Secant Method

Chemical Engineering Majors

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Secant Method – Derivation

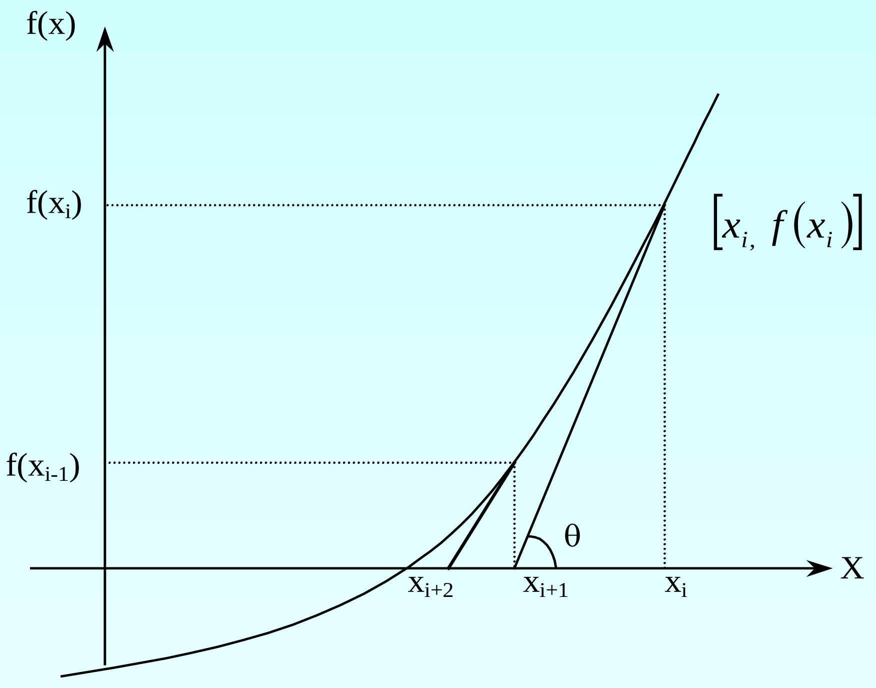


Figure 1 Geometrical illustration of the Newton-Raphson method.

Newton's Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \qquad (1)$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \qquad (2)$$

Substituting Equation (2) into Equation (1) gives the Secant method

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method – Derivation

The secant method can also be derived from geometry:

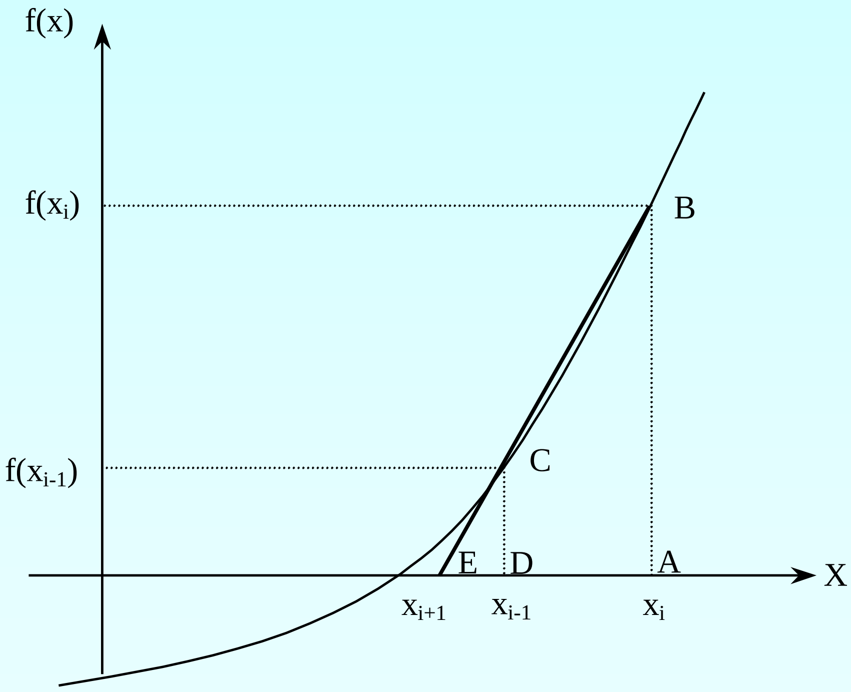


Figure 2 Geometrical representation of the Secant method.

The Geometric Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

can be written as

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

On rearranging, the secant method is given as

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$|\in_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

Step 2

Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example 1

You have a spherical storage tank containing oil. The tank has a diameter of 6 ft. You are asked to calculate the height, *h*, to which a dipstick 8 ft long would be wet with oil when immersed in the tank when it contains 4 ft^3 of oil.

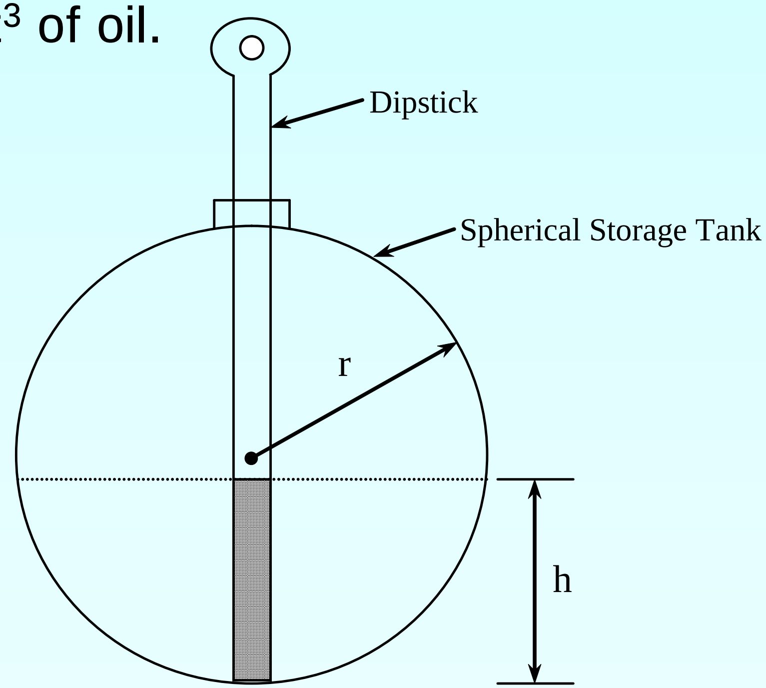


Figure 2 Spherical Storage tank problem.

Example 1 Cont.

The equation that gives the height, *h*, of liquid in the spherical tank for the given volume and radius is given by

$$f(h) = h^3 - 9h^2 + 3.8197 = 0$$

Use the secant method of finding roots of equations to find the height, *h*, to which the dipstick is wet with oil. Conduct three iterations to estimate the root of the above equation. Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

Example 1 Cont.

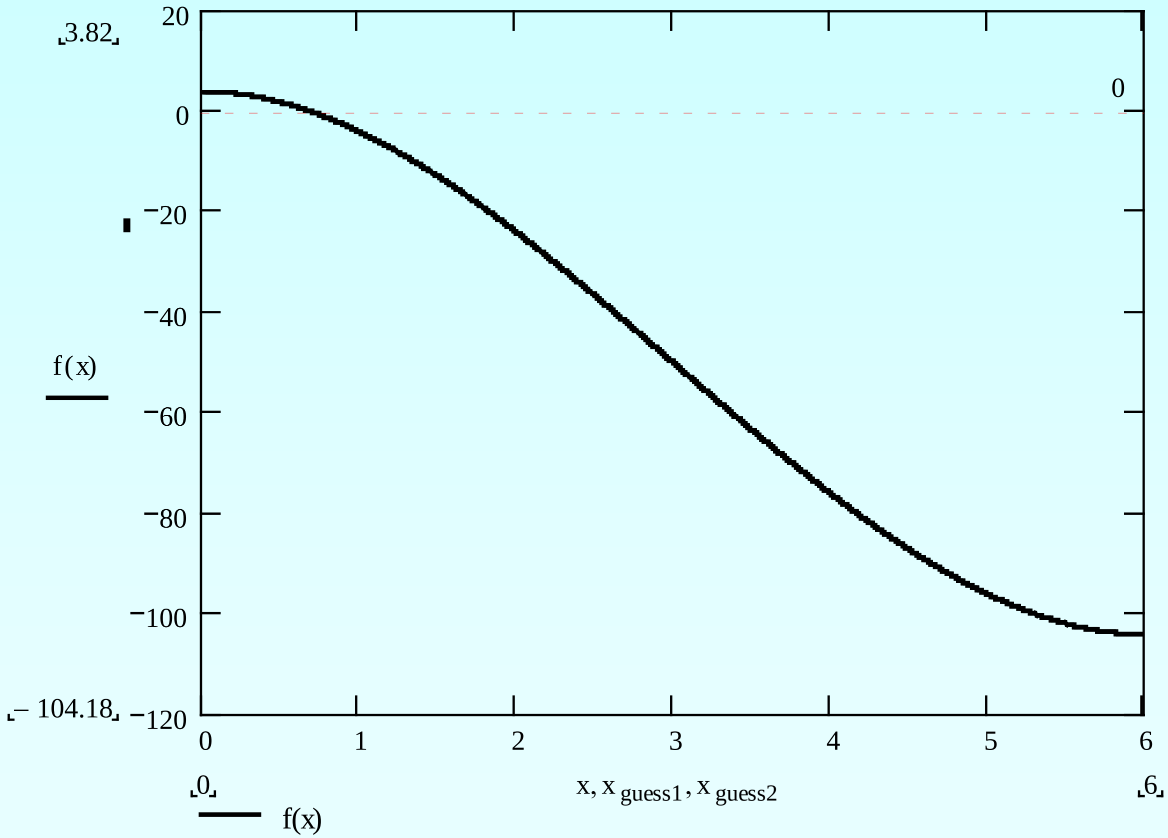


Figure 3 Graph of the function f(h)

$$f(h) = h^3 - 9h^2 + 3.8197 = 0$$

Example 1 Cont.

Solution

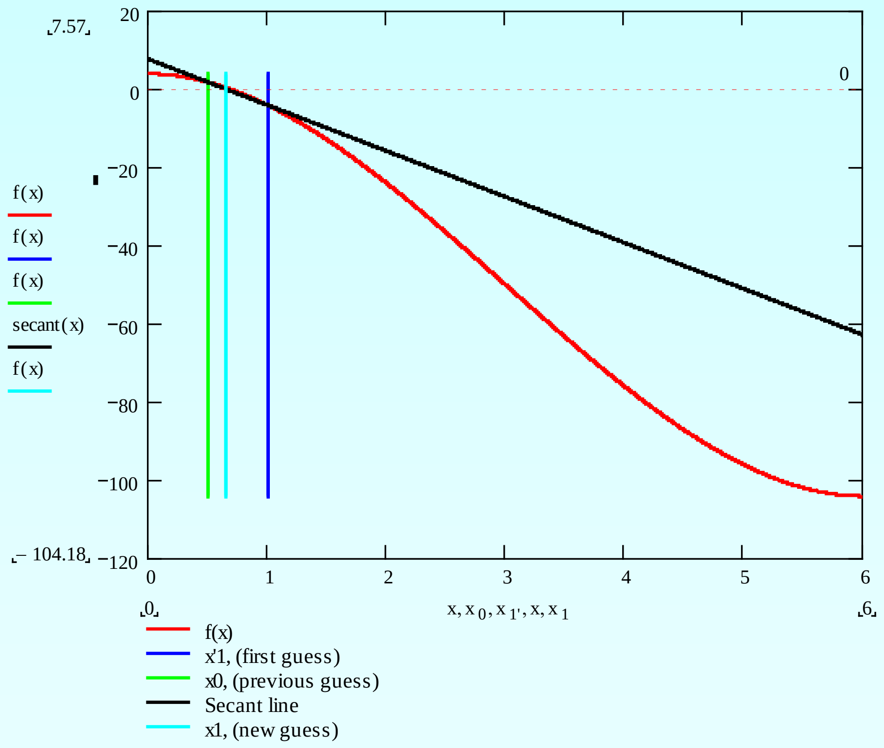


Figure 4 Graph of the estimated root of the equation after Iteration 1.

Initial guesses of the root:

$$h_{-1} = 0.5 \text{ and } h_0 = 1.$$

<u>Iteration 1</u>

The estimate of the root is

$$h_1 = h_0 - \frac{f(h_0)(h_0 - h_{-1})}{f(h_0) - f(h_{-1})}$$

$$= h_0 - \frac{\left(h_0^3 - 9h_0^2 + 3.8197\right)(h_0 - h_{-1})}{\left(h_0^3 - 9h_0^2 + 3.8197\right) - \left(h_{-1}^3 - 9h_{-1}^2 + 3.8197\right)}$$

$$= 0.64423$$

The absolute relative approximate error is

$$|\in_a| = \left|\frac{h_1 - h_0}{h_1}\right| \times 100$$

$$= 55.224\,\%$$

The number of significant digits at least correct is 0.

Example 1 Cont.

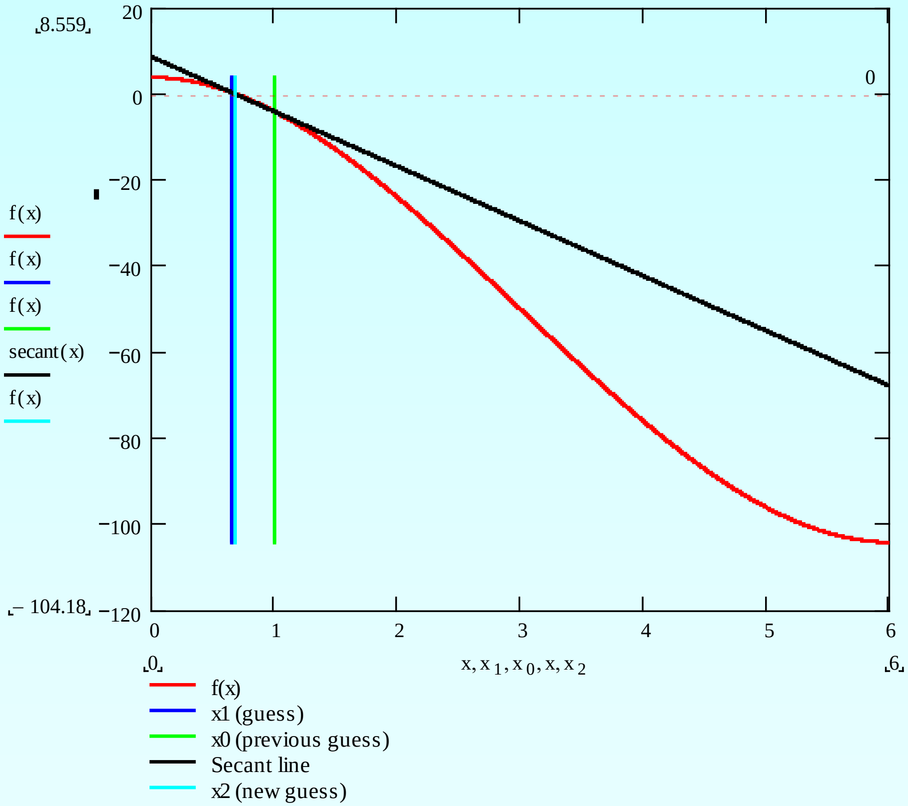


Figure 5 Graph of the estimated root after Iteration 2.

Iteration 2

The estimate of the root is

$$h_2 = h_1 - \frac{f(h_1)(h_1 - h_0)}{f(h_1) - f(h_0)}$$

$$= h_1 - \frac{\left(h_1^3 - 9h_1^2 + 3.8197\right)(h_1 - h_0)}{\left(h_1^3 - 9h_1^2 + 3.8197\right) - \left(h_0^3 - 9h_0^2 + 3.8197\right)}$$

$$= 0.67185$$

The absolute relative approximate error is

$$|\epsilon_a| = \left|\frac{h_2 - h_1}{h_2}\right| \times 100$$

$$= 4.1104\,\%$$

The number of significant digits at least correct is 1.

Example 1 Cont.

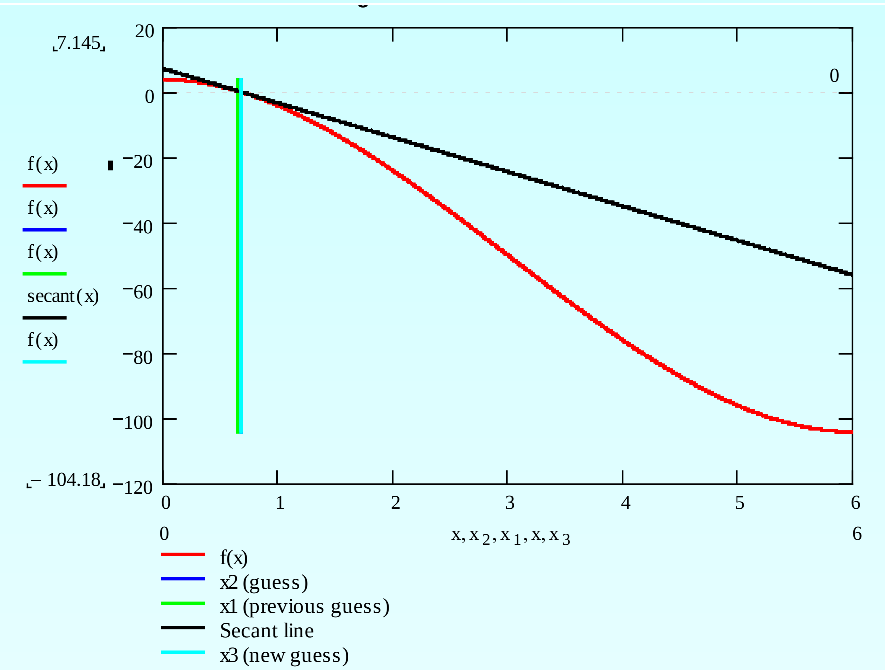


Figure 6 Graph of the estimated root after Iteration 3.

<u>Iteration 3</u>

The estimate of the root is

$$h_3 = h_2 - \frac{f(h_2)(h_2 - h_1)}{f(h_2) - f(h_1)}$$

$$= h_2 - \frac{\left(h_2^3 - 9h_2^2 + 3.8197\right)(h_2 - h_1)}{\left(h_2^3 - 9h_2^2 + 3.8197\right) - \left(h_1^3 - 9h_1^2 + 3.8197\right)}$$

$$= 0.67759$$

The absolute relative approximate error is

$$|\in_a| = \left|\frac{h_2 - h_1}{h_2}\right| \times 100$$

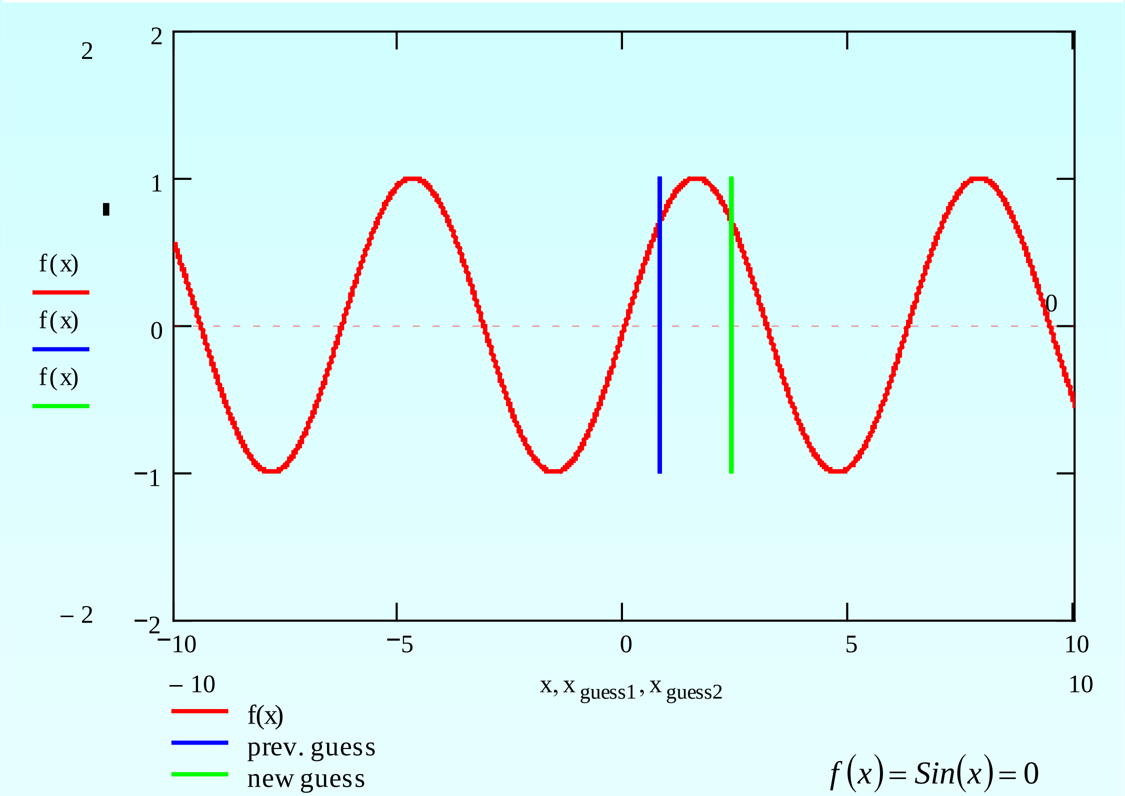
$$= 0.84768\,\%$$

The number of significant digits at least correct is 1.

Advantages

- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

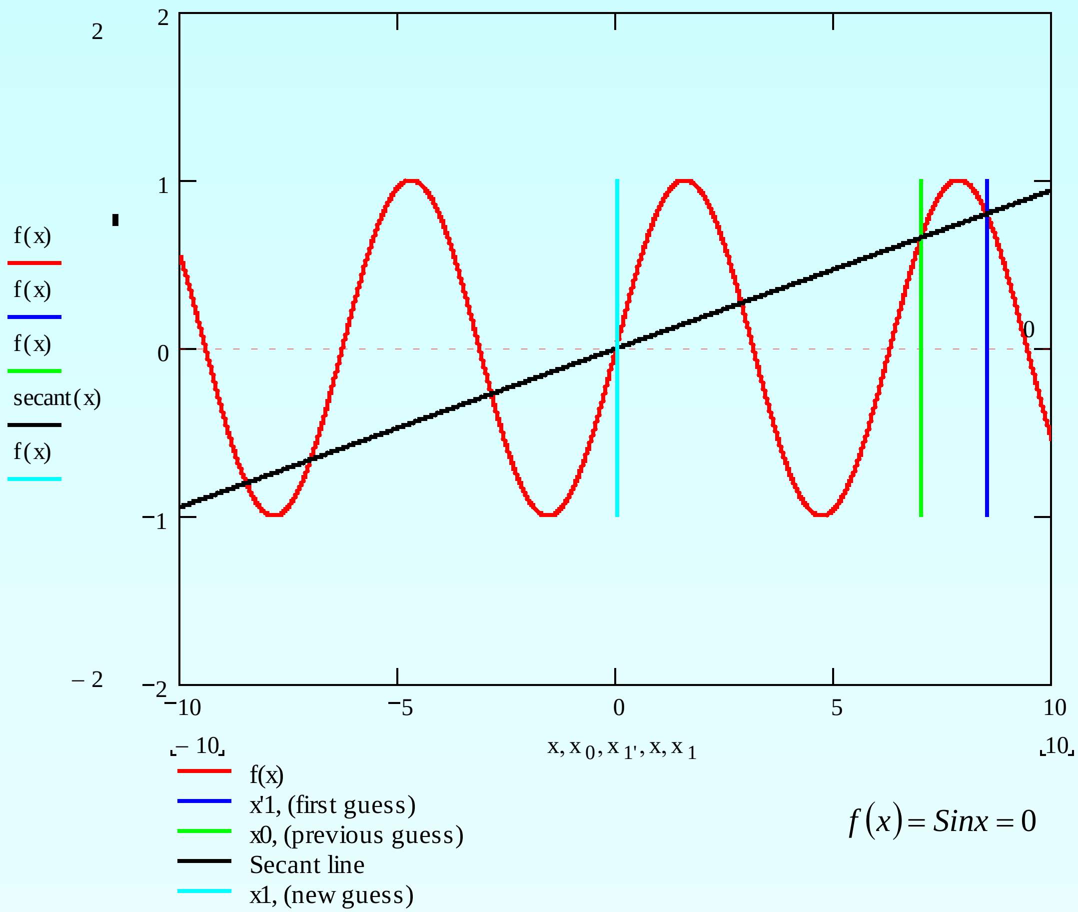
Drawbacks



$f(x) = Sin(x) = 0$

Division by zero

Drawbacks (continued)



Root Jumping

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/secant_method.html

THE END

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