Secant Method

Electrical Engineering Majors

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Secant Method – Derivation

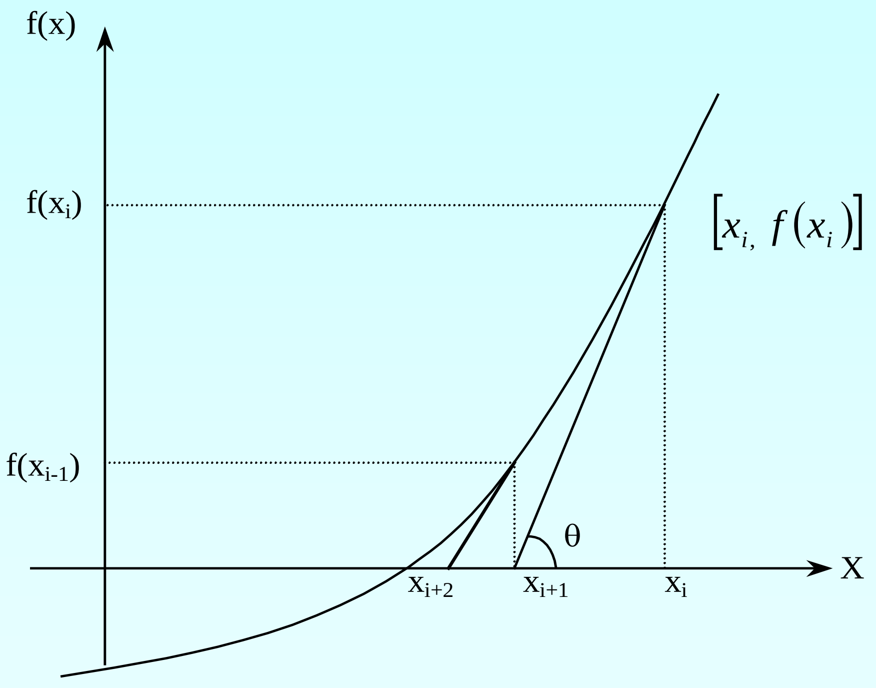


Figure 1 Geometrical illustration of the Newton-Raphson method.

Newton's Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad (1)$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \quad (2)$$

Substituting Equation (2) into Equation (1) gives the Secant method

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method – Derivation

The secant method can also be derived from geometry:

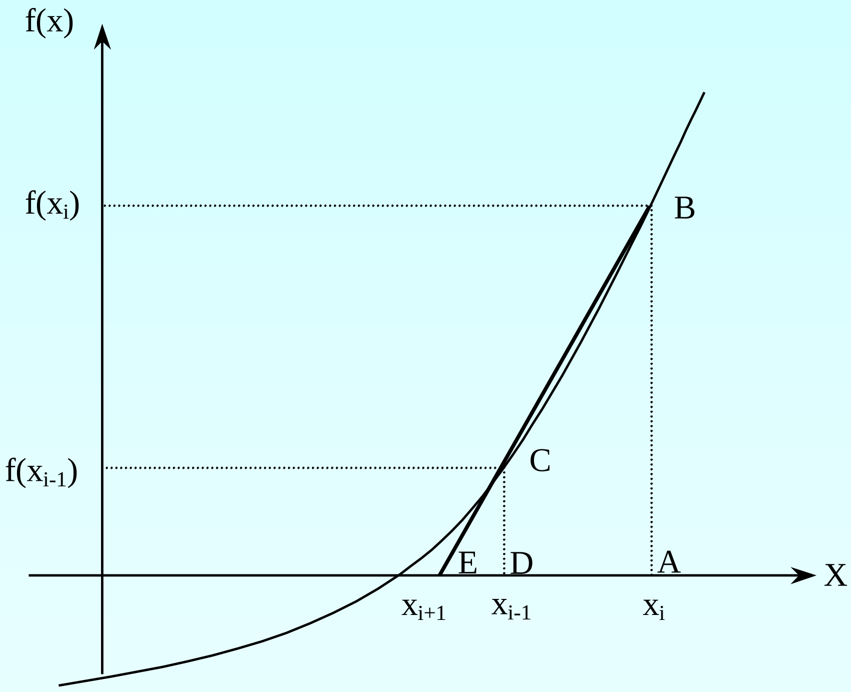


Figure 2 Geometrical representation of the Secant method.

The Geometric Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

can be written as

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

On rearranging, the secant method is given as

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$|\in_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

Step 2

Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example 1

Thermistors are temperature-measuring devices based on the principle that the thermistor material exhibits a change in electrical resistance with a change in temperature. By measuring the resistance of the thermistor material, one can then determine the temperature.

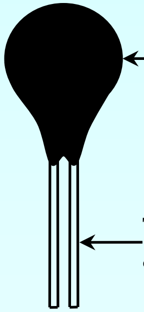


Figure 3 A typical thermistor.

For a 10K3A Betatherm thermistor, the relationship between the resistance, *R*, of the thermistor and the temperature is given by

$$\frac{1}{T} = 1.129241\times10^{-3} + 2.341077\times10^{-4}\ln(R) + 8.775468\times10^{-8}\{\ln(R)\}^3$$

where T is in Kelvin and R is in ohms.

Example 1 Cont.

For the thermistor, error of no more than ±0.01°C is acceptable. To find the range of the resistance that is within this acceptable limit at 19°C, we need to solve

$$\frac{1}{19.01+273.15} = 1.129241\times10^{-3} + 2.341077\times10^{-4}\ln(R) + 8.775468\times10^{-8}\{\ln(R)\}^3$$

and

$$\frac{1}{18.99+273.15} = 1.129241\times10^{-3} + 2.341077\times10^{-4}\ln(R) + 8.775468\times10^{-8}\{\ln(R)\}^3$$

Use the Newton-Raphson method of finding roots of equations to find the resistance R at 18.99°C.

a) Conduct three iterations to estimate the root of the above equation.

b) Find the absolute relative approximate error at the end of each iteration and the number of significant digits at least correct at the end of each iteration.

Example 1 Cont.

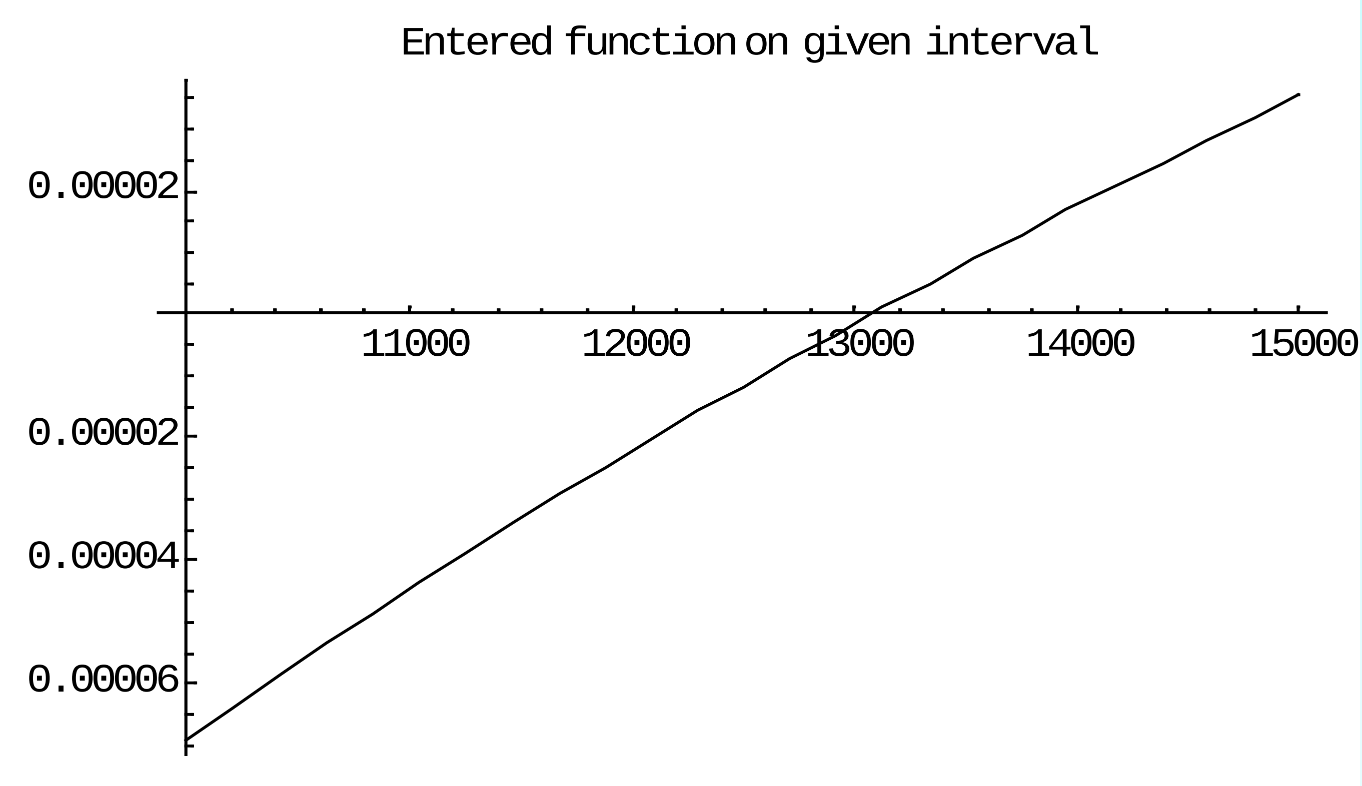


Figure 4 Graph of the function f(R).

$$f(R) = 2.341077\times10^{-4}\ln(R) + 8.775468\times10^{-8}\{\ln(R)\}^3 - 2.293775\times10^{-3}$$

Example 1 Cont.

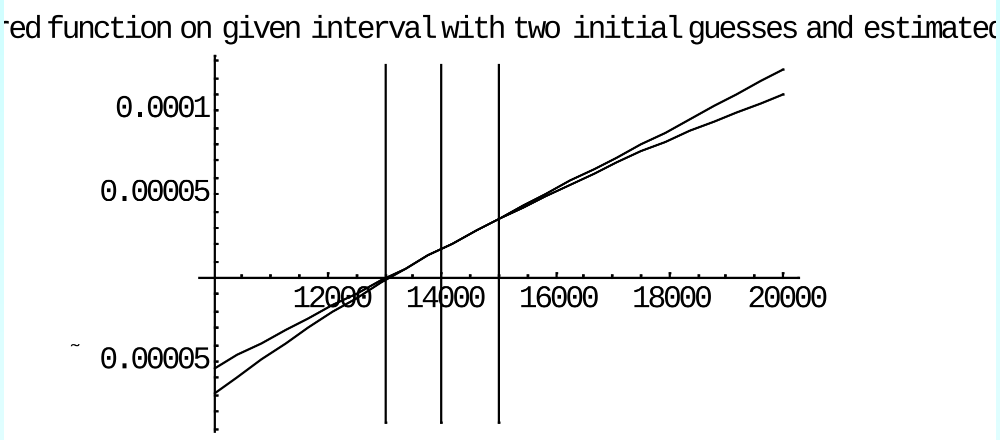


Figure 5 Graph of the estimate of the root after Iteration 1.

Initial guesses: $R_{-1} = 14000, R_0 = 15000$

<u>Iteration 1</u>

The estimate of the root is

$$R_1 = R_0 - \frac{f(R_0)(R_0 - R_{-1})}{f(R_0) - f(R_{-1})}$$

$$R_1 = 15000 - \frac{(3.5383\times10^{-5})(15000-14000)}{(3.5383\times10^{-5})-(1.7563\times10^{-5})}$$

$$= 13014$$

The absolute relative approximate error is

$$|\in_a| = \left|\frac{13014-15000}{13014}\right| \times 100$$

$$= 15.257\%$$

The number of significant digits at least correct is 0.

Example 1 Cont.

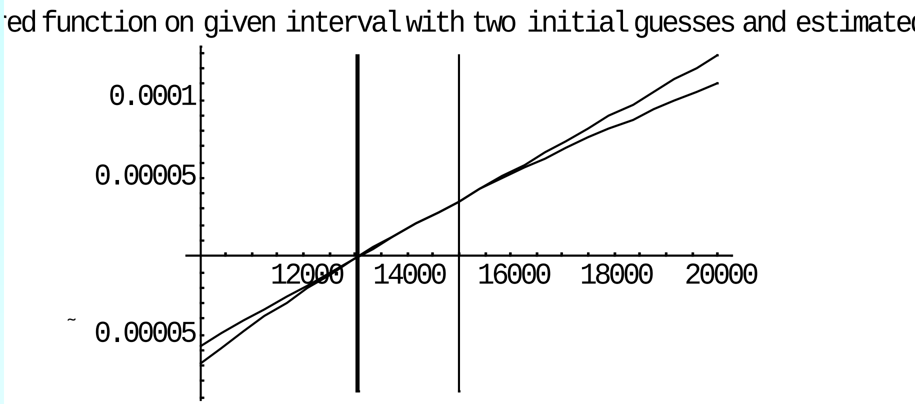


Figure 6 Graph of the estimate of the root after Iteration 2.

<u>Iteration 2</u>

The estimate of the root is

$$R_2 = R_1 - \frac{f(R_1)(R_1 - R_0)}{f(\mathrm{R}_1) - f(R_0)}$$

$$R_2 = 13014 - \frac{(-1.2658\times10^{-6})(13014-15000)}{(-1.2658\times10^{-6}) - (3.5383\times10^{-5})}$$

$$= 13083$$

The absolute relative approximate error is

$$|\in_a| = \left|\frac{13083-13014}{13083}\right|\times100$$

$$= 0.52422\%$$

The number of significant digits at least correct is 1.

Example 1 Cont.

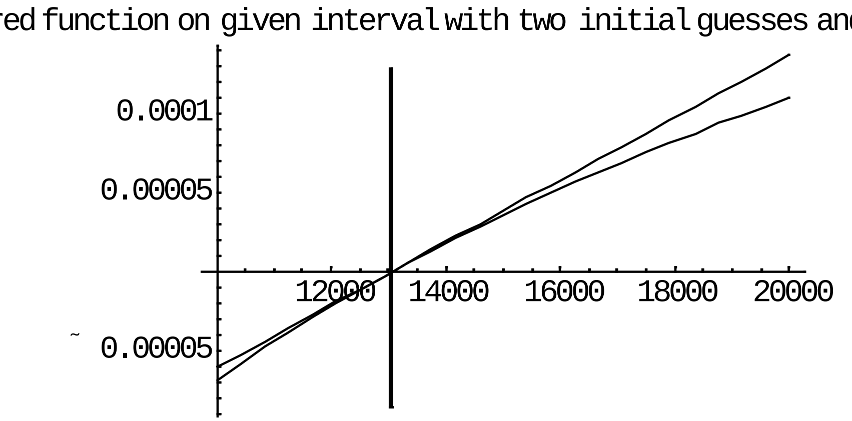


Figure 7 Graph of the estimate of the root after Iteration 3.

Iteration 3

The estimate of the root is

$$R_3 = R_2 - \frac{f(R_2)(R_2 - R_1)}{f(R_2) - f(R_1)}$$

$$R_3 = 13083 - \frac{(8.8911\times10^{-8})(13083 - 13014)}{(8.8911\times10^{-8}) - (-1.2658\times10^{-6})}$$

$$= 13078$$

The absolute relative approximate error is

$$|\epsilon_a| = \left|\frac{13078 - 13083}{13078}\right| \times 100$$

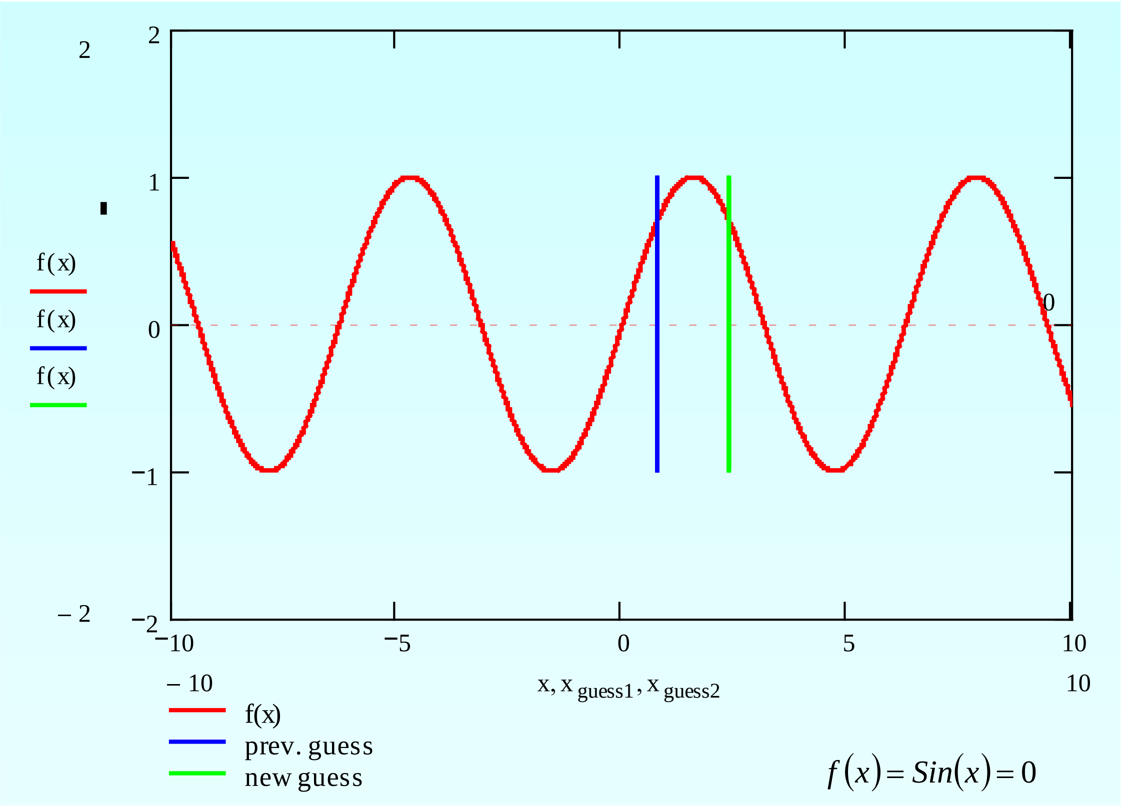
$$= 0.034415\%$$

The number of significant digits at least correct is 3.

Advantages

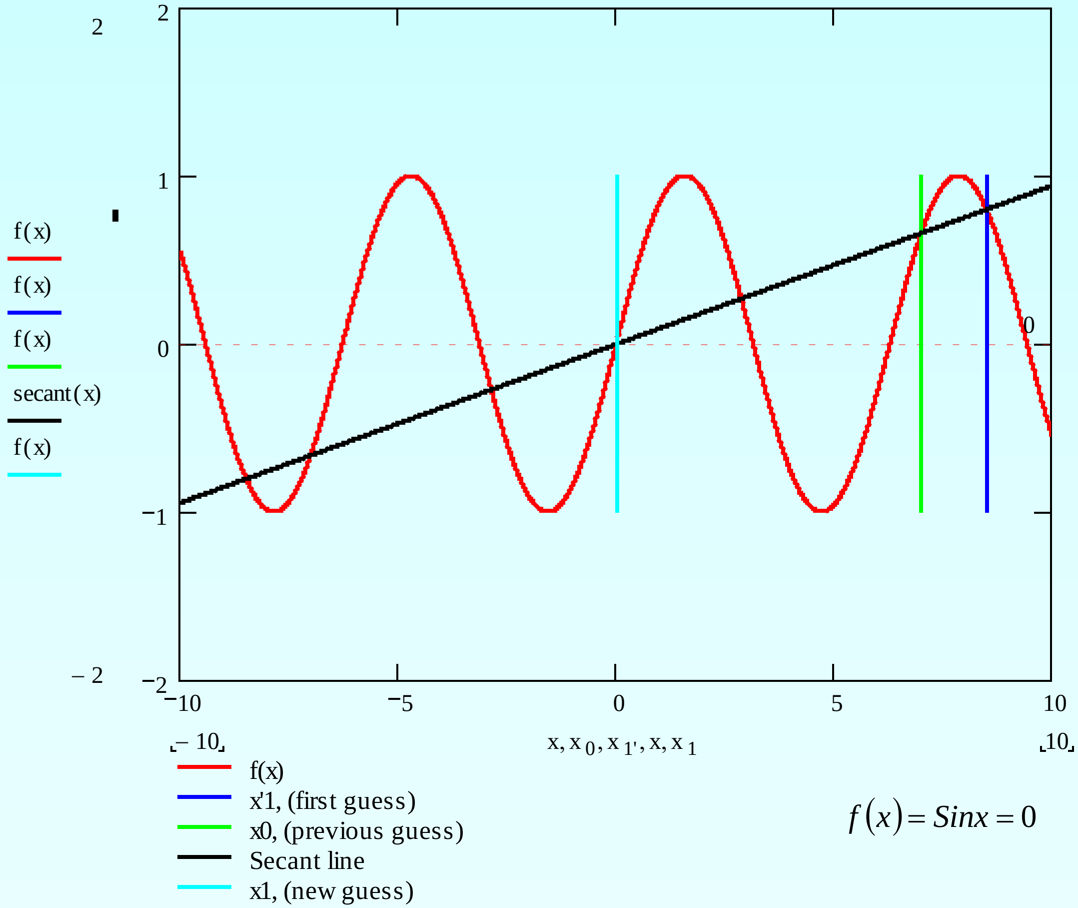
- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

Drawbacks



Division by zero

Drawbacks (continued)



$f(x) = Sinx = 0$

Root Jumping

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/secant_method.html

THE END

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