

Direct Method of Interpolation

Major: All Engineering Majors

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Transforming Numerical Methods Education for STEM Undergraduates

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What is Interpolation ?

Given $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$, find the value of 'y' at a value of 'x' that is not given.

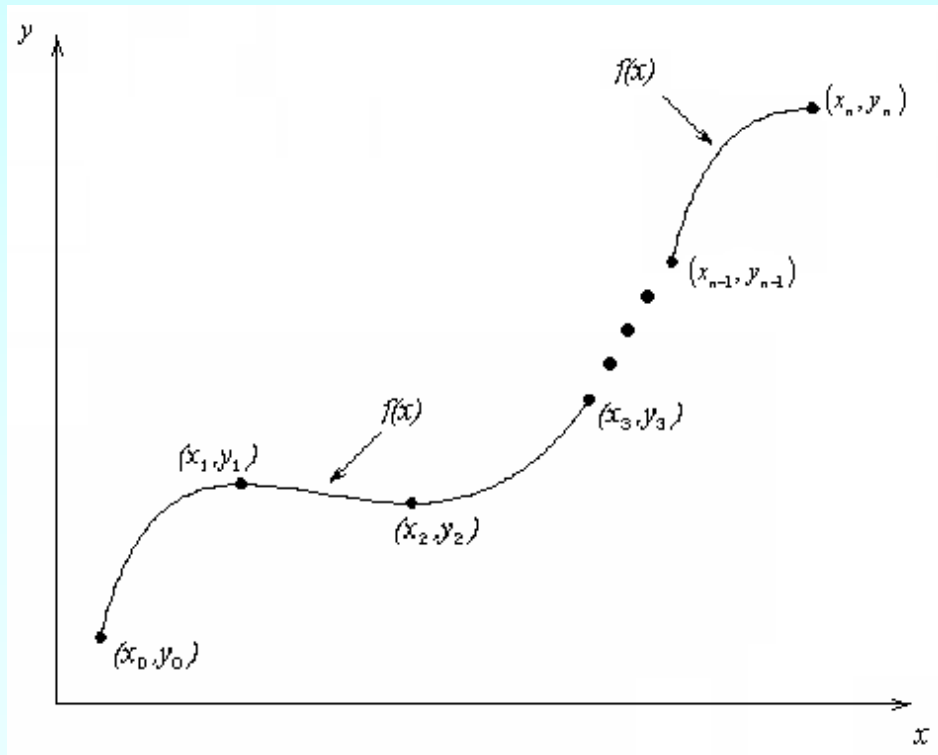


Figure 1 Interpolation of discrete.

Interpolants

Polynomials are the most common choice of interpolants because they are easy to:

- Evaluate
- Differentiate, and
- Integrate

Direct Method

Given 'n+1' data points $(x_0, y_0), (x_1, y_1), \dots, (x_n, y_n)$, pass a polynomial of order 'n' through the data as given below:

$$y = a_0 + a_1x + \dots + a_nx^n .$$

where a_0, a_1, \dots, a_n are real constants.

- Set up 'n+1' equations to find 'n+1' constants.
- To find the value 'y' at a given value of 'x', simply substitute the value of 'x' in the above polynomial.



Example 1

The upward velocity of a rocket is given as a function of time in Table 1.

Find the velocity at $t=16$ seconds using the direct method for linear interpolation.

Table 1 Velocity as a function of time.

$t, (s)$	$v(t), (m/s)$
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

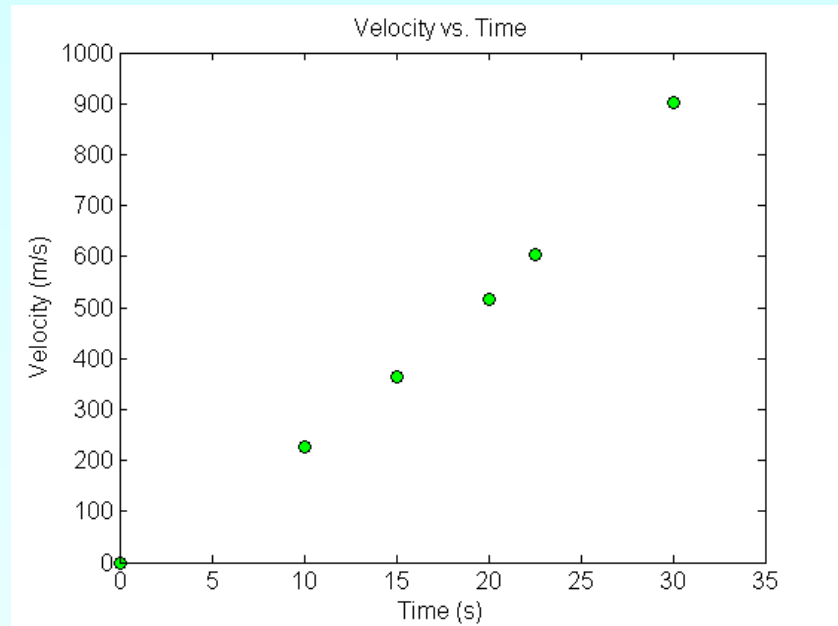


Figure 2 Velocity vs. time data for the rocket example

Linear Interpolation

$$v(t) = a_0 + a_1 t$$

$$v(15) = a_0 + a_1(15) = 362.78$$

$$v(20) = a_0 + a_1(20) = 517.35$$

Solving the above two equations gives,

$$a_0 = -100.93 \quad a_1 = 30.914$$

Hence

$$v(t) = -100.93 + 30.914t, \quad 15 \leq t \leq 20.$$

$$v(16) = -100.93 + 30.914(16) = 393.7 \text{ m/s}$$

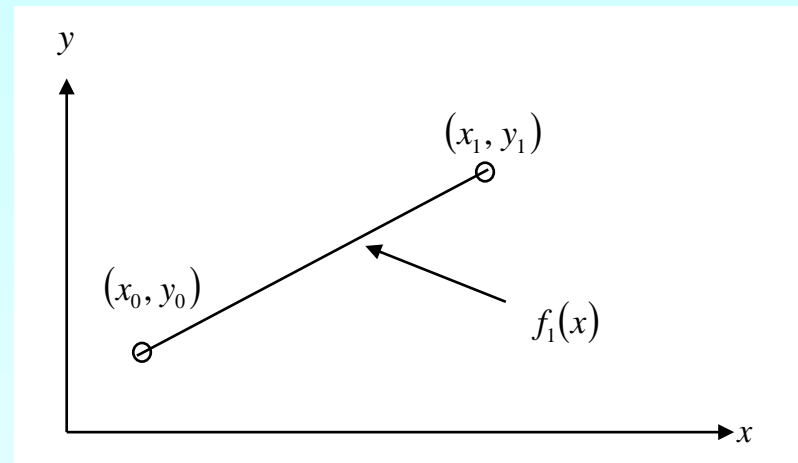


Figure 3 Linear interpolation.



Example 2

The upward velocity of a rocket is given as a function of time in Table 2.

Find the velocity at $t=16$ seconds using the direct method for quadratic interpolation.

Table 2 Velocity as a function of time.

$t, (s)$	$v(t), (m/s)$
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

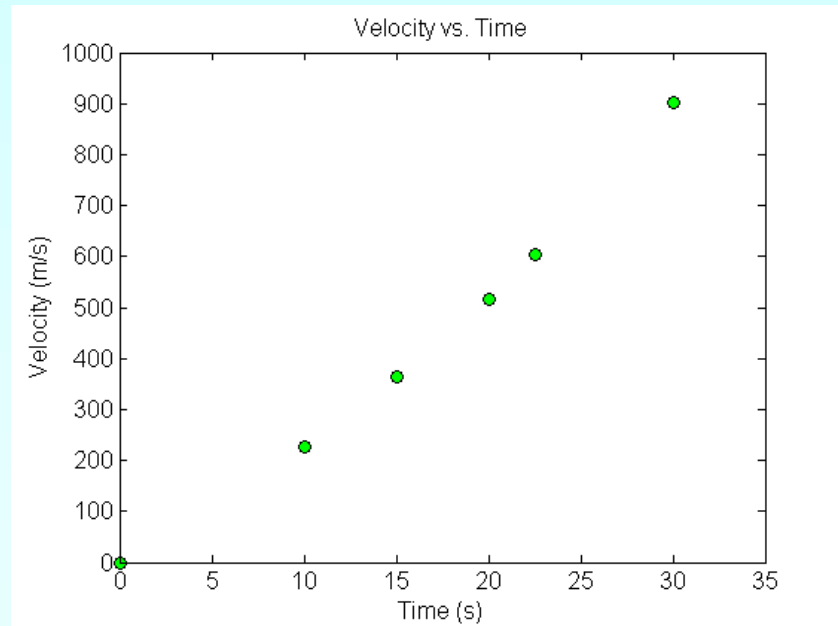


Figure 5 Velocity vs. time data for the rocket example

Quadratic Interpolation

$$v(t) = a_0 + a_1 t + a_2 t^2$$

$$v(10) = a_0 + a_1(10) + a_2(10)^2 = 227.04$$

$$v(15) = a_0 + a_1(15) + a_2(15)^2 = 362.78$$

$$v(20) = a_0 + a_1(20) + a_2(20)^2 = 517.35$$

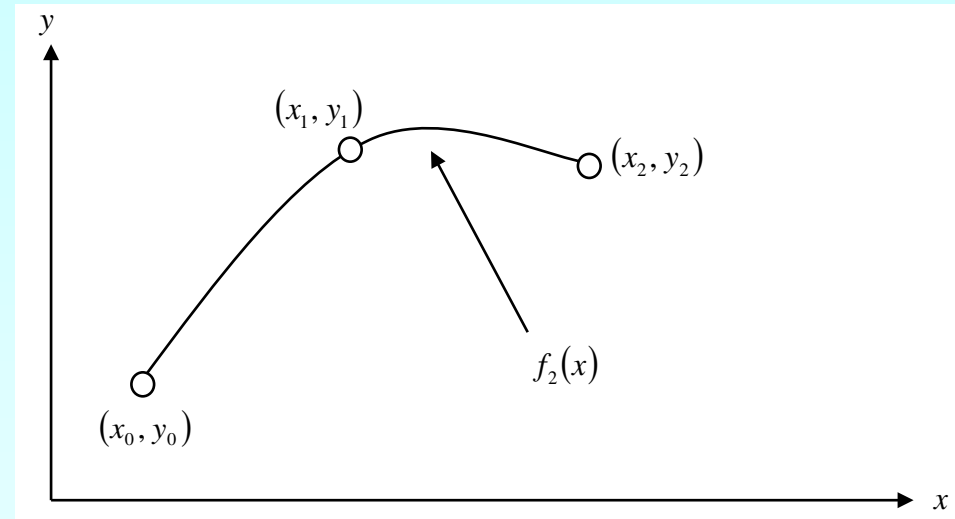


Figure 6 Quadratic interpolation.

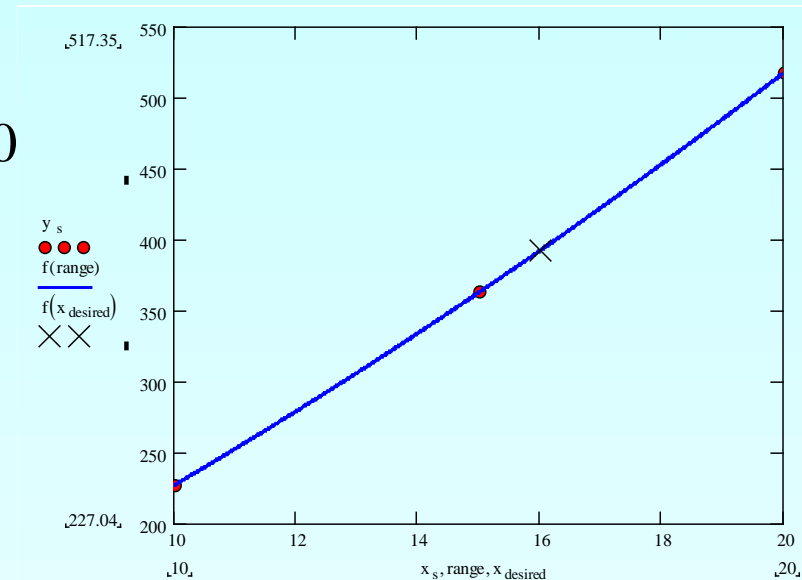
Solving the above three equations gives

$$a_0 = 12.05 \quad a_1 = 17.733 \quad a_2 = 0.3766$$

Quadratic Interpolation (cont.)

$$v(t) = 12.05 + 17.733t + 0.3766t^2, \quad 10 \leq t \leq 20$$

$$\begin{aligned} v(16) &= 12.05 + 17.733(16) + 0.3766(16)^2 \\ &= 392.19 \text{ m/s} \end{aligned}$$



The absolute relative approximate error $|\epsilon_a|$ obtained between the results from the first and second order polynomial is

$$\begin{aligned} |\epsilon_a| &= \left| \frac{392.19 - 393.70}{392.19} \right| \times 100 \\ &= 0.38410\% \end{aligned}$$



Example 3

The upward velocity of a rocket is given as a function of time in Table 3.

Find the velocity at $t=16$ seconds using the direct method for cubic interpolation.

Table 3 Velocity as a function of time.

$t, (s)$	$v(t), (m/s)$
0	0
10	227.04
15	362.78
20	517.35
22.5	602.97
30	901.67

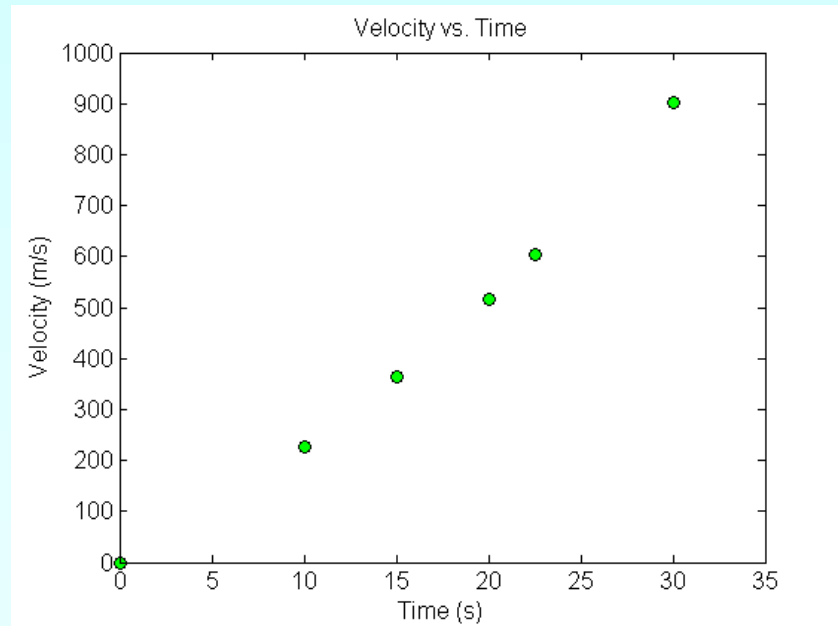


Figure 6 Velocity vs. time data for the rocket example

Cubic Interpolation

$$v(t) = a_0 + a_1t + a_2t^2 + a_3t^3$$

$$v(10) = 227.04 = a_0 + a_1(10) + a_2(10)^2 + a_3(10)^3$$

$$v(15) = 362.78 = a_0 + a_1(15) + a_2(15)^2 + a_3(15)^3$$

$$v(20) = 517.35 = a_0 + a_1(20) + a_2(20)^2 + a_3(20)^3$$

$$v(22.5) = 602.97 = a_0 + a_1(22.5) + a_2(22.5)^2 + a_3(22.5)^3$$

$$a_0 = -4.2540 \quad a_1 = 21.266 \quad a_2 = 0.13204 \quad a_3 = 0.0054347$$

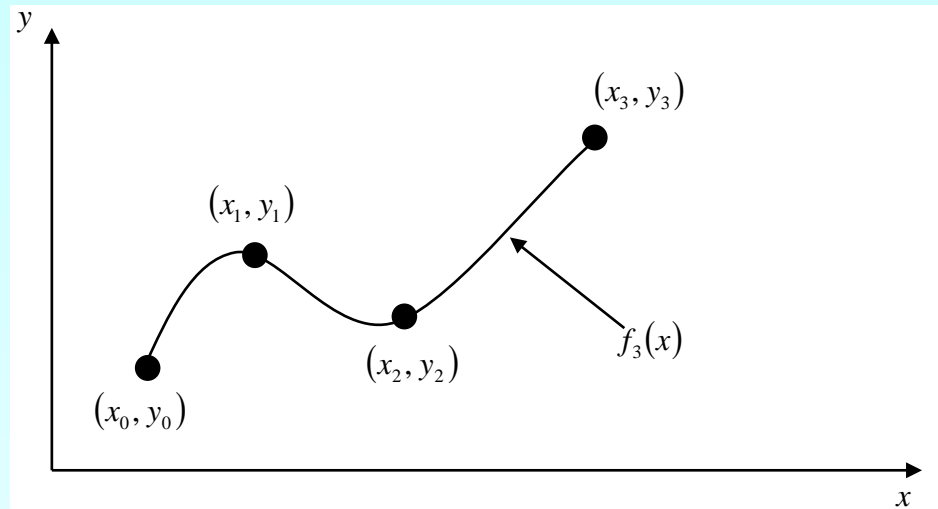
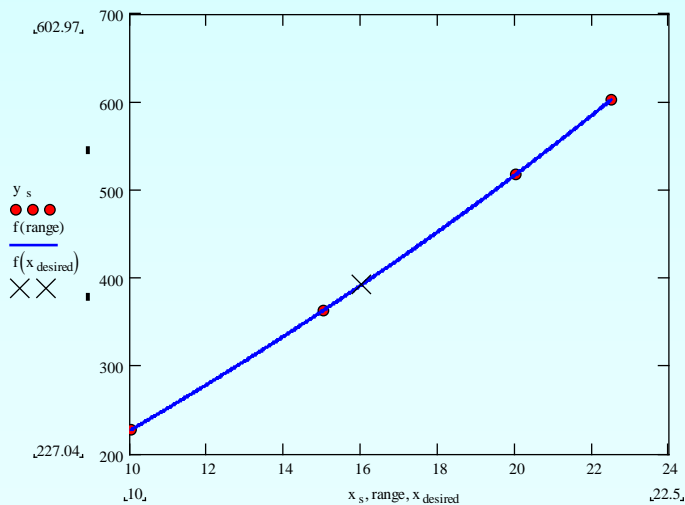


Figure 7 Cubic interpolation.

Cubic Interpolation (contd)

$$v(t) = -4.2540 + 21.266t + 0.13204t^2 + 0.0054347t^3, \quad 10 \leq t \leq 22.5$$

$$\begin{aligned} v(16) &= -4.2540 + 21.266(16) + 0.13204(16)^2 + 0.0054347(16)^3 \\ &= 392.06 \text{ m/s} \end{aligned}$$



The absolute percentage relative approximate error $|\epsilon_a|$ between second and third order polynomial is

$$\begin{aligned} |\epsilon_a| &= \left| \frac{392.06 - 392.19}{392.06} \right| \times 100 \\ &= 0.033269\% \end{aligned}$$

Comparison Table

Table 4 Comparison of different orders of the polynomial.

Order of Polynomial	1	2	3
$v(t = 16) \text{ m/s}$	393.7	392.19	392.06
Absolute Relative Approximate Error	-----	0.38410 %	0.033269 %

Distance from Velocity Profile

Find the distance covered by the rocket from $t=11\text{s}$ to $t=16\text{s}$?

$$v(t) = -4.3810 + 21.289t + 0.13064t^2 + 0.0054606t^3, \quad 10 \leq t \leq 22.5$$

$$\begin{aligned} s(16) - s(11) &= \int_{11}^{16} v(t) dt \\ &= \int_{11}^{16} \left(-4.2540 + 21.266t + 0.13204t^2 + 0.0054347t^3 \right) dt \\ &= \left[-4.2540t + 21.266 \frac{t^2}{2} + 0.13204 \frac{t^3}{3} + 0.0054347 \frac{t^4}{4} \right]_{11}^{16} \\ &= 1605 \text{ m} \end{aligned}$$

Acceleration from Velocity Profile

Find the acceleration of the rocket at $t=16$ s given that $v(t) = -4.2540 + 21.266t + 0.13204t^2 + 0.0054347t^3, 10 \leq t \leq 22.5$

$$\begin{aligned} a(t) &= \frac{d}{dt} v(t) \\ &= \frac{d}{dt} (-4.2540 + 21.266t + 0.13204t^2 + 0.0054347t^3) \\ &= 21.289 + 0.26130t + 0.016382t^2, \quad 10 \leq t \leq 22.5 \end{aligned}$$

$$\begin{aligned} a(16) &= 21.266 + 0.26408(16) + 0.016304(16)^2 \\ &= 29.665 \text{ m/s}^2 \end{aligned}$$

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/direct_method.html

THE END

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