Chapter 08.00C

Physical Problem for Civil Engineering Ordinary Differential Equations

When is it safe to return to the lake?

Pollution in lakes can be a serious issue as they are used for recreation use. Pollution resulting from sewage, runoff from suburban yards that is loaded with fertilizers and pesticides to keep the homeowner's associations off your back can be a safety hazard for people. One is generally interested in knowing that if the concentration of a particular pollutant is above acceptable levels, how long will it take for the pollution level to decrease to an acceptable level.

Figure 1 Pollutant in a lake

Mass of pollutant = Mass of pollutant entering – Mass of pollutant leaving

which also gives

Rate of change of mass of pollutant = Rate of change of mass of pollutant entering – Rate of change of mass of pollutant leaving.

If the concentration of pollutant is given by

$$C(t)=\frac{M(t)}{V}$$

where

$M(t)=$ mass of pollutant at time, t

$V=$ Volume of lake.

Rate of change of mass of pollutant entering is QC_o, where Q is the flow rate of the water into the lake, and the rate of change of mass of pollutant leaving the lake is $\frac{QM(t)}{V}$.

The above assumes that the flow rate of water going in and out of lake is the same. We also assuming that the pollutant is uniformly distributed in the lake. Also, no reaction is assumed. This gives

$$\frac{dM(t)}{dt}=QC_o-\frac{\dot{Q}M(t)}{V}$$

Now

$$M(t)=VC_o(t)$$

giving

$$V\frac{dC}{dt}=\dot{Q}C_o-\dot{Q}C$$

$$V\frac{dC}{dt}+\dot{Q}C=\dot{Q}C_o$$

Assume a weekly flow rate of fresh water as $1.5\times10^6\ \text{m}^3$. $C_o=0$ as we are assuming only fresh water coming in. The volume of the lake is $25\times10^6\ \text{m}^3$. If the initial concentration of the pollutant is $10^7\ \text{parts/m}^3$, and the acceptable level is $5\times10^6\ \text{parts/m}^3$, how much time would it take for the pollutant to reach acceptable levels.

$$25\times10^6\frac{dC}{dt}+1.5\times10^6C=0$$

$$\frac{dC}{dt}+0.06C=0, C(0)=10^7$$

ORDINARY DIFFERENTIAL EQUATION	
Topic	Ordinary Differential Equations
Summary	A physical problem of finding how much time it would take a lake to have safe levels of pollutant. To find the time, the problem is modeled as an ordinary differential equation.
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