# Direct Method of Interpolation

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# What is Interpolation ?

Given $(x_0,y_0)$, $(x_1,y_1)$, …… $(x_n,y_n)$, find the value of 'y' at a value of 'x' that is not given.

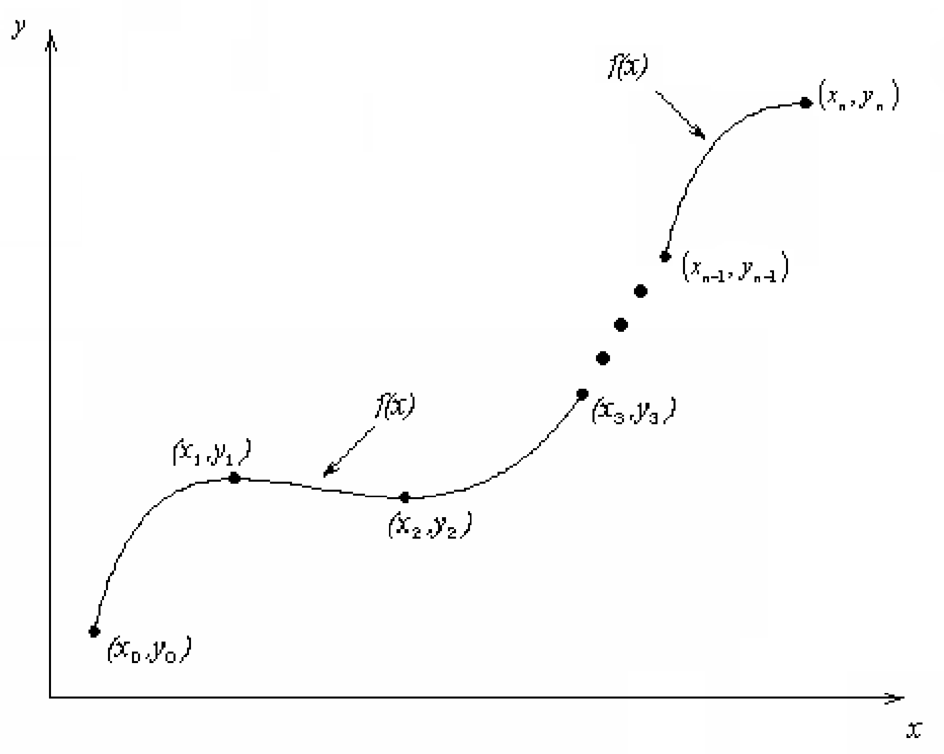



**Figure 1** Interpolation of discrete.

# Interpolants

Polynomials are the most common choice of interpolants because they are easy to:

- Evaluate
- Differentiate, and
- Integrate

# Direct Method

Given 'n+1' data points $(x_0,y_0)$, $(x_1,y_1)$,.............. $(x_n,y_n)$, pass a polynomial of order 'n' through the data as given below:

$$y = a_0 + a_1 x + \ldots\ldots\ldots\ldots\ldots\ldots\ldots + a_n x^n.$$

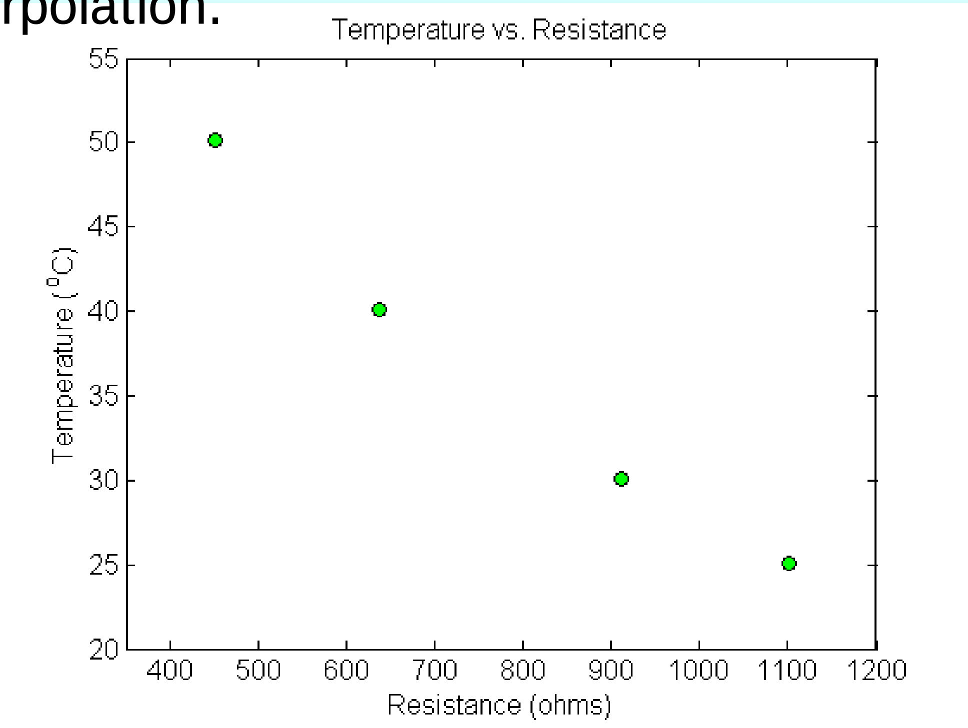
where $a_0$, $a_1$,.................. $a_n$ are real constants.

- Set up 'n+1' equations to find 'n+1' constants.
- To find the value 'y' at a given value of 'x', simply substitute the value of 'x' in the above polynomial.

# Example

Thermistors are based on materials' change in resistance with temperature. A manufacturer of thermistors makes the following observations on a thermistor. Determine the temperature corresponding to 754.8 ohms using the direct method for linear, quadratic, and cubic interpolation.

| R (Ω) | T( C) |
|---|---|
| 1101.0 | 25.113 |
| 911.3 | 30.131 |
| 636.0 | 40.120 |
| 451.1 | 50.128 |

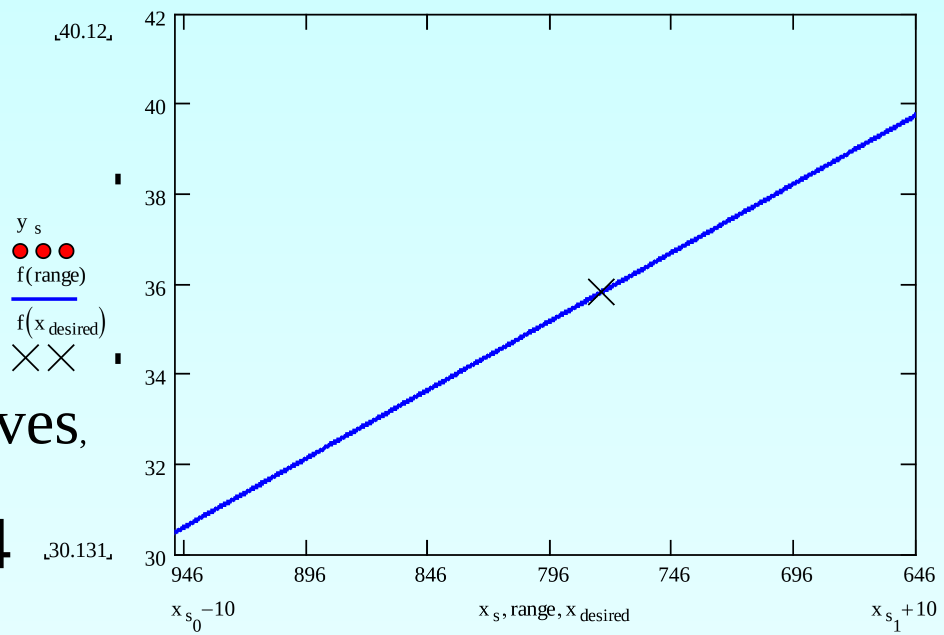


# Linear Interpolation

$$T(R) = a_0 + a_1 R$$

$$T(911.3) = a_0 + a_1(911.3) = 30.131$$

$$T(636.0) = a_0 + a_1(636.0) = 40.120$$




Solving the above two equations gives,

$$a_0 = 63.197 \quad a_1 = -0.036284$$

Hence

$$T(R) = 63.197 - 0.036284R, \quad 636.0 \le R \le 911.3$$

$$T(754.8) = 63.197 - 0.036284(754.8) = 35.809°\text{C}$$

# Quadratic Interpolation

$$T(R) = a_0 + a_1 R + a_2 R^2$$

$$T(911.3) = a_0 + a_1(911.3) + a_2(911.3)^2 = 30.131$$

$$T(636.0) = a_0 + a_1(636.0) + a_2(636.0)^2 = 40.120$$

$$T(451.1) = a_0 + a_1(451.1) + a_2(451.1)^2 = 50.128$$

Solving the above three equations gives

$$a_0 = 85.668 \quad a_1 = -0.096275 \quad a_2 = 3.8771 \times 10^{-5}$$

# Quadratic Interpolation (contd)

$$T(R) = 85.668 - 0.096275R + 3.8771\times10^{-5}R^2,\ \ 451.1 \le R \le 911.3$$

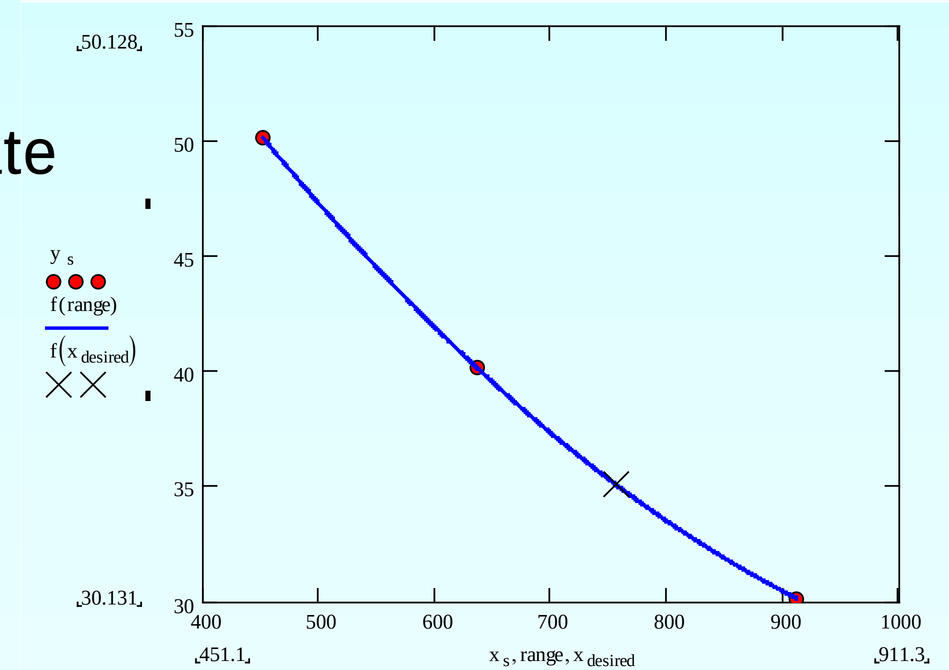
$$T(754.8) = 85.668 - 0.096275(754.8) + 3.8771\times10^{-5}(754.8)^2$$

$$= 35.089°\text{C}$$

The absolute relative approximate error obtained between the first and second order polynomial is

$$\left|\in_a\right| = \left|\frac{35.089 - 35.809}{35.089}\right| \times 100$$

$$= 2.0543\%$$



# Cubic Interpolation

$$T(R) = a_0 + a_1 R + a_2 R^2 + a_3 R^3$$

$$T(1101.0) = 25.113 = a_0 + a_1(1101.0) + a_2(1101.0)^2 + a_3(1101.0)^3$$

$$T(911.3) = 30.131 = a_0 + a_1(911.3) + a_2(911.3)^2 + a_3(911.3)^3$$

$$T(636.0) = 40.120 = a_0 + a_1(636.0) + a_2(636.0)^2 + a_3(636.0)^3$$

$$T(451.1) = 50.128 = a_0 + a_1(451.1) + a_2(451.1)^2 + a_3(451.1)^3$$

$$a_0 = 92.759 \qquad a_1 = -0.13093$$

$$a_2 = 9.2975 \times 10^{-5} \qquad a_3 = -2.7124 \times 10^{-8}$$

# Cubic Interpolation (contd)

$$T(R) = 92.759 - 0.13093R + 9.2975\times10^{-5}R^2 - 2.7124\times10^{-8}R^3,\quad 451.1 \le R \le 1101.0$$

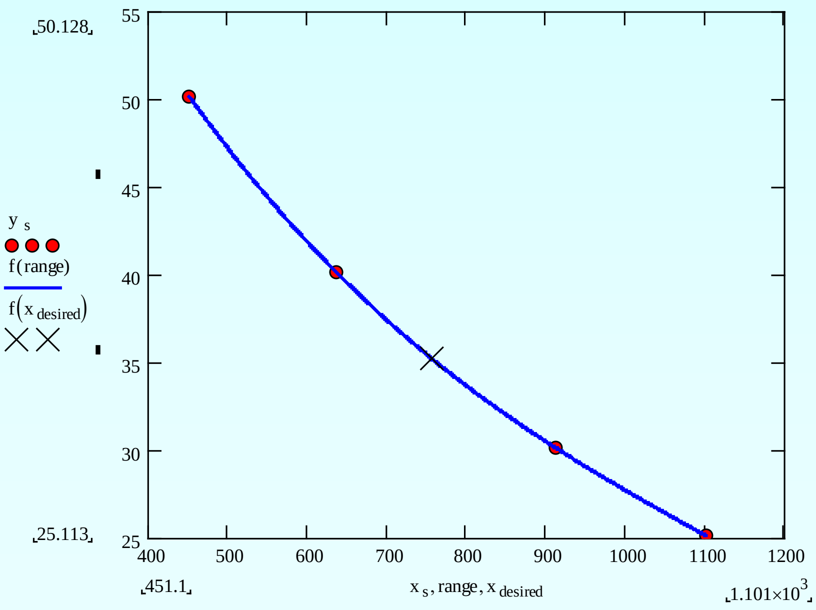
$$T(754.8) = 92.759 - 0.13093(754.8) + 9.2975\times10^{-5}(754.8)^2 - 2.7124\times10^{-8}(754.8)^3$$

$$= 35.242°\text{C}$$

The absolute relative approximate error between the second and third order polynomial is

$$\left|\in_a\right| = \left|\frac{35.242 - 35.089}{35.242}\right| \times 100$$

$$= 0.43458\%$$



# Comparison Table

| Order of Polynomial | 1 | 2 | 3 |
|---|---|---|---|
| Temperature $°C$ | 35.809 | 35.089 | 35.242 |
| Absolute Relative Approximate Error | ---------- | 2.0543% | 0.43458% |

# Actual Calibration

The actual calibration curve used is $\frac{1}{T} = a_0 + a_1[\ln R] + a_1[\ln R]^2 + a_1[\ln R]^3$

Substituting $y = \frac{1}{T}$, and $x = \ln R$ , gives the calibration curve as

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

Find the calibration curve and find the temperature corresponding to 754.8 ohms. What is the difference between the results from cubic interpolation? In which method is the difference larger, if the actual measured value at 754.8 ohms is 35.285°C

| $R$ (Ω) | $T$ ( C) | $x$ (ln$R$) | $y$ (1/$T$) |
|---|---|---|---|
| 1101.0 | 25.113 | 7.0040 | 0.039820 |
| 911.3 | 30.131 | 6.8149 | 0.033188 |
| 636.0 | 40.120 | 6.4552 | 0.024925 |
| 451.1 | 50.128 | 6.1117 | 0.019949 |

# Actual Calibration

$$y = a_0 + a_1 x + a_2 x^2 + a_3 x^3$$

$$y(7.0040) = 0.039820 = a_0 + a_1(7.0040) + a_2(7.0040)^2 + a_3(7.0040)^3$$

$$y(6.8149) = 0.033188 = a_0 + a_1(6.8149) + a_2(6.8149)^2 + a_3(6.8149)^3$$

$$y(6.4552) = 0.024925 = a_0 + a_1(6.4552) + a_2(6.4552)^2 + a_3(6.4552)^3$$

$$y(6.1117) = 0.019949 = a_0 + a_1(6.1117) + a_2(6.1117)^2 + a_3(6.1117)^3$$

$a_0 = -2.5964$ $a_1 = 1.2605$ $a_2 = -0.20448$ $a_3 = 0.011173$

# Actual Calibration

$$y(x) = -2.5964 + 1.2605x - 0.20448x^2 + 0.011173x^3, \; 6.1117 \le x \le 7.0040$$

However, since $y = \frac{1}{T}$ , and $x = \ln R$ ,

$$\frac{1}{T} = -2.5964 + 1.2605(\ln R) - 0.20448(\ln R)^2 + 0.011173(\ln R)^3, \; 451.1 \le R \le 1101.0$$

$$T(R) = \frac{1}{-2.5964 + 1.2605(\ln R) - 0.20448(\ln R)^2 + 0.011173(\ln R)^3}, \; 451.1 \le R \le 1101.0$$

At $R$ = 754.8,

$$T(754.8) = \frac{1}{-2.5964 + 1.2605(\ln(754.8)) - 0.20448(\ln(754.8))^2 + 0.011173(\ln(754.8))^3}$$

$$= 35.355°\mathrm{C}$$

# Actual Calibration

Since the actual measured value at 754.8 ohms is 35.285°C, the absolute relative true error between the value used for Cubic Interpolation is

$$\left|\in_t\right| = \left|\frac{35.285 - 35.242}{35.285}\right| \times 100$$

$$= 0.12253\%$$

and for actual calibration is

$$\left|\in_t\right| = \left|\frac{35.285 - 35.355}{35.285}\right| \times 100$$

$$= 0.19825\%$$

Therefore, the direct method of cubic polynomial interpolation obtained more accurate results than the actual calibration curve.

# Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/direct_method.html

# THE END

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