

Chapter 06.00D

Physical Problem for Regression Computer Engineering

Problem Statement

Automated cartography deals with building or updating maps automatically from aerial or satellite images. Gone are the days when map updating was a laborious process that took a long period to accomplish. There are now many computer vision methods that one can employ to assist and ease the job of the cartographer. For instance, consider the aerial image shown, which is of a park with a lake. One of the tasks is to extract the road network that is visible. The road network appears as bright linear strips. On the right of that figure is shown the road network



Figure 1 On left is an aerial image of park in Munich, Germany. We see the roads and a lake. On the right is the road network extracted from the image.

that is automatically extracted from the image. In this module, we will see how to achieve this by fitting models to the local intensities in the image.

Image is a collection of gray level values at set of predetermined sites known as pixels, arranged in an array. These gray level values are also known as image intensities. In the above figure we see the profile of the intensity value along a line (shown in red) cutting a road in an image. We represent small values to represent dark points in the image and large values to detect bright regions in the image. The value increases and then decreases. This intensity profile can be modeled as an inverted parabola. The road can be detected by identifying pixels in the image that exhibit this intensity profile around it. To detect all the road pixels, we would have to repeat this process for intensity profile along the vertical as well along the horizontal directions at each pixel.

Let the intensities profile at a pixel be denoted by the values: $y_{-K}, \dots, y_0, \dots, y_K$, the index 0 is used to denote the pixel location under consideration and we look at K pixels on either

sides of it. We will use $x = -K, \dots, 0, \dots, K$ to denote these pixel locations that are the independent variables.

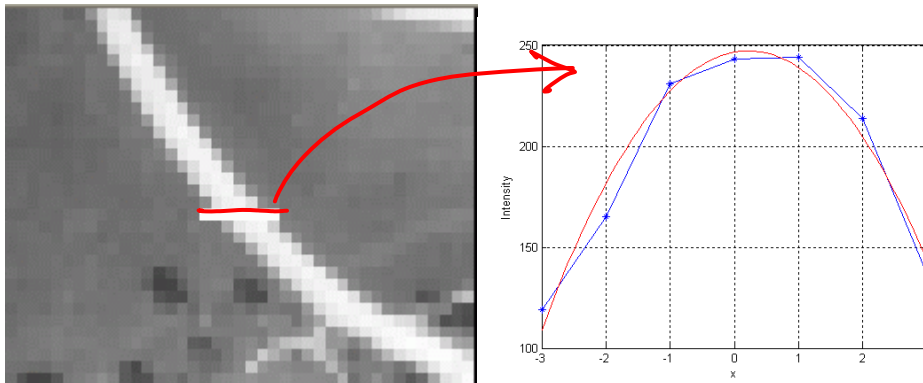


Figure 2 On left is a magnified portion of an image with a piece of the road. Note we can observe the individual pixels. On the right, we plot the intensities of the pixels marked by a red line. The raw intensities are plotted in blue, with fitted function in red.

Consider a polynomial model of the intensities.

$$y(x) = ax^2 + bx + c$$

We fit this second order polynomial model to the observed intensities to minimize the fit error denoted by

$$e = \sum_{k=-K}^{k=K} (y_k - y(k))^2 = \sum_{k=-K}^{k=K} (y_k - ak^2 - bk - c)^2$$

The unknown parameters, a , b , and c can be estimated to be the values that minimize the above error. The necessary condition is that the derivative of the error with respect to these parameters be zero.

$$\frac{\partial e}{\partial a} = -\sum 2k^2 (y_k - ak^2 - bk - c) = 0$$

$$\frac{\partial e}{\partial b} = -\sum 2k (y_k - ak^2 - bk - c) = 0$$

$$\frac{\partial e}{\partial c} = -\sum 2 (y_k - ak^2 - bk - c) = 0$$

The three equations can be expressed in matrix form as

$$\begin{bmatrix} \sum_k k^4 & \sum_k k^3 & \sum_k k^2 \\ \sum_k k^3 & \sum_k k^2 & \sum_k k^1 \\ \sum_k k^2 & \sum_k k^1 & \sum_k 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \sum_k k^2 y_k \\ \sum_k k y_k \\ \sum_k y_k \end{bmatrix}$$

We thus have three linear equations with three unknown, which can be solved using a variety of methods. The conditions corresponding to the road intensity profile, which is an inverted parabola, can be found by using the observation that the maximum of the fitted polynomial

should happen at or near $k = 0$. The condition for a maximum value is given by $a < 0$ and the location of the maximum is given by $-b/2a$, which should be between -0.5 and 0.5 pixels. Note that 0.5 denotes the halfway location between the pixel at location 0 and the next pixel, which is at 1. The value of the maximum is given by c , which should be high because we are looking of bright road edges.

Worked Out Example

Consider the following intensity profile: $\{y_k\} = 119, 165, 231, 243, 244, 214, 136$, corresponding to $k = -3, -2, -1, 0, 1, 2, 3$. This intensity profile corresponds to the one shown in Figure 2. The corresponding simultaneous equation for best least square fit is given by

$$\begin{bmatrix} 196 & 0 & 28 \\ 0 & 28 & 0 \\ 28 & 0 & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 4286 \\ 162 \\ 1352 \end{bmatrix}$$

The solution is given by $a = -13.3571$, $b = 5.7857$, and $c = 246.5714$. Since a is negative, we have a maximum value, which is located at $-b/2a = -0.2166$. This location is near 0 and the value at zero is 246.57, which denotes a bright value. All of these quantities suggest that we have a road at the pixel we are considering.

QUESTIONS

Instead of processing the an image, which is laid out in a 2D plane, along just the horizontal and vertical directions, it is more natural to consider fitting a 2D surface model, whose general form is given by

$$y(x, y) = ax^2 + by^2 + cxy + dx + ey + f$$

1. Work out the regression equation for fitting this surface model to intensities in a K by K neighborhood around a pixel.
2. How would you determine, based on the values of the estimated parameters, whether the pixel is on a road edge or not, that is, the fitted surface has a *roof-like* shape?

REGRESSION

Topic	Physical problem for regression
Summary	Extraction of road networks in images by function fitting
Major	Computer Science
Authors	Sudeep Sarkar
Date	December 23, 2009
Web Site	http://numericalmethods.eng.usf.edu
