

Secant Method

Civil Engineering Majors

Authors: Autar Kaw, Jai Paul

<http://numericalmethods.eng.usf.edu>

Transforming Numerical Methods Education for STEM
Undergraduates

Secant Method

<http://numericalmethods.eng.usf.edu>

Secant Method – Derivation

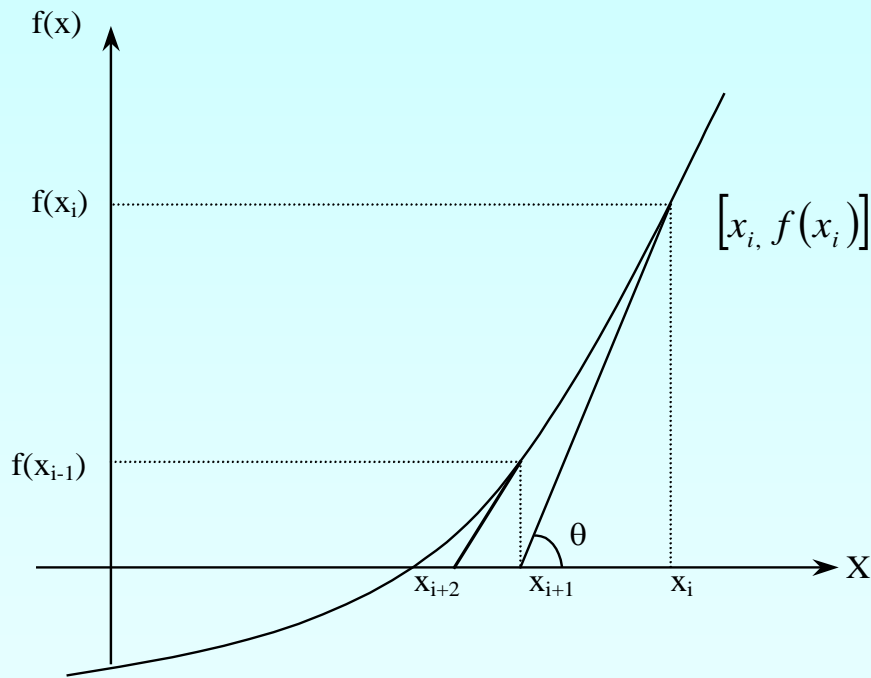


Figure 1 Geometrical illustration of the Newton-Raphson method.

Newton's Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad (1)$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \quad (2)$$

Substituting Equation (2) into Equation (1) gives the Secant method

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method – Derivation

The secant method can also be derived from geometry:

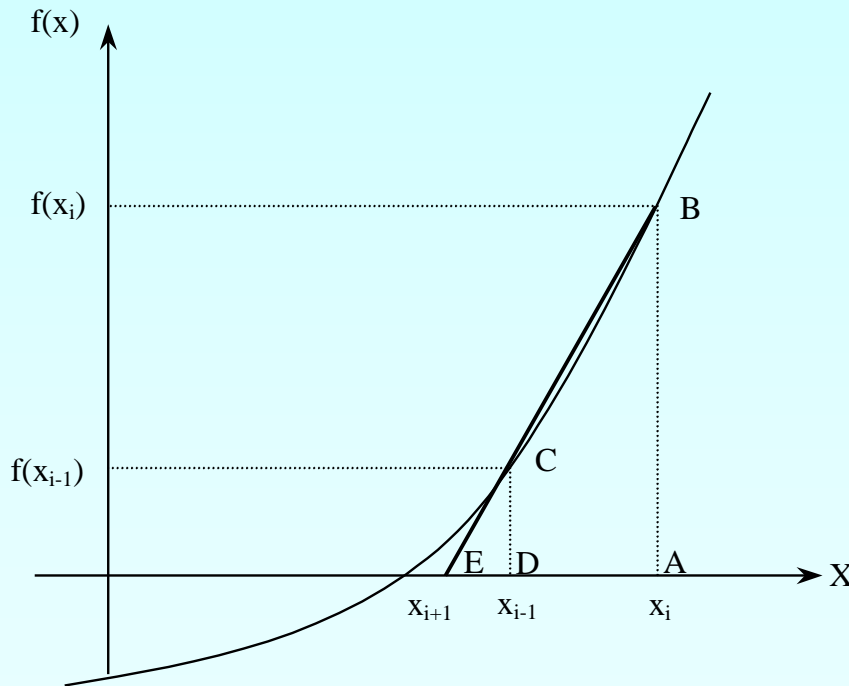


Figure 2 Geometrical representation of the Secant method.

The Geometric Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

can be written as

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

On rearranging, the secant method is given as

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$|\epsilon_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

Step 2

Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example 1

You are making a bookshelf to carry books that range from 8 ½ " to 11" in height and would take 29" of space along length. The material is wood having Young's Modulus 3.667 Msi, thickness 3/8 " and width 12". You want to find the maximum vertical deflection of the bookshelf. The vertical deflection of the shelf is given by

$$v(x) = 0.42493 \times 10^{-4} x^3 - 0.13533 \times 10^{-8} x^5 - 0.66722 \times 10^{-6} x^4 - 0.018507 x$$

where x is the position where the deflection is maximum. Hence to find the maximum deflection we need to find where $f(x) = \frac{dv}{dx} = 0$ and conduct the second derivative test.

Example 1 Cont.

The equation that gives the position x where the deflection is maximum is given by

$$f(x) = -0.67665 \times 10^{-8} x^4 - 0.26689 \times 10^{-5} x^3 + 0.12748 \times 10^{-3} x^2 - 0.018507 = 0$$

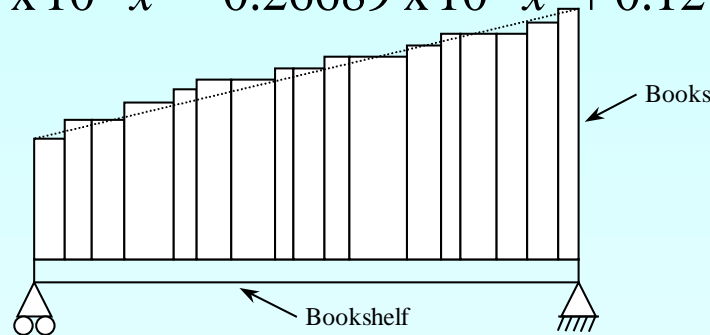


Figure 2 A loaded bookshelf.

Use the secant method of finding roots of equations to find the position where the deflection is maximum. Conduct three iterations to estimate the root of the above equation.

Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

Example 1 Cont.

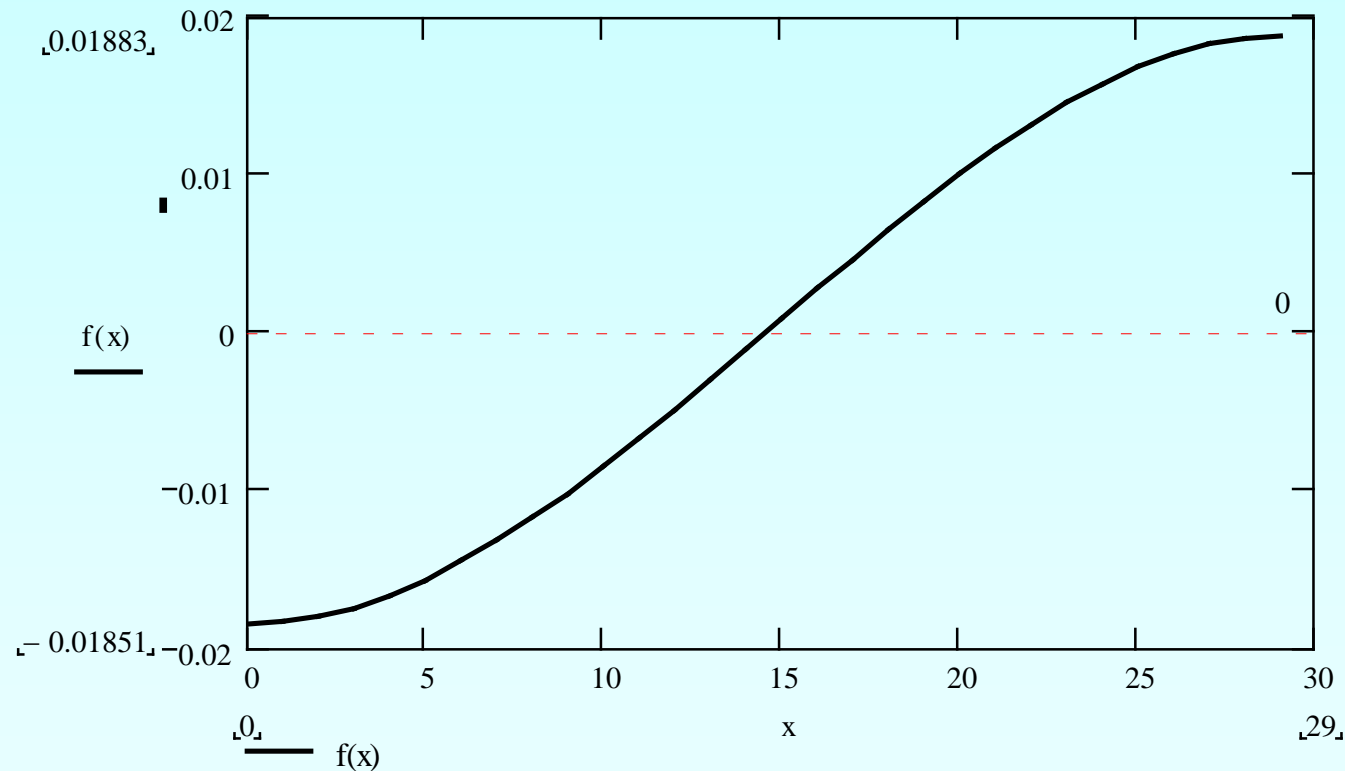


Figure 3 Graph of the function $f(x)$.

$$f(x) = -0.67665 \times 10^{-8} x^4 - 0.26689 \times 10^{-5} x^3 + 0.12748 \times 10^{-3} x^2 - 0.018507 = 0$$

Example 1 Cont.

Solution

Let us take the initial guesses of the root of $f(x) = 0$ as $x_{-1} = 10$ and $x_0 = 15$.

Iteration 1

The estimate of the root is

$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}$$

$$\begin{aligned} f(x_0) &= -0.67665 \times 10^{-8} x_0^4 - 2.6689 \times 10^{-5} x_0^3 + 0.12748 \times 10^{-3} x_0^2 - 0.018507 \\ &= -0.67665 \times 10^{-8} (15)^4 - 2.6689 \times 10^{-5} (15)^3 + 0.12748 \times 10^{-3} (15)^2 - 0.018507 \\ &= 8.2591 \times 10^{-4} \end{aligned}$$

$$\begin{aligned} f(x_{-1}) &= -0.67665 \times 10^{-8} x_{-1}^4 - 2.6689 \times 10^{-5} x_{-1}^3 + 0.12748 \times 10^{-3} x_{-1}^2 - 0.018507 \\ &= -0.67665 \times 10^{-8} (10)^4 - 2.6689 \times 10^{-5} (10)^3 + 0.12748 \times 10^{-3} (10)^2 - 0.018507 \\ &= -8.4956 \times 10^{-3} \end{aligned}$$

Example 1 Cont.

$$x_1 = 15 - \frac{(8.2591 \times 10^{-4})(15 - 10)}{(8.2591 \times 10^{-4}) - (-8.4956 \times 10^{-3})}$$

$$= 14.557$$

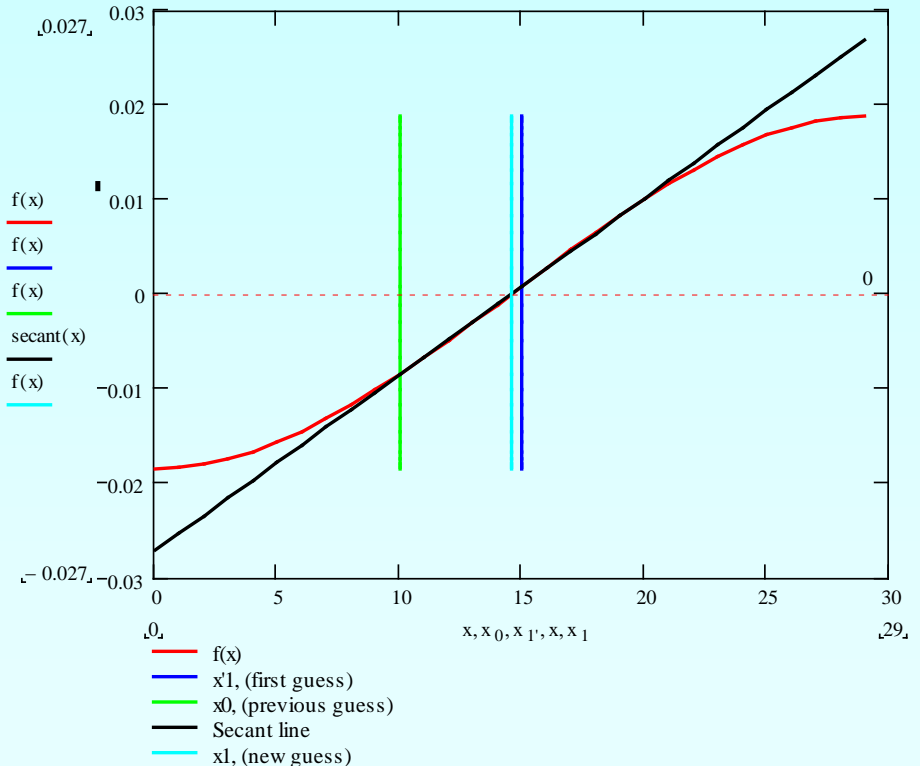


Figure 4 Graph of the estimated root after Iteration 1.

Example 1 Cont.

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 1 is

$$\begin{aligned} |\epsilon_a| &= \left| \frac{x_1 - x_0}{x_1} \right| \times 100 \\ &= \left| \frac{14.557 - 15}{14.557} \right| \times 100 \\ &= 3.0433\% \end{aligned}$$

The number of significant digits at least correct is 1, because the absolute relative approximate error is less than 5%.

Example 1 Cont.

Iteration 2

The estimate of the root is

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

$$f(x_1) = -0.67665 \times 10^{-8} x_1^4 - 2.6689 \times 10^{-5} x_1^3 + 0.12748 \times 10^{-3} x_1^2 - 0.018507$$

$$= -0.67665 \times 10^{-8} (14.557)^4 - 2.6689 \times 10^{-5} (14.557)^3 + 0.12748 \times 10^{-3} (14.557)^2 - 0.018507$$

$$= -2.9870 \times 10^{-5}$$

$$x_2 = 15 - \frac{(-2.9870 \times 10^{-5})(14.557 - 15)}{(-2.9870 \times 10^{-5}) - (8.2591 \times 10^{-4})}$$

$$= 14.572$$

Example 1 Cont.

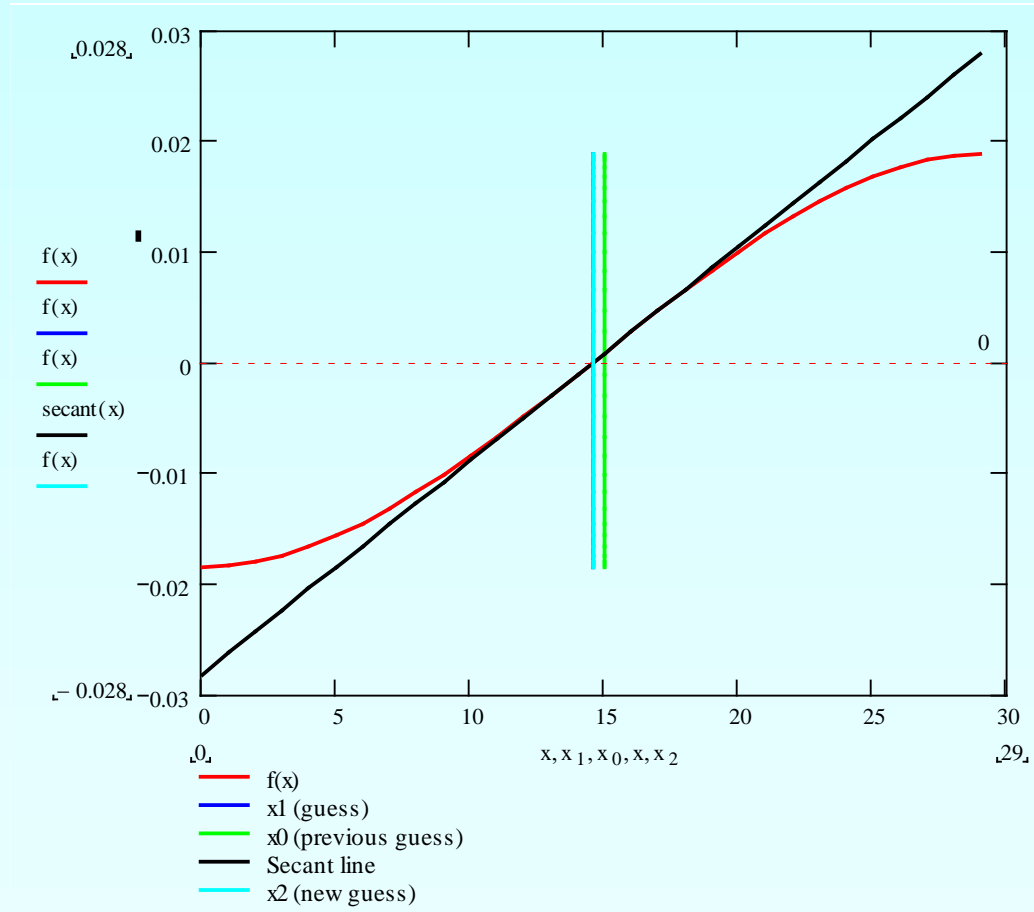


Figure 5 Graph of the estimate root after Iteration 2.

Example 1 Cont.

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 2 is

$$\begin{aligned} |\epsilon_a| &= \left| \frac{x_2 - x_1}{x_2} \right| \times 100 \\ &= \left| \frac{14.572 - 14.557}{14.572} \right| \times 100 \\ &= 0.10611\% \end{aligned}$$

The number of significant digits at least correct is 2, because the absolute relative approximate error is less than 0.5%.

Example 1 Cont.

Iteration 3

The estimate of the root is

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$f(x_2) = -0.67665 \times 10^{-8} x_2^4 - 2.6689 \times 10^{-5} x_2^3 + 0.12748 \times 10^{-3} x_2^2 - 0.018507$$

$$= -0.67665 \times 10^{-8} (14.572)^4 - 2.6689 \times 10^{-5} (14.572)^3 + 0.12748 \times 10^{-3} (14.572)^2 - 0.018507$$

$$= -6.0676 \times 10^{-9}$$

$$x_2 = 14.572 - \frac{(-6.0676 \times 10^{-9})(14.572 - 14.557)}{(-6.0676 \times 10^{-9}) - (-2.9870 \times 10^{-5})}$$

$$= 14.572$$

Example 1 Cont.

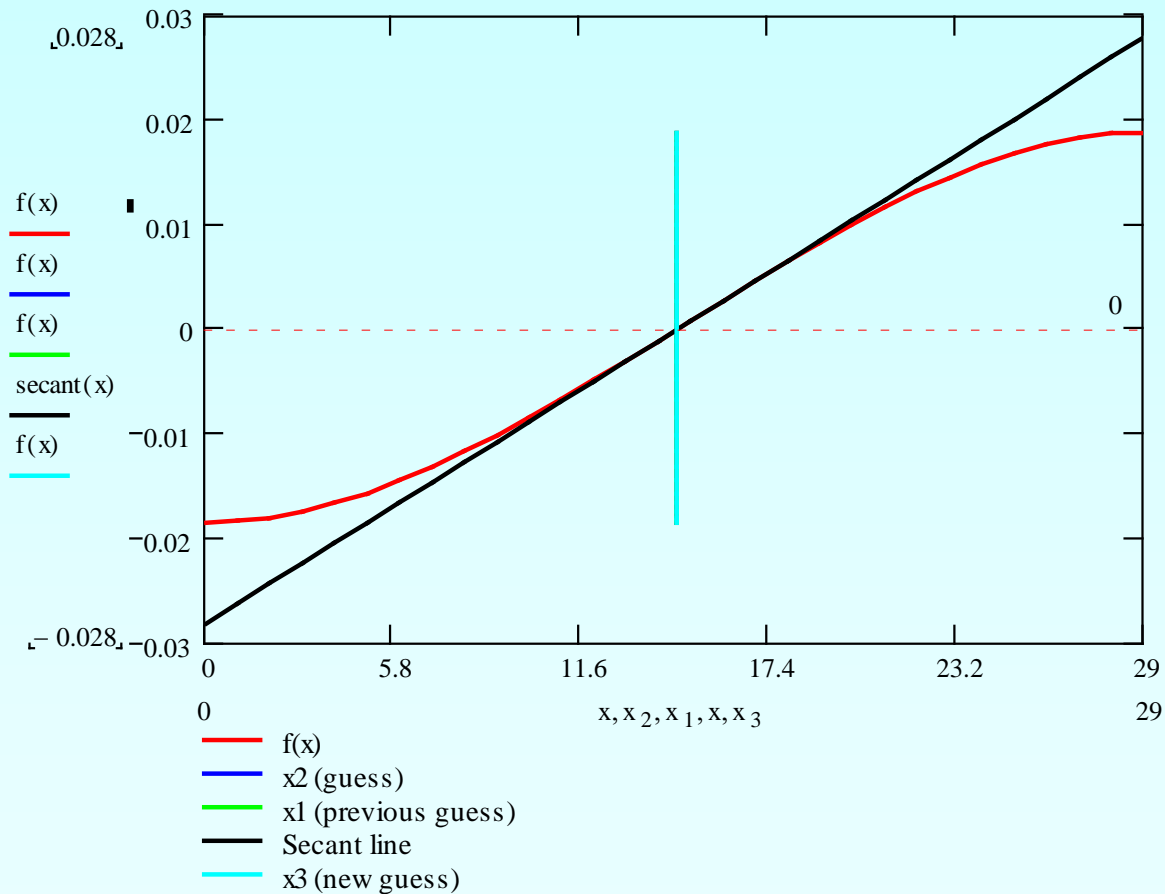


Figure 6 Graph of the estimate root after Iteration 3.

Example 1 Cont.

The absolute relative approximate error $|\epsilon_a|$ at the end of Iteration 3 is

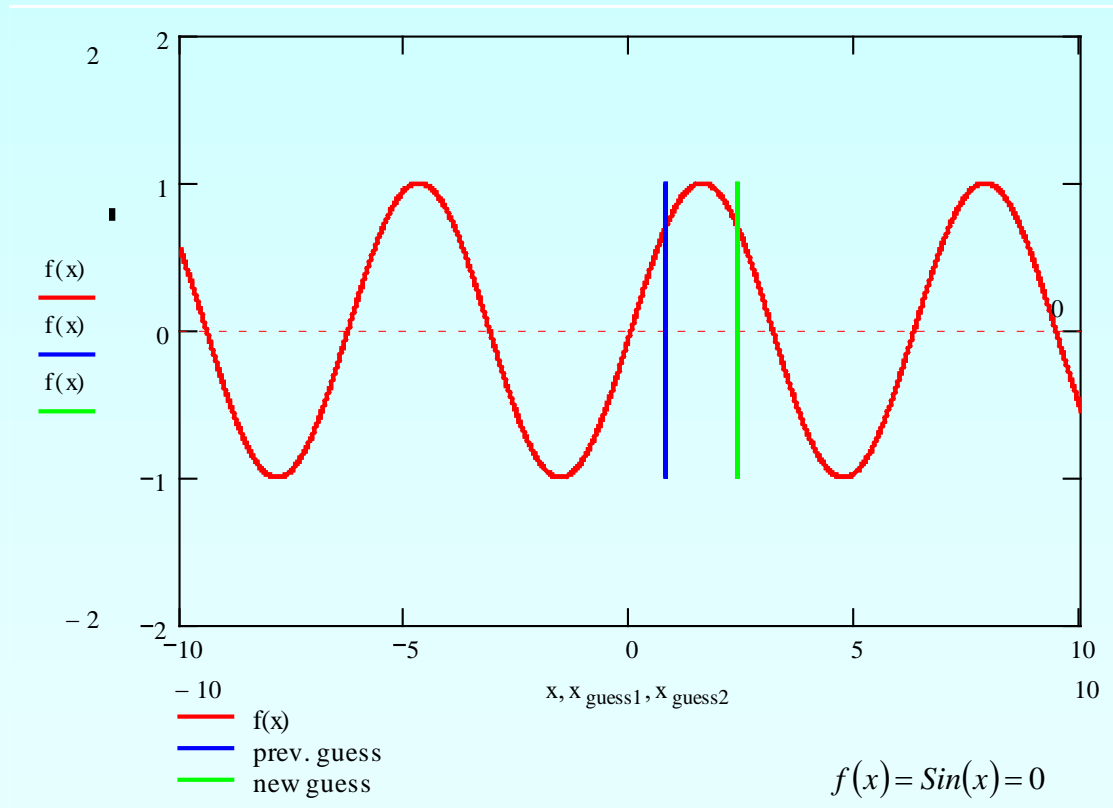
$$\begin{aligned} |\epsilon_a| &= \left| \frac{x_2 - x_1}{x_2} \right| \times 100 \\ &= \left| \frac{14.572 - 14.572}{14.572} \right| \times 100 \\ &= 2.1559 \times 10^{-5} \% \end{aligned}$$

The number of significant digits at least correct is 6, because the absolute relative approximate error is less than 0.00005%.

Advantages

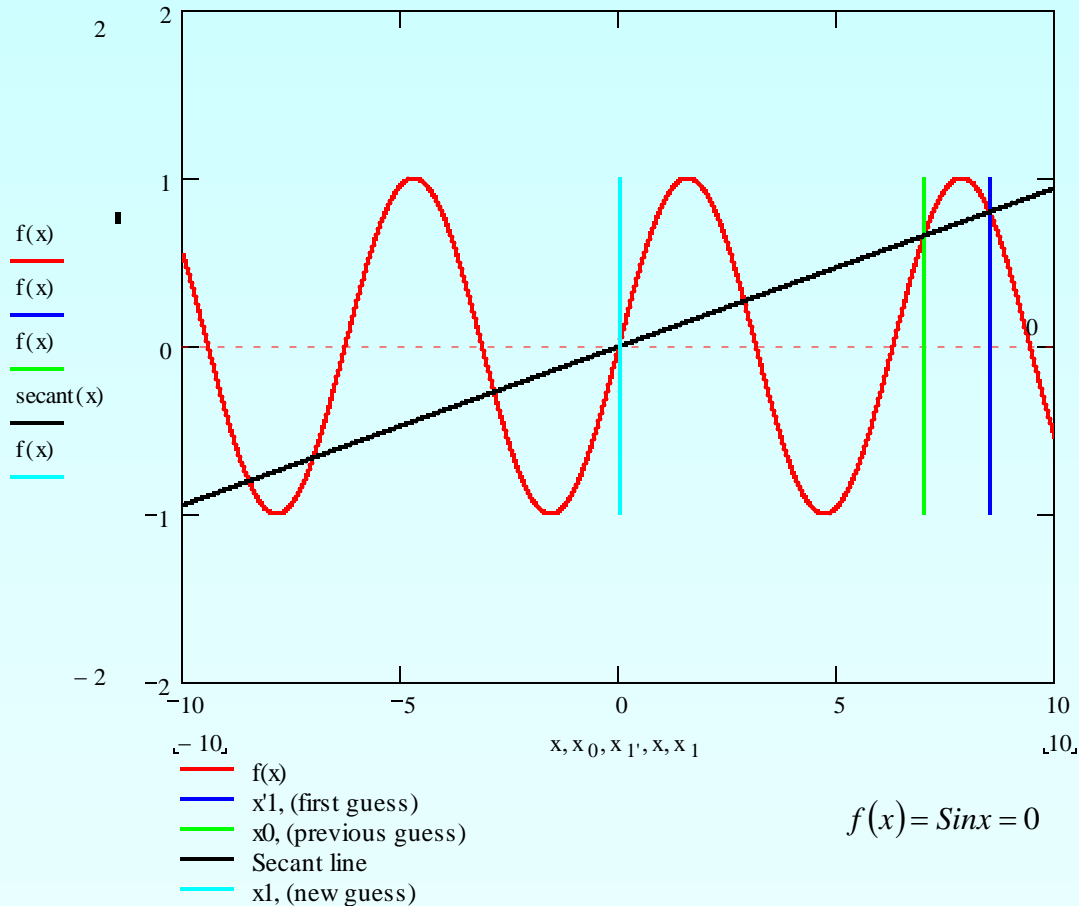
- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

Drawbacks



Division by zero

Drawbacks (continued)



Root Jumping

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/secant_method.html

THE END

<http://numericalmethods.eng.usf.edu>