Multiple-Choice Test
Fourier Transform Pair: Frequency and Time Domain
Chapter 11.03
COMPLETE SOLUTION SET

1. Given two complex numbers: \( C_1 = 2 - 3i \), and \( C_2 = 1 + 4i \). The product \( P = C_1 \times C_2 \) can be computed as

   (A) \( 2 + 5i \)
   (B) \( -10 + 5i \)
   (C) \( -14 + 5i \)
   (D) \( 14 + 5i \)

Solution
The correct answer is (D).

\[
P = C_1 \times C_2 \\
= (2 - 3i) \times (1 + 4i) \\
= 2 + 8i - 3i - 12i^2 \\
P = 2 + 5i + 12 \\
= 14 + 5i
\]
2. Given the complex number $C_1 = 3 + 4i$. In polar coordinates, the above complex number can be expressed as $C_1 = Ae^{i\theta}$, where $A$ and $\theta$ is called the amplitude and phase angle of $C_1$, respectively. The amplitude $A$ can be computed as

(A) 3  
(B) 4  
(C) 5  
(D) 7

Solution

The correct answer is (C).

Given

$C_1 = 3 + 4i = Ae^{i\theta}$

where

$A = \text{Amplitude} = \sqrt{3^2 + 4^2} = 5$
3. Given the complex number \( C_1 = 3 + 4i \). In polar coordinates, the above complex number can be expressed as \( C_1 = Ae^{i\theta} \), where \( A \) and \( \theta \) is called the amplitude and phase angle of \( C_1 \), respectively. The phase angle \( \theta \) in radians can be computed as

\[
\begin{align*}
(A) & \quad 0.6435 \\
(B) & \quad 0.9273 \\
(C) & \quad 2.864 \\
(D) & \quad 5.454
\end{align*}
\]

**Solution**

The correct answer is (B).

For the given complex number \( C_1 = 3 + 4i \), then

\[
\text{real } = 3; \text{ imaginary } = 4;
\]

\[
\theta = \cos^{-1}\left(\frac{\text{real}}{A}\right) = \cos^{-1}\left(\frac{3}{5}\right) = 53.13^\circ
\]

\[
\theta = \sin^{-1}\left(\frac{\text{imaginary}}{A}\right) = \sin^{-1}\left(\frac{4}{5}\right) = 53.13^\circ
\]

Note: Since both the Real and Imaginary components are Positive, hence the phase angle \( \theta \) has to be in the 1-st quadrant (between \( 0 - 90^\circ \))

Hence, \( \theta = 53.13^\circ = 0.9273 \text{ radians} \) = in the 1-st quadrant
4. For the complex number $C = -3 + 4i$, the phase angle $\theta$ in radians can be computed as

\[
\begin{align*}
(A) & \quad 0.6435 \\
(B) & \quad 0.9273 \\
(C) & \quad 1.206 \\
(D) & \quad 2.2143
\end{align*}
\]

**Solution**

*The correct answer is (D).*

Given $C = -3 + 4i$

Hence,

real $= -3; \text{imaginary} = 4$

Thus, angle $\theta$ should be in 2-nd quadrant.

\[
\theta = \cos^{-1}\left(\frac{\text{real}}{A}\right) = \cos^{-1}\left(\frac{-3}{5}\right) = 126.87^\circ, \text{ hence in 2-nd quadrant}
\]

\[
\theta = \sin^{-1}\left(\frac{\text{imaginary}}{A}\right) = \sin^{-1}\left(\frac{4}{5}\right) = 53.13^\circ, \text{ hence in 1-st quadrant}
\]

Thus, $\theta$ should be $126.87^\circ = 2.2143 \text{ radians}$
5. Given the function \( f_{np}(t) = \delta(t-a) = \begin{cases} 1, & \text{if } t = a \\ 0, & \text{elsewhere} \end{cases} \). The Fourier transform \( \hat{F}(iw_o) \) which will transform the function from time domain to frequency domain can be computed as

\[
\begin{align*}
(A) & \quad \delta(a + t) \\
(B) & \quad e^{-i(2\pi)at} \\
(C) & \quad 1 \\
(D) & \quad \delta(t - a)
\end{align*}
\]

**Solution**

*The correct answer is (B).*

Given \( f(t) = \delta(t - a) \); then the Fourier transform can be found/computed with MATLAB, as following:

```matlab
>> syms x y z t u v a w 
>> f = dirac(t-a) 
>> answer1 = fourier(f) = exp(-i*w*a) ........... using MATLAB 
```

```matlab
>> or, answer1 = int(f*exp(-i*w*t), t, -inf, inf) = exp(-i*w*a) 
```

Notes:
- `int` = take the INTEGRAL
- `t` = with respect to time variable "t"
- `inf` = infinitive
6. Given the function $\hat{F}(i\omega_0) = 1$. The inverse Fourier transform $f_{np}(t)$ which will transform the function from frequency domain to time domain can be computed as

(A) $e^{it}$
(B) $e^{-it}$
(C) $\delta(t - 0)$
(D) $e^{-i(2\pi)t}$

Solution
The correct answer is (C).

Given the function in frequency domain $\hat{f} = 1$, the inverse Fourier transform can be computed with MATLAB, as following:

```matlab
>> syms x y z t u v a w
>> f = dirac(t)
>> answer1 = fourier(f) = 1
>> answer2 = ifourier(answer1) = dirac(x)
```

Notes:
ifourier = inverse fourier transform ......... MATLAB command