116

Solution
The correct answer is (C).

The coefficient a_1 of the continuous Fourier series associated with the above given function $f(t)$ can be computed from Eq.(3), with $k=1$ *and* $T=6$ as following:

$$w_0=\left(\frac{2\pi}{T}\right)=\left(\frac{2\times 3.1416}{6}\right)=1.0472$$

$$a_1\times\left(\frac{T}{2}\right)=\int_0^2((2t)\cos(1.0472t))\,dt+\int_2^6((4)\cos(1.0472t))\,dt$$

$$a_1=-0.9119$$

6. For the given periodic function $f(t)=\begin{cases} 2t \text{ for } 0 \le t \le 2 \\ 4 \text{ for } 2 \le t \le 6 \end{cases}$ with a period $T = 6$ as shown in Problem 5. The complex form of the Fourier series can be expressed as $f(t)=\sum_{k=-\infty}^{\infty}\tilde{C}_k e^{ikw_0 t}$. The complex coefficient $\tilde{C}_1$ can be expressed as

(A) $0.4560 + 0.3734i$
(B) $0.4560 - 0.3734i$
(C) $-0.4560 + 0.3734i$
(D) $0.3734 - 0.4560i$

Solution
The correct answer is (C).

The coefficient $\tilde{C}_1$ (corresponding to $k = 1$) can be expressed/computed from Eq.(15) as:

$$\tilde{C}_1 = \frac{(a_1 - ib_1)}{2}$$

Using the CORRECT results in multiple choice questions # 4-5, one obtains

$$\tilde{C}_1 = \frac{(-0.9119 - i(-0.7468))}{2}$$

$$\tilde{C}_1 = -0.4560 + 0.3734i$$