



Backward Divided Difference



Topic: Differentiation

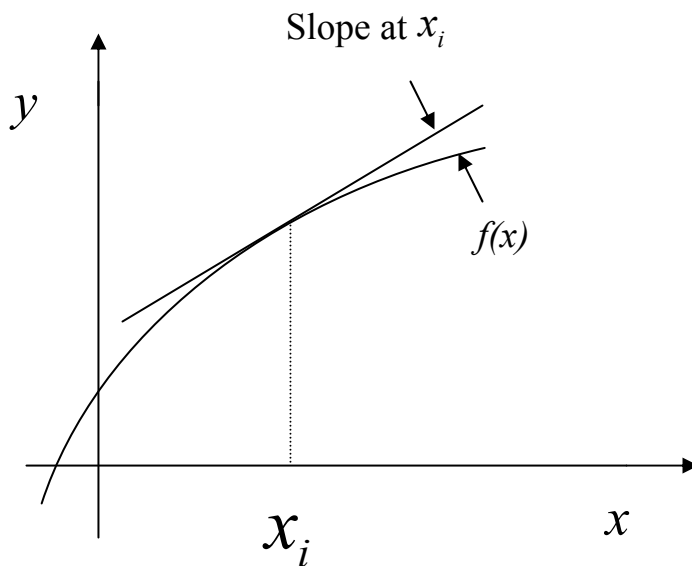
Major: General Engineering

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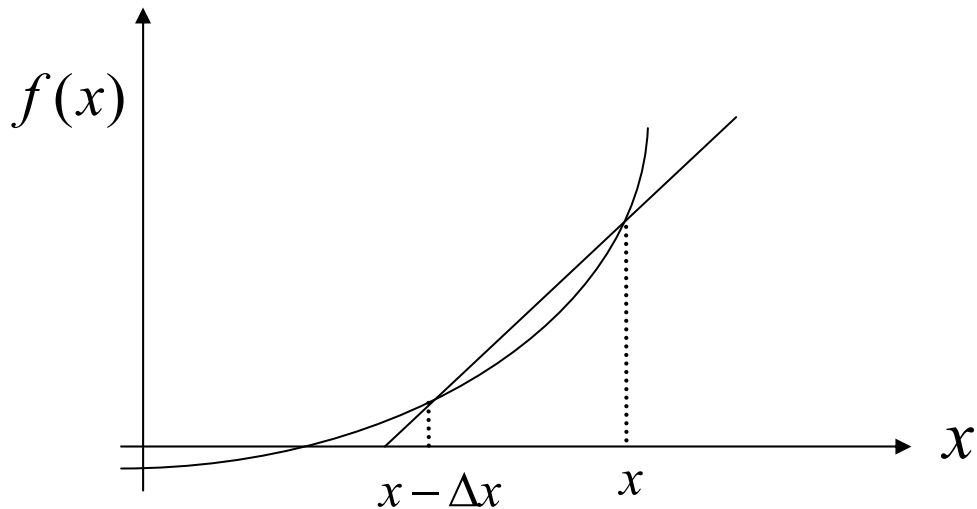
Definition

$$f'(x) = \lim_{\Delta x \rightarrow 0} \frac{f(x) - f(x - \Delta x)}{\Delta x}$$



Backward Divided Difference

$$f'(x) \cong \frac{f(x) - f(x - \Delta x)}{\Delta x}$$



$$f'(x_i) \cong \frac{f(x_i) - f(x_i - \Delta x)}{\Delta x}$$



Example

Example:

The velocity of a rocket is given by

$$v(t) = 2000 \ln \left[\frac{14 \times 10^4}{14 \times 10^4 - 2100 t} \right] - 9.8 t, 0 \leq t \leq 30$$

where v given in m/s and t is given in seconds. Use backward difference approximation of the first derivative of $v(t)$ to calculate the acceleration at $t = 16s$. Use a step size of $\Delta t = 2s$.

Solution:

$$a(t_i) \cong \frac{v(t_i) - v(t_{i-1})}{\Delta t}$$

$$t_i = 16$$



Example (contd.)

$$\Delta t = 2$$

$$t_{i-1} = t_i - \Delta t = 16 - 2 = 14$$

$$a(16) = \frac{v(16) - v(14)}{2}$$

$$v(16) = 2000 \ln \left[\frac{14 \times 10^4}{14 \times 10^4 - 2100(16)} \right] - 9.8(16) = 392.07 \text{ m/s}$$

$$v(14) = 2000 \ln \left[\frac{14 \times 10^4}{14 \times 10^4 - 2100(14)} \right] - 9.8(14) = 334.24 \text{ m/s}$$



Example (contd.)

Hence

$$a(16) = \frac{v(16) - v(14)}{2} = 392.07 - 334.24 = 28.915m/s^2$$

The exact value of $a(16)$ can be calculated by differentiating

$$v(t) = 2000 \ln \left[\frac{14 \times 10^4}{14 \times 10^4 - 2100t} \right] - 9.8t$$

as

$$a(t) = \frac{d}{dt} [v(t)] = \frac{-4040 - 29.4t}{-200 + 3t}$$

$$a(16) = 29.674m/s^2$$



Example (contd.)

The absolute relative true error is

$$\begin{aligned} |\varepsilon_t| &= \left| \frac{\textit{TrueValue} - \textit{ApproximateValue}}{\textit{TrueValue}} \right| \times 100 \\ &= \left| \frac{29.674 - 28.915}{29.674} \right| \times 100 \\ &= 2.557\% \end{aligned}$$



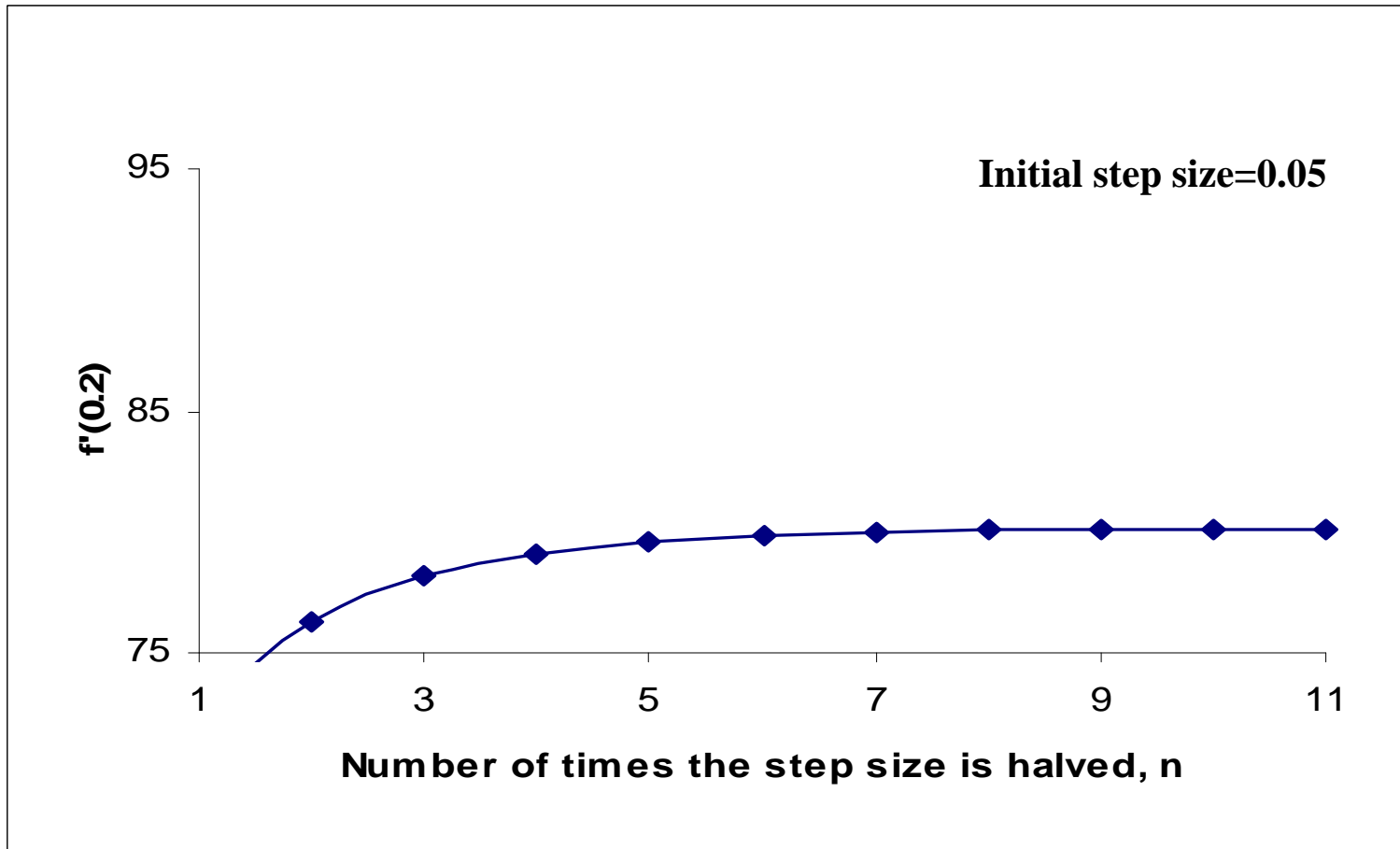
Effect Of Step Size

$$f(x) = 9e^{4x}$$

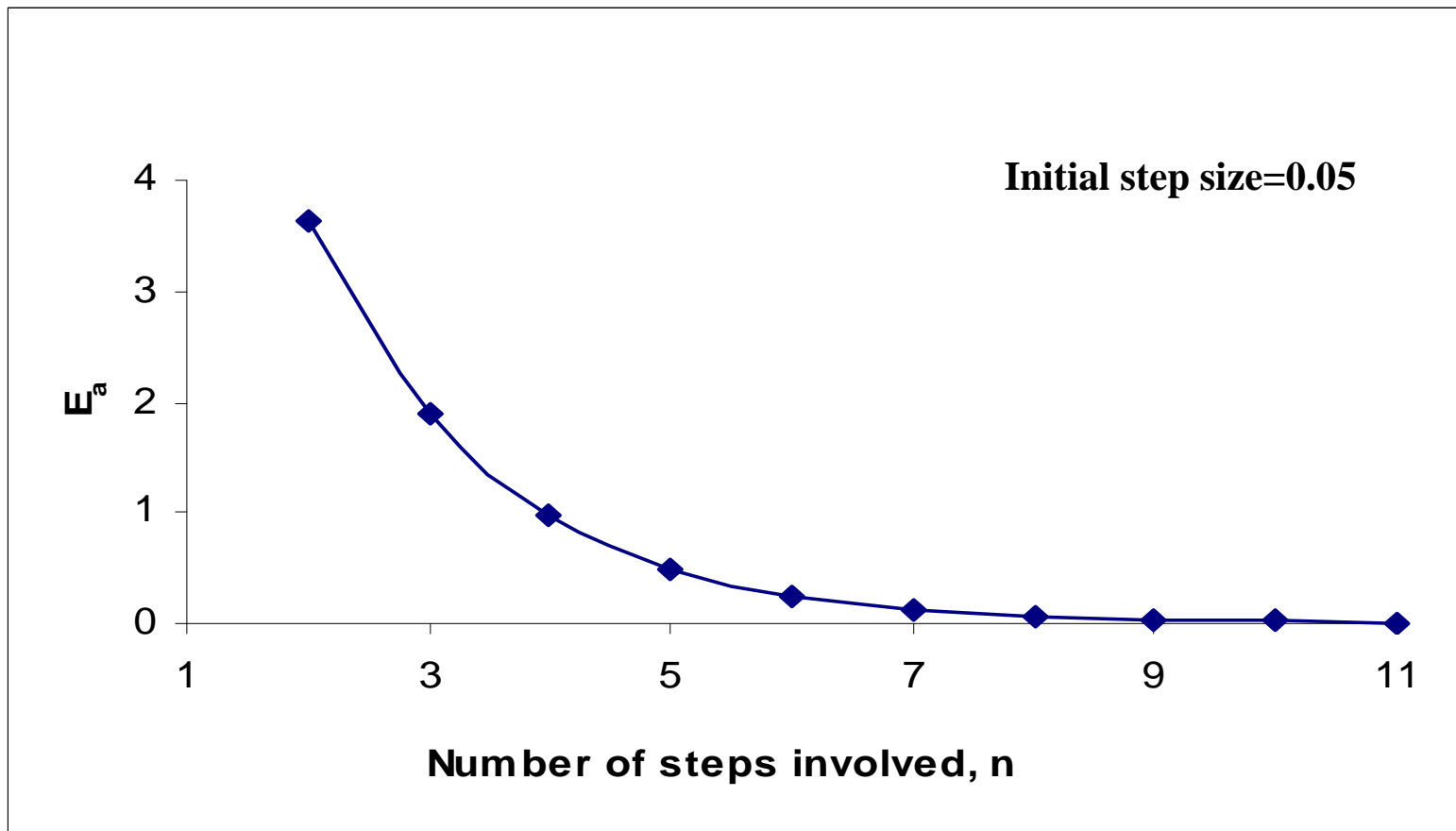
Value of $f'(0.2)$ Using backward Divided difference method.

h	$f'(0.2)$	E_a	$ \varepsilon_a \%$	Significant digits	E_t	$ \varepsilon_t \%$
0.05	72.61598				7.50349	9.365377
0.025	76.24376	3.627777	4.758129	1	3.87571	4.837418
0.0125	78.14946	1.905697	2.438529	1	1.97002	2.458849
0.00625	79.12627	0.976817	1.234504	1	0.99320	1.239648
0.003125	79.62081	0.494533	0.62111	1	0.49867	0.622404
0.001563	79.86962	0.248814	0.311525	2	0.24985	0.31185
0.000781	79.99442	0.124796	0.156006	2	0.12506	0.156087
0.000391	80.05691	0.062496	0.078064	2	0.06256	0.078084
0.000195	80.08818	0.031272	0.039047	3	0.03129	0.039052
9.77E-05	80.10383	0.015642	0.019527	3	0.01565	0.019529
4.88E-05	80.11165	0.007823	0.009765	3	0.00782	0.009765

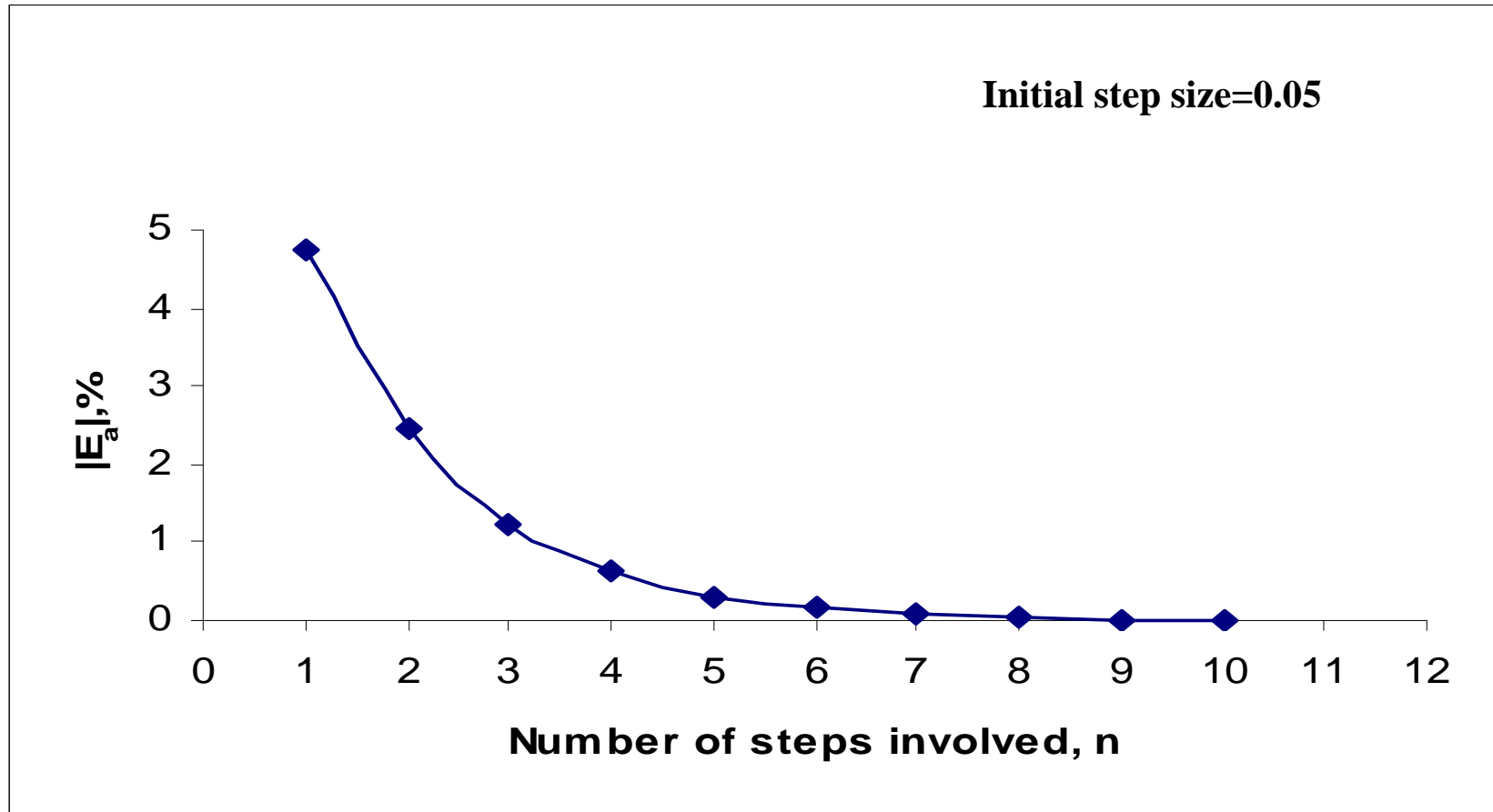
Effect of Step Size in Backward Divided Difference Method



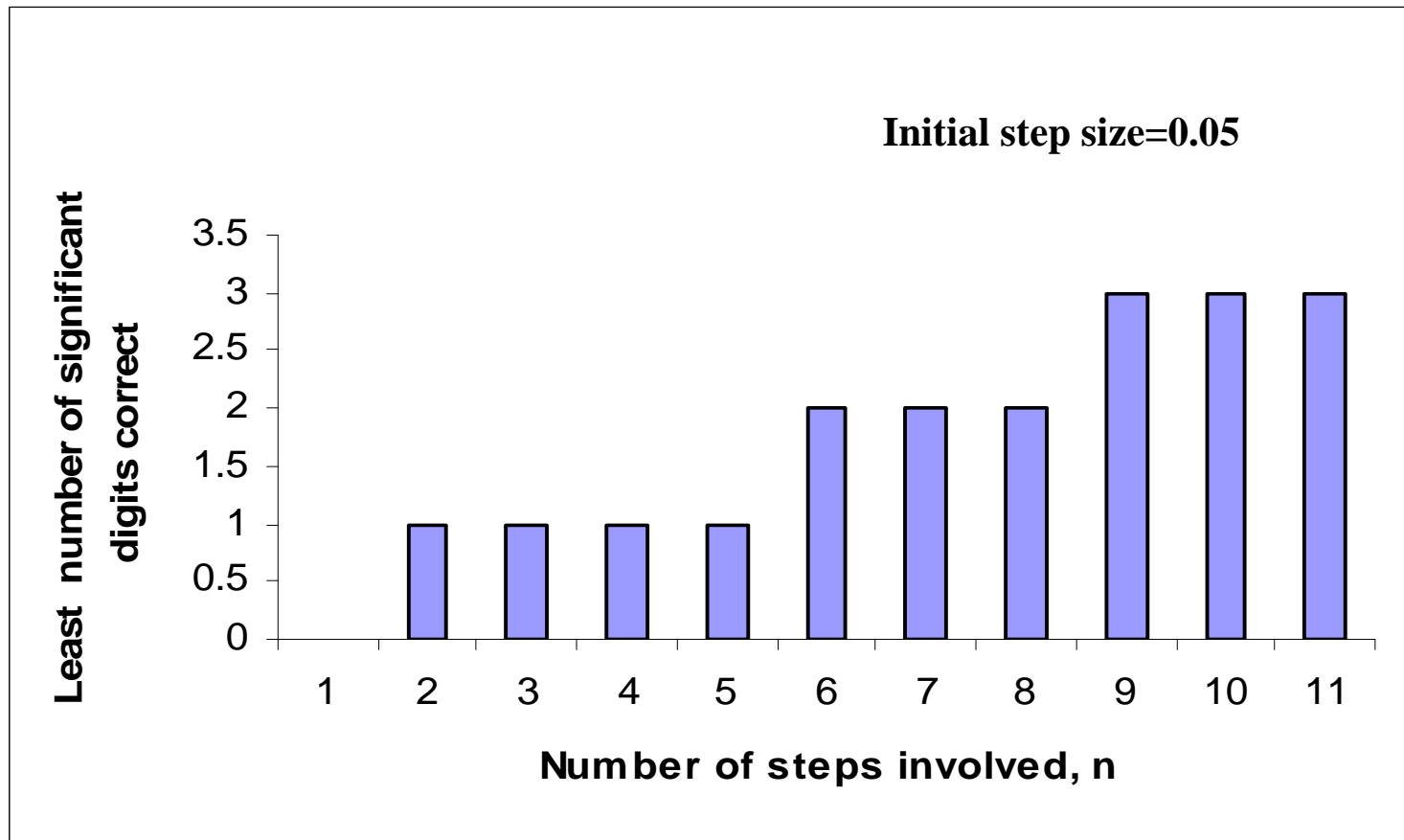
Effect of Step Size on Approximate Error



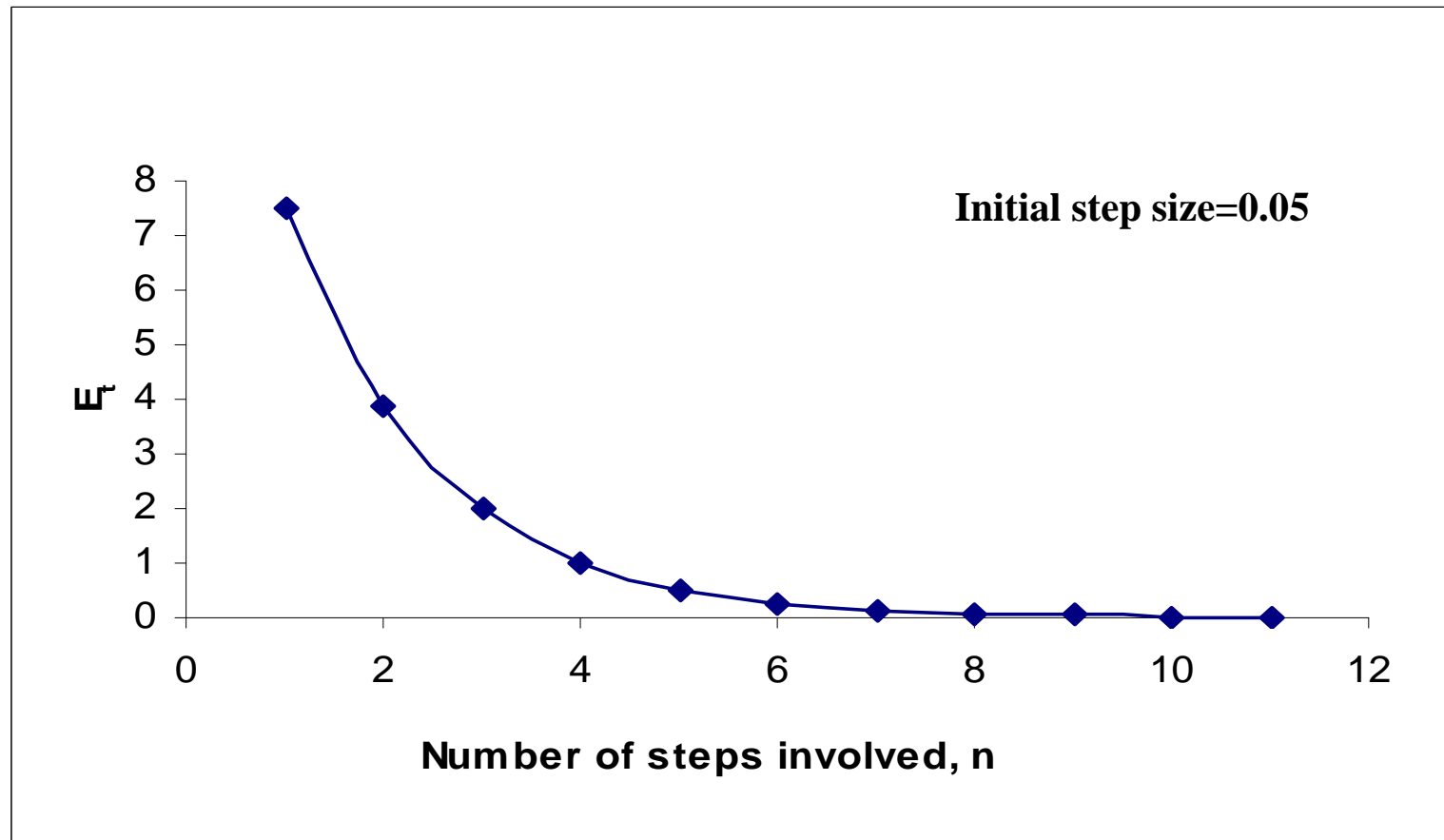
Effect of Step Size on Absolute Relative Approximate Error



Effect of Step Size on Least Number of Significant Digits Correct



Effect of Step Size on True Error



Effect of Step Size on Absolute Relative True Error

