Gauss-Siedel Method

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Transforming Numerical Methods Education for STEM Undergraduates

Gauss-Seidel Method

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Gauss-Seidel Method

An ***iterative*** method.

Basic Procedure:

-Algebraically solve each linear equation for x_i

-Assume an initial guess solution array

-Solve for each x_i and repeat

-Use absolute relative approximate error after each iteration to check if error is within a pre-specified tolerance.

Gauss-Seidel Method

Why?

The Gauss-Seidel Method allows the user to control round-off error.

Elimination methods such as Gaussian Elimination and LU Decomposition are prone to prone to round-off error.

Also: If the physics of the problem are understood, a close initial guess can be made, decreasing the number of iterations needed.

Gauss-Seidel Method

Algorithm

A set of *n* equations and *n* unknowns:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + \ldots + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + \ldots + a_{2n}x_n = b_2$$

$$\vdots \qquad \vdots$$

$$a_{n1}x_1 + a_{n2}x_2 + a_{n3}x_3 + \ldots + a_{nn}x_n = b_n$$

If: the diagonal elements are non-zero

Rewrite each equation solving for the corresponding unknown

ex:

First equation, solve for x_1

Second equation, solve for x_2

Gauss-Seidel Method

Algorithm

Rewriting each equation

$$x_1 = \frac{c_1 - a_{12}x_2 - a_{13}x_3 \ldots\ldots - a_{1n}x_n}{a_{11}}$$ From Equation 1

$$x_2 = \frac{c_2 - a_{21}x_1 - a_{23}x_3 \ldots\ldots - a_{2n}x_n}{a_{22}}$$ From equation 2

$$\vdots \qquad \vdots \qquad \vdots$$

$$x_{n-1} = \frac{c_{n-1} - a_{n-1,1}x_1 - a_{n-1,2}x_2 \ldots\ldots - a_{n-1,n-2}x_{n-2} - a_{n-1,n}x_n}{a_{n-1,n-1}}$$ From equation n-1

$$x_n = \frac{c_n - a_{n1}x_1 - a_{n2}x_2 - \ldots\ldots - a_{n,n-1}x_{n-1}}{a_{nn}}$$ From equation n

Gauss-Seidel Method

Algorithm

General Form of each equation

$$x_1 = \frac{c_1 - \sum_{\substack{j=1 \\ j\neq 1}}^{n} a_{1j}x_j}{a_{11}}$$

$$x_{n-1} = \frac{c_{n-1} - \sum_{\substack{j=1 \\ j\neq n-1}}^{n} a_{n-1,j}x_j}{a_{n-1,n-1}}$$

$$x_2 = \frac{c_2 - \sum_{\substack{j=1 \\ j\neq 2}}^{n} a_{2j}x_j}{a_{22}}$$

$$x_n = \frac{c_n - \sum_{\substack{j=1 \\ j\neq n}}^{n} a_{nj}x_j}{a_{nn}}$$

Gauss-Seidel Method

Algorithm

General Form for any row 'i'

$$x_i = \frac{c_i - \sum_{\substack{j=1 \\ j \neq i}}^{n} a_{ij} x_j}{a_{ii}}, i = 1,2,\ldots,n.$$

How or where can this equation be used?

Gauss-Seidel Method

Solve for the unknowns

Assume an initial guess for [X]

$$\begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_{n-1} \\ x_n \end{bmatrix}$$

Use rewritten equations to solve for each value of x_i.

Important: Remember to use the most recent value of x_i. Which means to apply values calculated to the calculations remaining in the **current** iteration.

Gauss-Seidel Method

Calculate the Absolute Relative Approximate Error

$$\left|\in_a\right|_i = \left|\frac{x_i^{new} - x_i^{old}}{x_i^{new}}\right| \times 100$$

So when has the answer been found?

The iterations are stopped when the absolute relative approximate error is less than a prespecified tolerance for all unknowns.

Example: Unbalanced three phase load

Three-phase loads are common in AC systems. When the system is balanced the analysis can be simplified to a single equivalent circuit model. However, when it is unbalanced the only practical solution involves the solution of simultaneous linear equations. In a model the following equations need to be solved.

$$\begin{bmatrix} 0.7460 & -0.4516 & 0.0100 & -0.0080 & 0.0100 & -0.0080 \\ 0.4516 & 0.7460 & 0.0080 & 0.0100 & 0.0080 & 0.0100 \\ 0.0100 & -0.0080 & 0.7787 & -0.5205 & 0.0100 & -0.0080 \\ 0.0080 & 0.0100 & 0.5205 & 0.7787 & 0.0080 & 0.0100 \\ 0.0100 & -0.0080 & 0.0100 & -0.0080 & 0.8080 & -0.6040 \\ 0.0080 & 0.0100 & 0.0080 & 0.0100 & 0.6040 & 0.8080 \end{bmatrix} \begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 120 \\ 0.000 \\ -60.00 \\ -103.9 \\ -60.00 \\ 103.9 \end{bmatrix}$$

Find the values of I_{ar}, I_{ai}, I_{br}, I_{bi}, I_{cr}, and I_{ci} using the Gauss-Seidel method.

Example: Unbalanced three phase load

Rewrite each equation to solve for each of the unknowns

$$I_{ar} = \frac{120.00 - (-0.4516)I_{ai} - 0.0100 I_{br} - (-0.0080)I_{bi} - 0.0100 I_{cr} - (-0.0080)I_{ci}}{0.7460}$$

$$I_{ai} = \frac{0.000 - 0.4516 I_{ar} - 0.0080 I_{br} - 0.0100 I_{bi} - 0.0080 I_{cr} - 0.0100 I_{ci}}{0.7460}$$

$$I_{br} = \frac{-60.00 - 0.0100 I_{ar} - (-0.0080)I_{ai} - (-0.5205)I_{bi} - 0.0100 I_{cr} - (-0.0080)I_{ci}}{0.7787}$$

$$I_{bi} = \frac{-103.9 - 0.0080 I_{ar} - 0.0100 I_{ai} - 0.5205 I_{br} - 0.0080 I_{cr} - 0.0100 I_{ci}}{0.7787}$$

$$I_{cr} = \frac{-60.00 - 0.0100 I_{ar} - (-0.0080)I_{ai} - 0.0100 I_{br} - (-0.0080)I_{bi} - (-0.6040)I_{ci}}{0.8080}$$

$$I_{ci} = \frac{103.9 - 0.0080 I_{ar} - 0.0100 I_{ai} - 0.0080 I_{br} - 0.0100 I_{bi} - 0.6040 I_{cr}}{0.8080}$$

Example: Unbalanced three phase load

For iteration 1, start with an initial guess value

Initial Guess:

$$\begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 20 \\ 20 \\ 20 \\ 20 \\ 20 \\ 20 \end{bmatrix}$$

Example: Unbalanced three phase load

Substituting the guess values into the first equation

$$I_{ar} = \frac{120 - \left(-0.4516\right) I_{ai} - 0.0100 I_{br} - \left(-0.0080\right) I_{bi} - 0.0100 I_{cr} - \left(-0.0080\right) I_{ci}}{0.7460}$$

$$= 172.86$$

Substituting the new value of I_{ar} and the remaining guess values into the second equation

$$I_{ai} = \frac{0.00 - 0.4516 I_{ar} - 0.0080 I_{br} - 0.0100 I_{bi} - 0.0080 I_{cr} - 0.0100 I_{ci}}{0.7460}$$

$$= -105.61$$

Example: Unbalanced three phase load

Substituting the new values I_{ar}, I_{ai}, and the remaining guess values into the third equation

$$I_{br} = \frac{-60.00 - 0.0100 I_{ar} - (-0.0080) I_{ai} - (-0.5205) I_{bi} - 0.0100 I_{cr} - (-0.0080) I_{ci}}{0.7787}$$

$$= -67.039$$

Substituting the new values I_{ar}, I_{ai}, I_{br}, and the remaining guess values into the fourth equation

$$I_{bi} = \frac{-103.9 - 0.0080 I_{ar} - 0.0100 I_{ai} - 0.5205 I_{br} - 0.0080 I_{cr} - 0.0100 I_{ci}}{0.7787}$$

$$= -89.499$$

Example: Unbalanced three phase load

Substituting the new values I_{ar}, I_{ai}, I_{br}, I_{bi}, and the remaining guess values into the fifth equation

$$I_{cr} = \frac{-60.00 - 0.0100 I_{ar} - (-0.0080) I_{ai} - 0.0100 I_{br} - (-0.0080) I_{bi} - (-0.6040) I_{ci}}{0.8080}$$

$$= -62.548$$

Substituting the new values I_{ar}, I_{ai}, I_{br}, I_{bi}, I_{cr}, and the remaining guess value into the sixth equation

$$I_{ci} = \frac{103.9 - 0.0080 I_{ar} - 0.0100 I_{ai} - 0.0080 I_{br} - 0.0100 I_{bi} - 0.6040 I_{cr}}{0.8080}$$

$$= 176.71$$

Example: Unbalanced three phase load

At the end of the first iteration, the solution matrix is:

$$\begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 172.86 \\ -105.61 \\ -67.039 \\ -89.499 \\ -62.548 \\ 176.71 \end{bmatrix}$$

How accurate is the solution? Find the absolute relative approximate error using:

$$\left|\in_a\right|_i = \left|\frac{x_i^{new} - x_i^{old}}{x_i^{new}}\right| \times 100$$

Example: Unbalanced three phase load

Calculating the absolute relative approximate errors

$$\left|\in_a\right|_1 = \left|\frac{172.86-20}{172.86}\right|\times 100 = 88.430\%$$

$$\left|\in_a\right|_2 = \left|\frac{-105.61-20}{-105.61}\right|\times 100 = 118.94\%$$

$$\left|\in_a\right|_3 = \left|\frac{-67.039-20}{-67.039}\right|\times 100 = 129.83\%$$

$$\left|\in_a\right|_4 = \left|\frac{-89.499-20}{-89.499}\right|\times 100 = 122.35\%$$

$$\left|\in_a\right|_5 = \left|\frac{-62.548-20}{-62.548}\right|\times 100 = 131.98\%$$

$$\left|\in_a\right|_6 = \left|\frac{176.71-20}{176.71}\right|\times 100 = 88.682\%$$

The maximum error after the first iteration is:

131.98%

Another iteration is needed!

Example: Unbalanced three phase load

Starting with the values obtained in iteration #1

$$\begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 172.86 \\ -105.61 \\ -67.039 \\ -89.499 \\ -62.548 \\ 176.71 \end{bmatrix}$$

Substituting the values from Iteration 1 into the first equation

$$I_{ar} = \frac{120.00 - (-0.4516)I_{ai} - 0.0100 I_{br} - (-0.0080)I_{bi} - 0.0100 I_{cr} - (-0.0080)I_{ci}}{0.7460}$$

$$= 99.600$$

Example: Unbalanced three phase load

Substituting the new value of I_{ar} and the remaining values from Iteration 1 into the second equation

$$I_{ai} = \frac{0.00 - 0.4516 I_{ar} - 0.0080 I_{br} - 0.0100 I_{bi} - 0.0080 I_{cr} - 0.0100 I_{ci}}{0.7460}$$

$$= -60.073$$

Substituting the new values I_{ar}, I_{ai}, and the remaining values from Iteration 1 into the third equation

$$I_{br} = \frac{-60.00 - 0.0100 I_{ar} - (-0.0080) I_{ai} - (-0.5205) I_{bi} - 0.0100 I_{cr} - (-0.0080) I_{ci}}{0.7787}$$

$$= -136.15$$

Example: Unbalanced three phase load

Substituting the new values I_{ar}, I_{ai}, I_{br}, and the remaining values from Iteration 1 into the fourth equation

$$I_{bi} = \frac{-103.9 - 0.0080 I_{ar} - 0.0100 I_{ai} - 0.5205 I_{br} - 0.0080 I_{cr} - 0.0100 I_{ci}}{0.7787}$$

$$= -44.299$$

Substituting the new values I_{ar}, I_{ai}, I_{br}, I_{bi}, and the remaining values From Iteration 1 into the fifth equation

$$I_{cr} = \frac{-60.00 - 0.0100 I_{ar} - (-0.0080) I_{ai} - 0.0100 I_{br} - (-0.0080) I_{bi} - (-0.6040) I_{ci}}{0.8080}$$

$$= 57.259$$

Example: Unbalanced three phase load

Substituting the new values I_{ar}, I_{ai}, I_{br}, I_{bi}, I_{cr}, and the remaining value from Iteration 1 into the sixth equation

$$I_{ci} = \frac{103.9 - 0.0080 I_{ar} - 0.0100 I_{ai} - 0.0080 I_{br} - 0.0100 I_{bi} - 0.6040 I_{cr}}{0.8080}$$

$$= 87.441$$

The solution matrix at the end of the second iteration is:

$$\begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 99.600 \\ -60.073 \\ -136.15 \\ -44.299 \\ 57.259 \\ 87.441 \end{bmatrix}$$

Example: Unbalanced three phase load

Calculating the absolute relative approximate errors for the second iteration

$$\left|\in_a\right|_1 = \left|\frac{99.600 - 172.86}{99.600}\right| \times 100 = 73.552\%$$

$$\left|\in_a\right|_2 = \left|\frac{-60.073 - (-105.61)}{-60.073}\right| \times 100 = 75.796\%$$

$$\left|\in_a\right|_3 = \left|\frac{-136.35 - (-67.039)}{-136.35}\right| \times 100 = 50.762\%$$

$$\left|\in_a\right|_4 = \left|\frac{-44.299 - (-89.499)}{-44.299}\right| \times 100 = 102.03\%$$

$$\left|\in_a\right|_5 = \left|\frac{57.259 - (-62.548)}{57.259}\right| \times 100 = 209.24\%$$

$$\left|\in_a\right|_6 = \left|\frac{87.441 - 176.71}{87.441}\right| \times 100 = 102.09\%$$

The maximum error after the second iteration is:

209.24%

More iterations are needed!

Example: Unbalanced three phase load

Repeating more iterations, the following values are obtained

Iteration	I_{ar}	I_{ai}	I_{br}	I_{bi}	I_{cr}	I_{ci}
1	172.86	−105.61	−67.039	−89.499	−62.548	176.71
2	99.600	−60.073	−136.15	−44.299	57.259	87.441
3	126.01	−76.015	−108.90	−62.667	−10.478	137.97
4	117.25	−70.707	−119.62	−55.432	27.658	109.45
5	119.87	−72.301	−115.62	−58.141	6.2513	125.49
6	119.28	−71.936	−116.98	−57.216	18.241	116.53

Iteration	$\left\|\in_a\right\|_1 \%$	$\left\|\in_a\right\|_2 \%$	$\left\|\in_a\right\|_3 \%$	$\left\|\in_a\right\|_4 \%$	$\left\|\in_a\right\|_5 \%$	$\left\|\in_a\right\|_6 \%$
1	88.430	118.94	129.83	122.35	131.98	88.682
2	73.552	75.796	50.762	102.03	209.24	102.09
3	20.960	20.972	25.027	29.311	646.45	36.623
4	7.4738	7.5067	8.9631	13.053	137.89	26.001
5	2.1840	2.2048	3.4633	4.6595	342.43	12.742
6	0.49408	0.50789	1.1629	1.6170	65.729	7.6884

Example: Unbalanced three phase load

After six iterations, the solution matrix is

$$\begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 119.28 \\ -71.936 \\ -116.98 \\ 57.216 \\ 18.241 \\ 116.53 \end{bmatrix}$$

The maximum error after the sixth iteration is:

65.729%

The absolute relative approximate error is still high, but allowing for more iterations, the error quickly begins to converge to zero.

What could have been done differently to allow for a faster convergence?

Example: Unbalanced three phase load

Repeating more iterations, the following values are obtained

Iteration	I_{ar}	I_{ai}	I_{br}	I_{bi}	I_{cr}	I_{ci}
32	119.33	−71.973	−116.66	−57.432	13.940	119.74
33	119.33	−71.973	−116.66	−57.432	13.940	119.74

Iteration	$\left\|\in_a\right\|_1 \%$	$\left\|\in_a\right\|_2 \%$	$\left\|\in_a\right\|_3 \%$	$\left\|\in_a\right\|_4 \%$	$\left\|\in_a\right\|_5 \%$	$\left\|\in_a\right\|_6 \%$
32	3.0666×10^{-7}	3.0047×10^{-7}	4.2389×10^{-7}	5.7116×10^{-7}	2.0941×10^{-5}	1.8238×10^{-6}
33	1.7062×10^{-7}	1.6718×10^{-7}	2.3601×10^{-7}	3.1801×10^{-7}	1.1647×10^{-5}	1.0144×10^{-6}

Example: Unbalanced three phase load

After 33 iterations, the solution matrix is

$$\begin{bmatrix} I_{ar} \\ I_{ai} \\ I_{br} \\ I_{bi} \\ I_{cr} \\ I_{ci} \end{bmatrix} = \begin{bmatrix} 119.33 \\ -71.973 \\ -116.66 \\ -57.432 \\ 13.940 \\ 119.74 \end{bmatrix}$$

The maximum absolute relative approximate error is $1.1647 \times 10^{-5}\%$.

Gauss-Seidel Method: Pitfall

Even though done correctly, the answer may not converge to the correct answer.

This is a pitfall of the Gauss-Siedel method: not all systems of equations will converge.

Is there a fix?

One class of system of equations always converges: One with a *diagonally dominant* coefficient matrix.

Diagonally dominant: [A] in [A] [X] = [C] is diagonally dominant if:

$$\left|a_{\mathrm{ii}}\right| \geq \sum_{\substack{j=1 \\ j \neq \mathrm{i}}}^{n} \left|a_{ij}\right| \quad \text{for all 'i'} \qquad \text{and} \qquad \left|a_{ii}\right| > \sum_{\substack{j=1 \\ j \neq i}}^{n} \left|a_{ij}\right| \quad \text{for at least one 'i'}$$

Gauss-Seidel Method: Pitfall

Diagonally dominant: The coefficient on the diagonal must be at least equal to the sum of the other coefficients in that row and at least one row with a diagonal coefficient greater than the sum of the other coefficients in that row.

Which coefficient matrix is diagonally dominant?

$$[A] = \begin{bmatrix} 2 & 5.81 & 34 \\ 45 & 43 & 1 \\ 123 & 16 & 1 \end{bmatrix}$$

$$[B] = \begin{bmatrix} 124 & 34 & 56 \\ 23 & 53 & 5 \\ 96 & 34 & 129 \end{bmatrix}$$

Most physical systems do result in simultaneous linear equations that have diagonally dominant coefficient matrices.

Gauss-Seidel Method: Example 2

Given the system of equations

$$12x_1 + 3x_2 - 5x_3 = 1$$

$$x_1 + 5x_2 + 3x_3 = 28$$

$$3x_1 + 7x_2 + 13x_3 = 76$$

The coefficient matrix is:

$$[A] = \begin{bmatrix} 12 & 3 & -5 \\ 1 & 5 & 3 \\ 3 & 7 & 13 \end{bmatrix}$$

With an initial guess of

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Will the solution converge using the Gauss-Siedel method?

Gauss-Seidel Method: Example 2

Checking if the coefficient matrix is diagonally dominant

$$[A]=\begin{bmatrix}12 & 3 & -5\\ 1 & 5 & 3\\ 3 & 7 & 13\end{bmatrix}$$

$$\left|a_{11}\right| = \left|12\right| = 12 \geq \left|a_{12}\right| + \left|a_{13}\right| = \left|3\right| + \left|-5\right| = 8$$

$$\left|a_{22}\right| = \left|5\right| = 5 \geq \left|a_{21}\right| + \left|a_{23}\right| = \left|1\right| + \left|3\right| = 4$$

$$\left|a_{33}\right| = \left|13\right| = 13 \geq \left|a_{31}\right| + \left|a_{32}\right| = \left|3\right| + \left|7\right| = 10$$

The inequalities are all true and at least one row is *strictly* greater than:

Therefore: The solution should converge using the Gauss-Siedel Method

Gauss-Seidel Method: Example 2

Rewriting each equation

$$\begin{bmatrix} 12 & 3 & -5 \\ 1 & 5 & 3 \\ 3 & 7 & 13 \end{bmatrix} \begin{bmatrix} a_1 \\ a_2 \\ a_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 28 \\ 76 \end{bmatrix}$$

$$x_1 = \frac{1 - 3x_2 + 5x_3}{12}$$

$$x_2 = \frac{28 - x_1 - 3x_3}{5}$$

$$x_3 = \frac{76 - 3x_1 - 7x_2}{13}$$

With an initial guess of

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$x_1 = \frac{1 - 3(0) + 5(1)}{12} = 0.50000$$

$$x_2 = \frac{28 - (0.5) - 3(1)}{5} = 4.9000$$

$$x_3 = \frac{76 - 3(0.50000) - 7(4.9000)}{13} = 3.0923$$

Gauss-Seidel Method: Example 2

The absolute relative approximate error

$$\left|\in_a\right|_1 = \left|\frac{0.50000 - 1.0000}{0.50000}\right| \times 100 = 100.00\%$$

$$\left|\in_a\right|_2 = \left|\frac{4.9000 - 0}{4.9000}\right| \times 100 = 100.00\%$$

$$\left|\in_a\right|_3 = \left|\frac{3.0923 - 1.0000}{3.0923}\right| \times 100 = 67.662\%$$

The maximum absolute relative error after the first iteration is 100%

Gauss-Seidel Method: Example 2

After Iteration #1

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.5000 \\ 4.9000 \\ 3.0923 \end{bmatrix}$$

Substituting the x values into the equations

$$x_1 = \frac{1 - 3(4.9000) + 5(3.0923)}{12} = 0.14679$$

$$x_2 = \frac{28 - (0.14679) - 3(3.0923)}{5} = 3.7153$$

$$x_3 = \frac{76 - 3(0.14679) - 7(4.900)}{13} = 3.8118$$

After Iteration #2

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.14679 \\ 3.7153 \\ 3.8118 \end{bmatrix}$$

Gauss-Seidel Method: Example 2

Iteration #2 absolute relative approximate error

$$\left|\in_a\right|_1 = \left|\frac{0.14679 - 0.50000}{0.14679}\right| \times 100 = 240.61\%$$

$$\left|\in_a\right|_2 = \left|\frac{3.7153 - 4.9000}{3.7153}\right| \times 100 = 31.889\%$$

$$\left|\in_a\right|_3 = \left|\frac{3.8118 - 3.0923}{3.8118}\right| \times 100 = 18.874\%$$

The maximum absolute relative error after the first iteration is 240.61%

This is much larger than the maximum absolute relative error obtained in iteration #1. Is this a problem?

Gauss-Seidel Method: Example 2

Repeating more iterations, the following values are obtained

Iteration	a_1	$\left\|\in_a\right\|_1 \%$	a_2	$\left\|\in_a\right\|_2 \%$	a_3	$\left\|\in_a\right\|_3 \%$
1	0.50000	100.00	4.9000	100.00	3.0923	67.662
2	0.14679	240.61	3.7153	31.889	3.8118	18.876
3	0.74275	80.236	3.1644	17.408	3.9708	4.0042
4	0.94675	21.546	3.0281	4.4996	3.9971	0.65772
5	0.99177	4.5391	3.0034	0.82499	4.0001	0.074383
6	0.99919	0.74307	3.0001	0.10856	4.0001	0.00101

The solution obtained $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0.99919 \\ 3.0001 \\ 4.0001 \end{bmatrix}$ is close to the exact solution of $\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 4 \end{bmatrix}$.

Gauss-Seidel Method: Example 3

Given the system of equations

$$3x_1 + 7x_2 + 13x_3 = 76$$

$$x_1 + 5x_2 + 3x_3 = 28$$

$$12x_1 + 3x_2 - 5x_3 = 1$$

With an initial guess of

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

Rewriting the equations

$$x_1 = \frac{76 - 7x_2 - 13x_3}{3}$$

$$x_2 = \frac{28 - x_1 - 3x_3}{5}$$

$$x_3 = \frac{1 - 12x_1 - 3x_2}{-5}$$

Gauss-Seidel Method: Example 3

Conducting six iterations, the following values are obtained

Iteration	a_1	$\left\|\in_a\right\|_1 \%$	A_2	$\left\|\in_a\right\|_2 \%$	a_3	$\left\|\in_a\right\|_3 \%$
1	21.000	95.238	0.80000	100.00	50.680	98.027
2	−196.15	110.71	14.421	94.453	−462.30	110.96
3	−1995.0	109.83	−116.02	112.43	4718.1	109.80
4	−20149	109.90	1204.6	109.63	−47636	109.90
5	2.0364 10^5	109.89	−12140	109.92	4.8144 10^5	109.89
6	−2.0579 10^5	109.89	1.2272 10^5	109.89	−4.8653 10^6	109.89

The values are not converging.

Does this mean that the Gauss-Seidel method cannot be used?

Gauss-Seidel Method

The Gauss-Seidel Method can still be used

The coefficient matrix is not diagonally dominant

$$[A] = \begin{bmatrix} 3 & 7 & 13 \\ 1 & 5 & 3 \\ 12 & 3 & -5 \end{bmatrix}$$

But this is the same set of equations used in example #2, which did converge.

$$[A] = \begin{bmatrix} 12 & 3 & -5 \\ 1 & 5 & 3 \\ 3 & 7 & 13 \end{bmatrix}$$

If a system of linear equations is not diagonally dominant, check to see if rearranging the equations can form a diagonally dominant matrix.

Gauss-Seidel Method

Not every system of equations can be rearranged to have a diagonally dominant coefficient matrix.

Observe the set of equations

$$x_1 + x_2 + x_3 = 3$$

$$2x_1 + 3x_2 + 4x_3 = 9$$

$$x_1 + 7x_2 + x_3 = 9$$

Which equation(s) prevents this set of equation from having a diagonally dominant coefficient matrix?

Gauss-Seidel Method

Summary

-Advantages of the Gauss-Seidel Method

-Algorithm for the Gauss-Seidel Method

-Pitfalls of the Gauss-Seidel Method

Gauss-Seidel Method

Questions?

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/gauss_seidel.html

THE END

http://numericalmethods.eng.usf.edu