

Secant Method

Industrial Engineering Majors

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Secant Method – Derivation

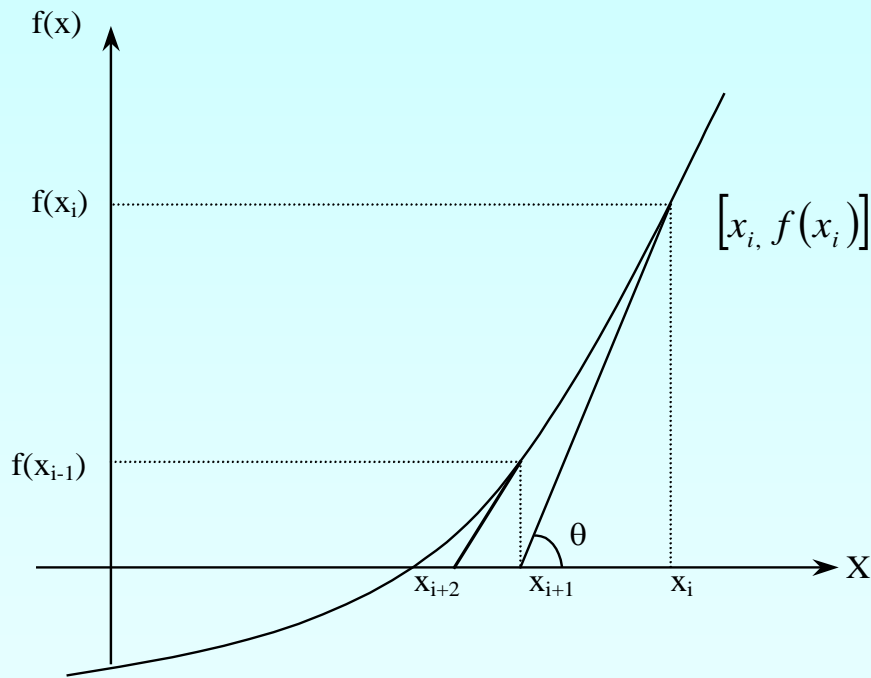


Figure 1 Geometrical illustration of the Newton-Raphson method.

Newton's Method

$$x_{i+1} = x_i - \frac{f(x_i)}{f'(x_i)} \quad (1)$$

Approximate the derivative

$$f'(x_i) = \frac{f(x_i) - f(x_{i-1})}{x_i - x_{i-1}} \quad (2)$$

Substituting Equation (2) into Equation (1) gives the Secant method

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Secant Method – Derivation

The secant method can also be derived from geometry:

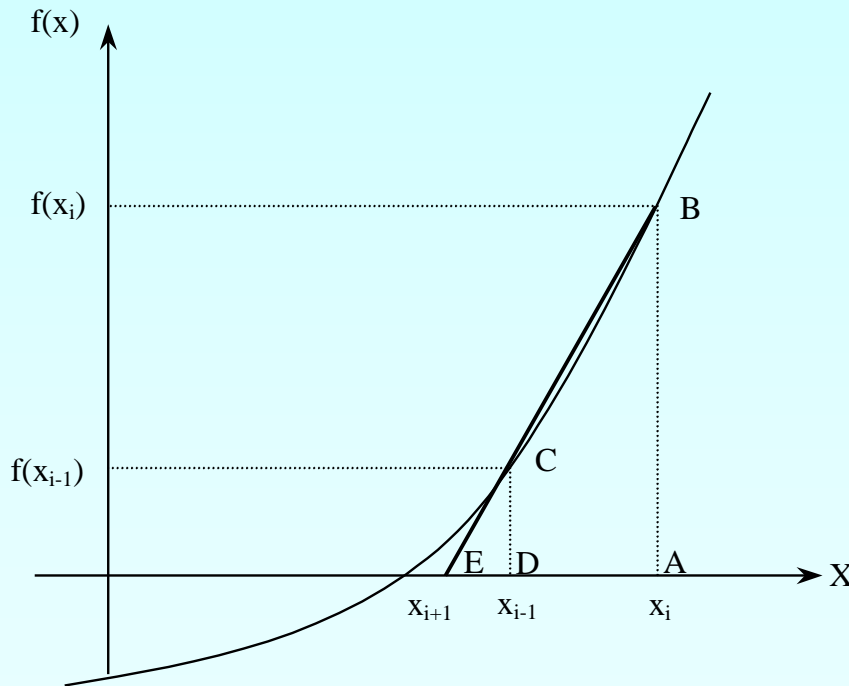


Figure 2 Geometrical representation of the Secant method.

The Geometric Similar Triangles

$$\frac{AB}{AE} = \frac{DC}{DE}$$

can be written as

$$\frac{f(x_i)}{x_i - x_{i+1}} = \frac{f(x_{i-1})}{x_{i-1} - x_{i+1}}$$

On rearranging, the secant method is given as

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Algorithm for Secant Method

Step 1

Calculate the next estimate of the root from two initial guesses

$$x_{i+1} = x_i - \frac{f(x_i)(x_i - x_{i-1})}{f(x_i) - f(x_{i-1})}$$

Find the absolute relative approximate error

$$|\epsilon_a| = \left| \frac{x_{i+1} - x_i}{x_{i+1}} \right| \times 100$$

Step 2

Find if the absolute relative approximate error is greater than the prespecified relative error tolerance.

If so, go back to step 1, else stop the algorithm.

Also check if the number of iterations has exceeded the maximum number of iterations.

Example

You are working for a start-up computer assembly company and have been asked to determine the minimum number of computers that the shop will have to sell to make a profit.

The equation that gives the minimum number of computers 'x' to be sold after considering the total costs and the total sales is:

$$f(x) = 40x^{1.5} - 875x + 35000 = 0$$

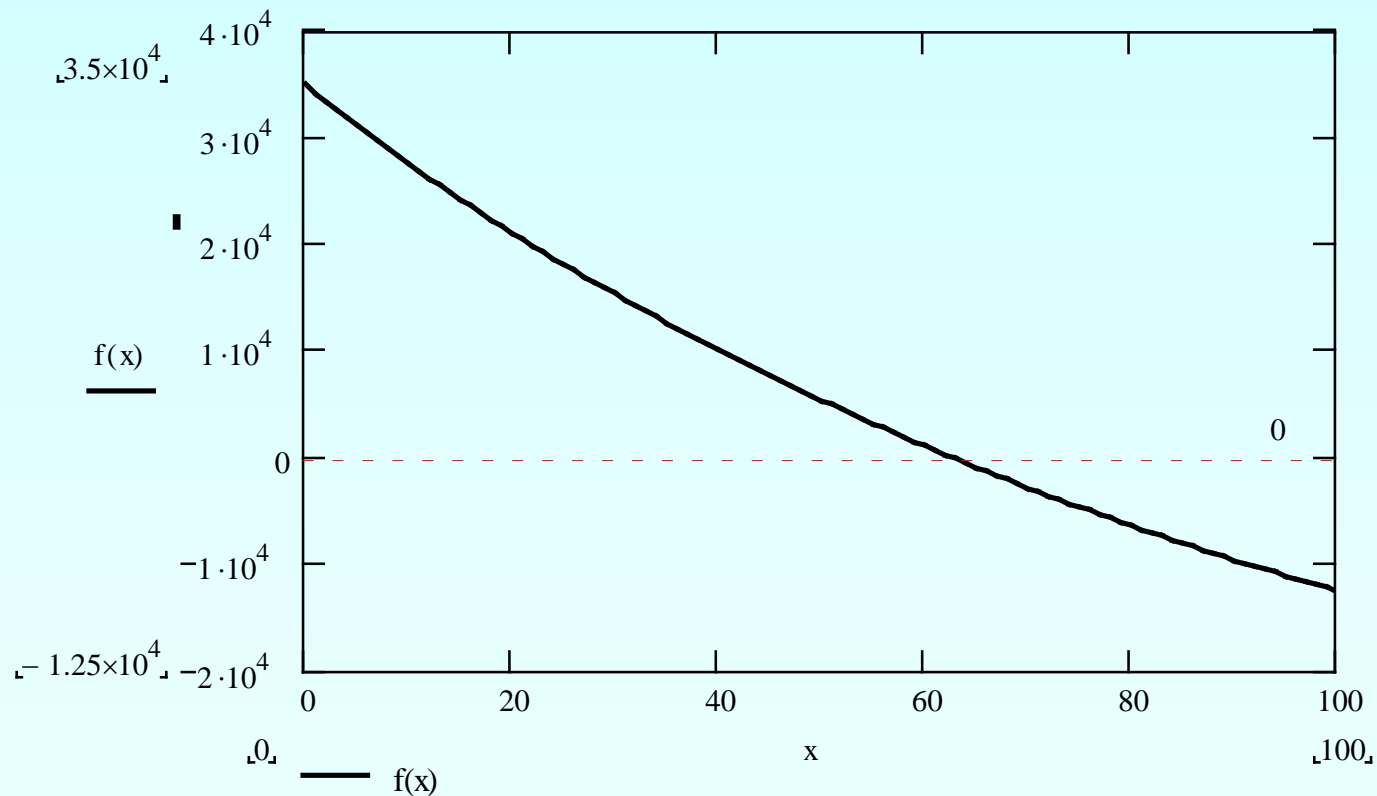
Solution

Use the Secant method of finding roots of equations to find

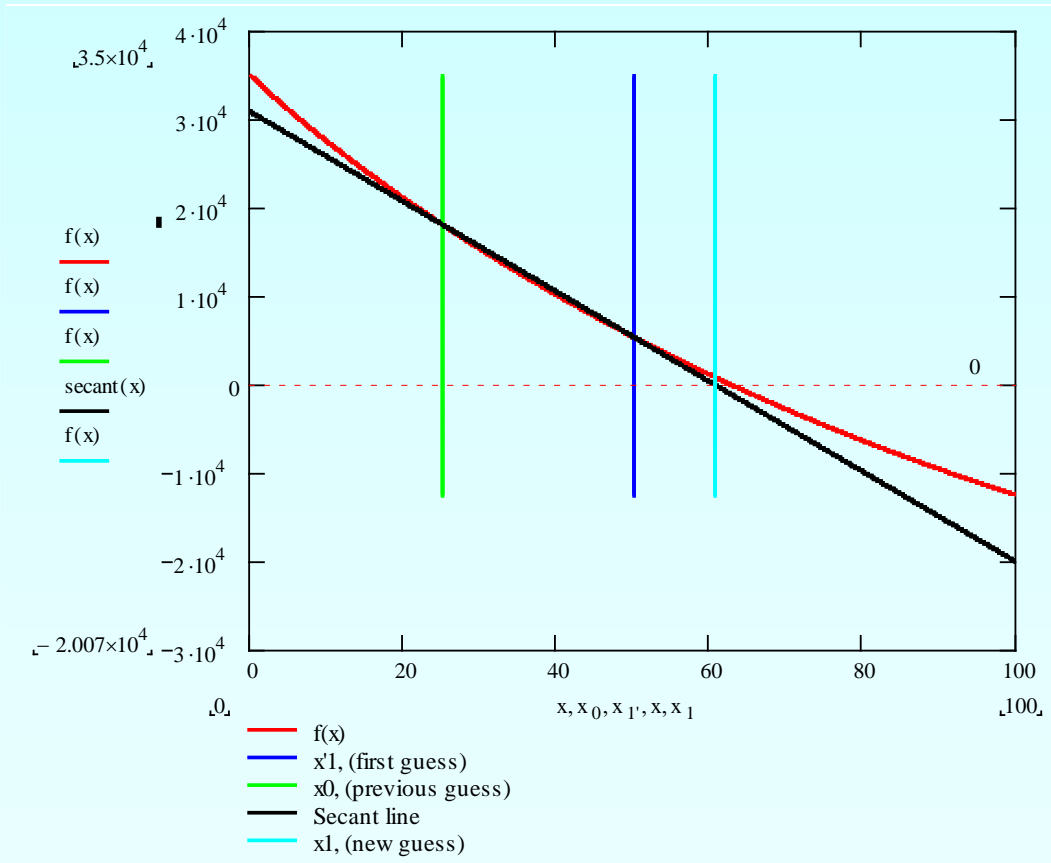
- The minimum number of computers that need to be sold to make a profit. Conduct three iterations to estimate the root of the above equation.
- Find the absolute relative approximate error at the end of each iteration, and
- The number of significant digits at least correct at the end of each iteration.

Graph of function $f(x)$

$$f(x) = 40x^{1.5} - 875x + 35000 = 0$$



Iteration #1



$$x_{-1} = 25, x_0 = 50$$

$$x_1 = x_0 - \frac{f(x_0)(x_0 - x_{-1})}{f(x_0) - f(x_{-1})}$$

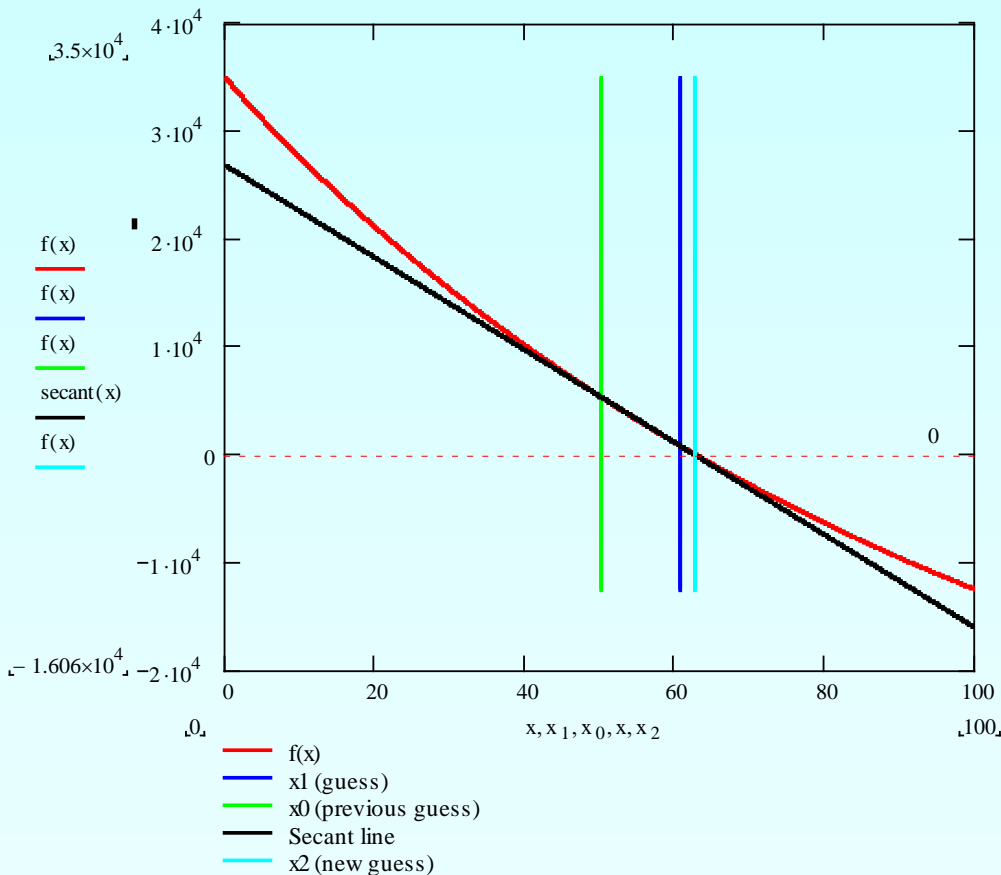
$$x_1 = 50 - \frac{(5392)(50 - 25)}{(5392) - (18125)}$$

$$= 60.587$$

$$|\epsilon_a| = 17.474\%$$

The number of significant digits at least correct is 0.

Iteration #2



$$x_0 = 50, x_1 = 60.587$$

$$x_2 = x_1 - \frac{f(x_1)(x_1 - x_0)}{f(x_1) - f(x_0)}$$

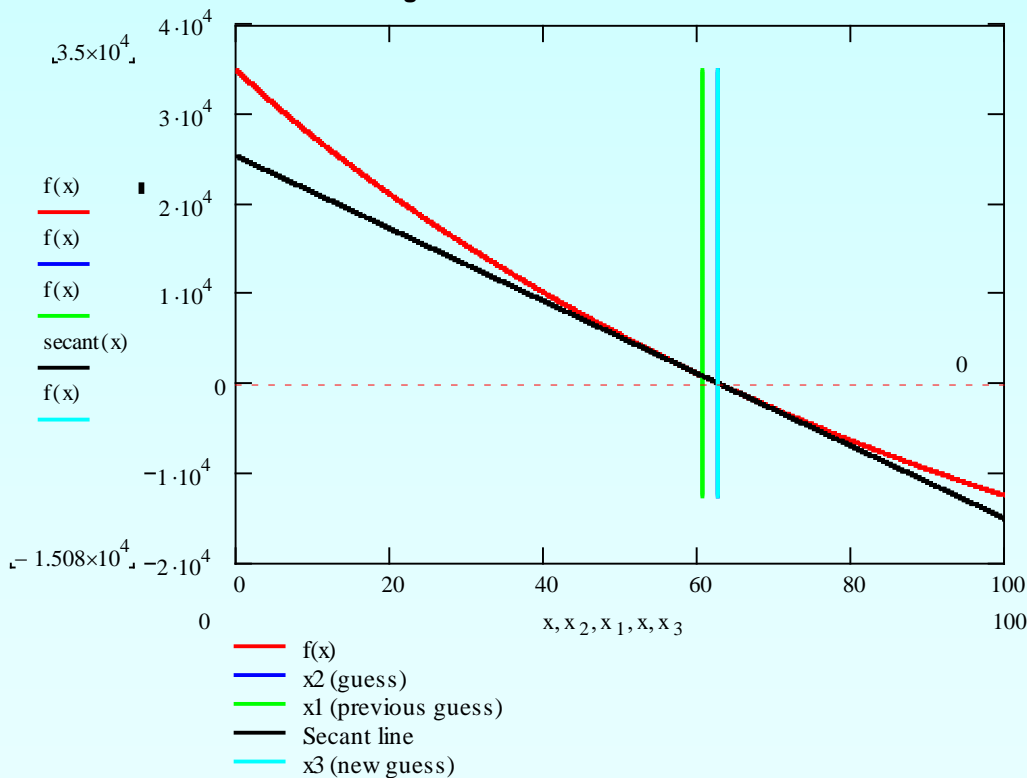
$$x_2 = 60.5871 - \frac{850.133(60.587 - 50)}{(850.133) - (5392)}$$

$$= 62.569$$

$$|\epsilon_a| = 3.1672\%$$

The number of significant digits at least correct is 1.

Iteration #3



$$x_1 = 60.587, x_2 = 62.569$$

$$x_3 = x_2 - \frac{f(x_2)(x_2 - x_1)}{f(x_2) - f(x_1)}$$

$$x_3 = 62.569 - \frac{(49.219)(62.569 - 60.587)}{(49.219) - (850.133)}$$

$$= 62.690$$

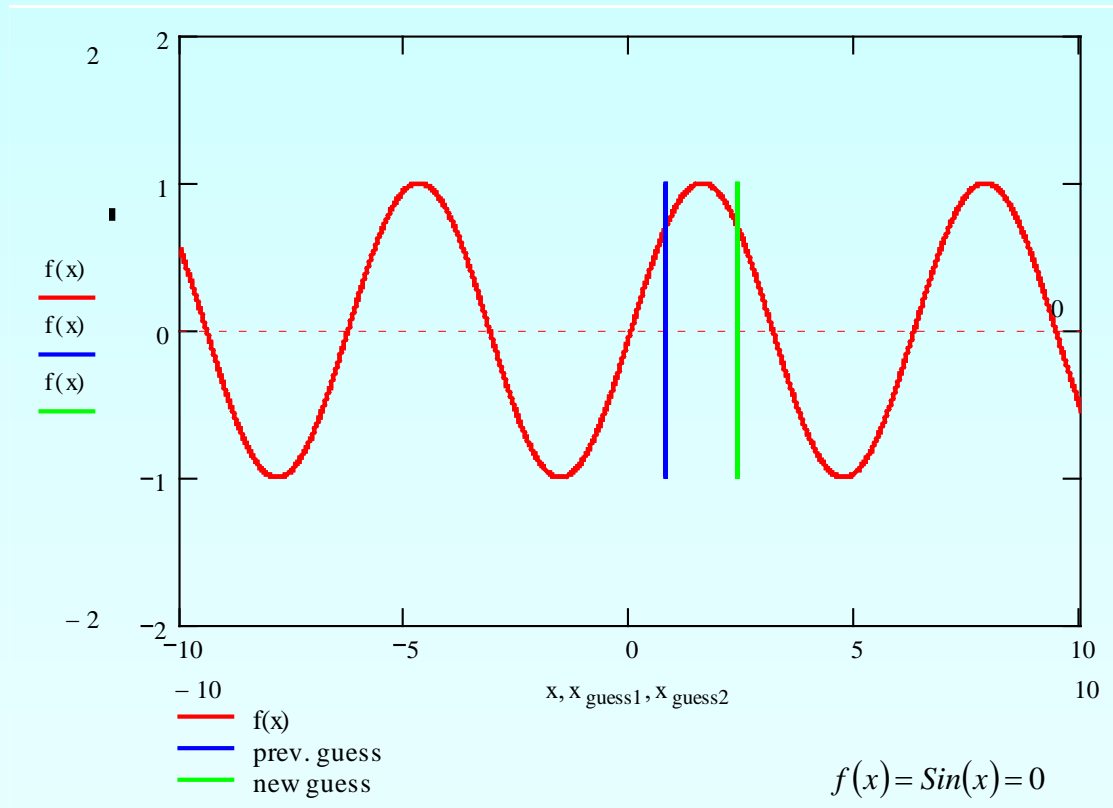
$$|\epsilon_a| = 0.19425\%$$

The number of significant digits at least correct is 2.

Advantages

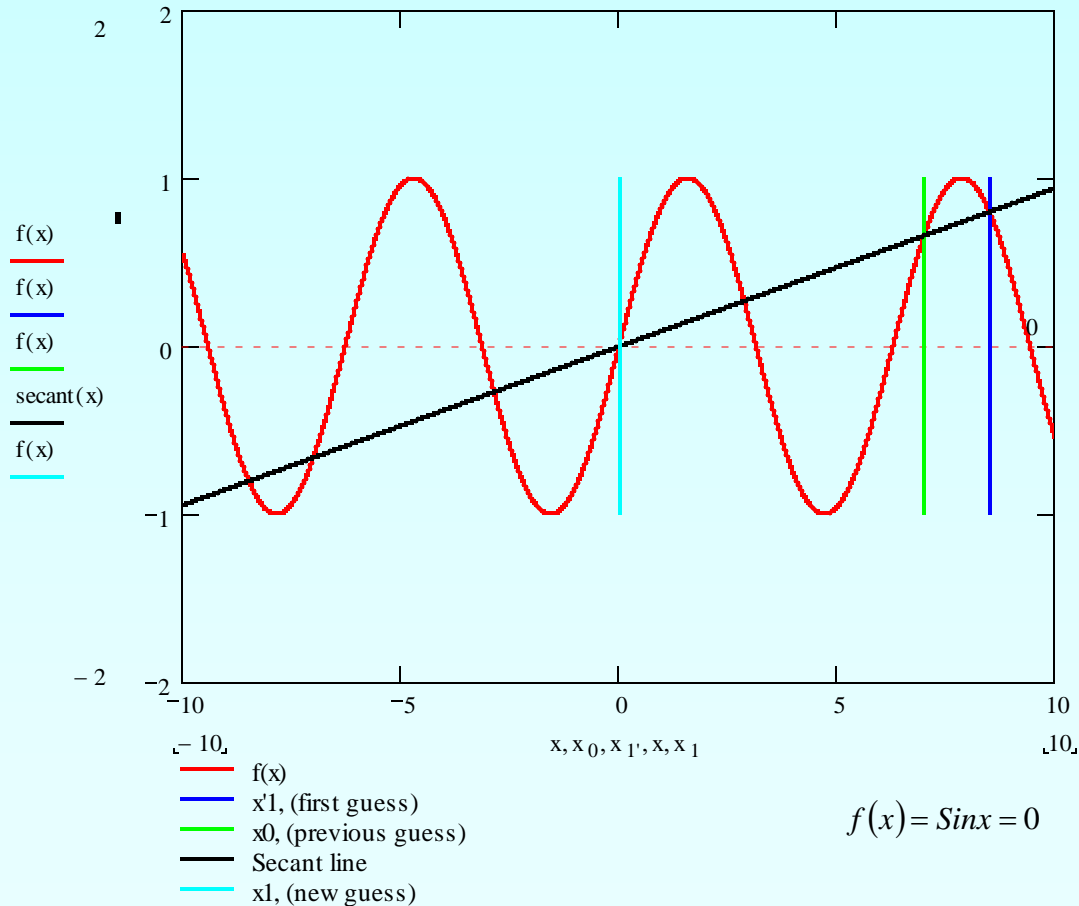
- Converges fast, if it converges
- Requires two guesses that do not need to bracket the root

Drawbacks



Division by zero

Drawbacks (continued)



Root Jumping

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/secant_method.html

THE END

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