Holistic Numerical Methods Institute
committed to bringing numerical methods to undergraduates

Multiple-Choice Test
Fourier Transform Pair: Frequency and Time Domain
Chapter 11.03
COMPLETE SOLUTION SET

1. Given two complex numbers: $C_1 = 2 - 3i$, and $C_2 = 1 + 4i$. The product $P = C_1 \times C_2$ can be computed as

   (A) $2 + 5i$
   (B) $-10 + 5i$
   (C) $-14 + 5i$
   (D) $14 + 5i$

Solution
The correct answer is (D).

$$\begin{aligned} P &= C_1 \times C_2 \\ &= (2 - 3i) \times (1 + 4i) \\ &= 2 + 8i - 3i - 12i^2 \end{aligned}$$

$$\begin{aligned} P &= 2 + 5i + 12 \\ &= 14 + 5i \end{aligned}$$

2. Given the complex number $C_1 = 3 + 4i$. In polar coordinates, the above complex number can be expressed as $C_1 = Ae^{i\theta}$, where A and θ is called the amplitude and phase angle of C_1, respectively. The amplitude A can be computed as

(A) 3
(B) 4
(C) 5
(D) 7

Solution
The correct answer is (C).

Given

$$C_1 = 3 + 4i = Ae^{(i\theta)}$$

where

$$A = Amplitude = \sqrt{\left(3^2 + 4^2\right)} = 5$$

3. Given the complex number $C_1 = 3 + 4i$. In polar coordinates, the above complex number can be expressed as $C_1 = Ae^{i\theta}$, where A and θ is called the amplitude and phase angle of C_1, respectively. The phase angle θ in radians can be computed as

(A) 0.6435
(B) 0.9273
(C) 2.864
(D) 5.454

Solution
The correct answer is (B).

For the given complex number $C_1 = 3 + 4i$, then

$real = 3; imaginary = 4;$

$$\theta = \cos^{-1}\left(\frac{real}{A}\right) = \cos^{-1}\left(\frac{3}{5}\right) = 53.13^o$$

$$\theta = \sin^{-1}\left(\frac{imaginary}{A}\right) = \sin^{-1}\left(\frac{4}{5}\right) = 53.13^o$$

Note: Since both the Real and Imaginary components are Positive, hence the phase angle θ has to be in the 1-st quadrant (between $0 - 90^o$)

Hence, $\theta = 53.13^o = 0.9273\, radians$ = in the 1-st quadrant

4. For the complex number $C = -3 + 4i$, the phase angle θ in radians can be computed as

(A) 0.6435
(B) 0.9273
(C) 1.206
(D) 2.2143

Solution
The correct answer is (D).

Given $C = -3 + 4i$

Hence,

$real = -3;\ imaginary = 4;$

Thus, angle θ should be in 2-nd quadrant.

$\theta = \cos^{-1}\left(\frac{real}{A}\right) = \cos^{-1}\left(\frac{-3}{5}\right) = 126.87^o$, hence in 2-nd quadrant

$\theta = \sin^{-1}\left(\frac{imaginary}{A}\right) = \sin^{-1}\left(\frac{4}{5}\right) = 53.13^o$, hence in 1-st quadrant

Thus, θ should be $126.87^o = 2.2143\ radians$

5. Given the function $f_{np}(t) = \delta(t-a) = \begin{cases} 1, \, if \;\; t = a \\ 0, \, elsewhere \end{cases}$. The Fourier transform $\hat{F}(iw_0)$ which will transform the function from time domain to frequency domain can be computed as

(A) $\delta(a+t)$
(B) $e^{-i(2\pi f)a}$
(C) 1
(D) $\delta(t-a)$

Solution
The correct answer is (B).

Given $f(t) = \delta(t-a)$; then the Fourier transform can be found/computed with MATLAB, as following:

```
>> syms x y z t u v a w
>> f = dirac(t-a)
>> answer1 = fourier(f) = exp(-i*w*a)  ........... using MATLAB

>> or,  answer1 = int(f*exp(-i*w*t), t, -inf, inf) = exp(-i*w*a)
            Notes:
            int = take the INTEGRAL
            t   = with respect to time variable "t"
            inf = infinitive
```

6. Given the function $\hat{F}(iw_0) = 1$. The inverse Fourier transform $f_{np}(t)$ which will transform the function from frequency domain to time domain can be computed as

(A) e^{it}
(B) e^{-it}
(C) $\delta(t-0)$
(D) $e^{-i(2\pi f)t}$

Solution
The correct answer is (C).

Given the function in frequency domain $\hat{f} = 1$, the inverse Fourier transform can be computed with MATLAB, as following:

```
>> syms x y z t u v a w
>> f = dirac(t)
>> answer1 = fourier(f) = 1
>> answer2 = ifourier(answer1) = dirac(x)
```

Notes:
ifourier = inverse fourier transform ..... MATLAB command