Chapter 02.00G

Physical Problem for Differentiation Mechanical Engineering

Problem Statement

To make the fulcrum (Figure 1) of a bascule bridge, a long hollow steel shaft called the trunnion is shrink fit into a steel hub.

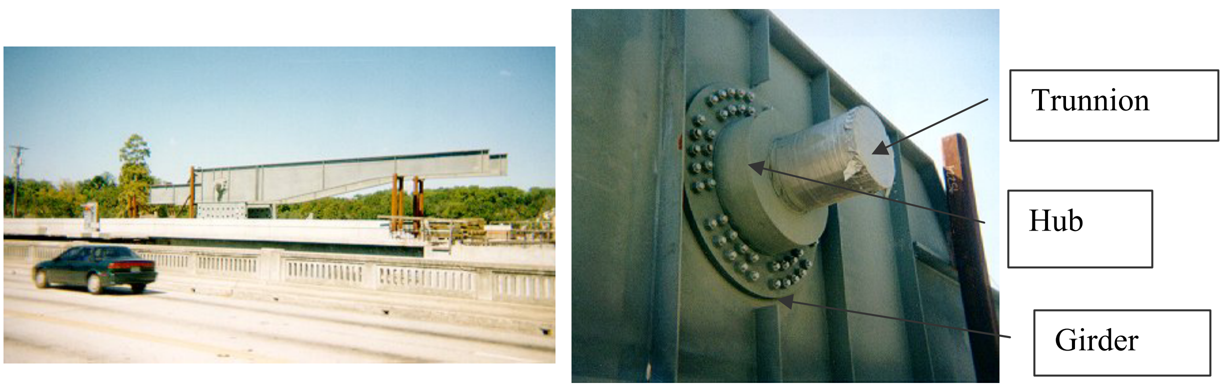

Figure 1 Trunnion-Hub-Girder (THG) assembly.

This is done by first immersing the trunnion in a cold medium such as dry-ice/alcohol mixture. After the trunnion reaches a steady state temperature of the cold medium, the trunnion outer diameter contracts, is taken out and slid though the hole of the hub (Figure 2).

When the trunnion heats up, it expands and creates an interference fit with the hub. In 1995, on one of the bridges in Florida, this assembly procedure did not work as designed. Before the trunnion could be inserted fully into the hub, the trunnion got stuck. So a new trunnion and hub had to be ordered worth $50,000. Coupled with construction delays, the total loss ran into more than hundred thousand dollars.

Why did the trunnion get stuck? This was because the trunnion had not contracted enough to slide through the hole.

Now the same designer is working on making the fulcrum for another bascule bridge. Can you help him so that he does not make the same mistake?

For this new bridge, he needs to fit a hollow trunnion of outside diameter 12.363" in a hub of inner diameter 12.358". His plan is to put the trunnion in dry ice/alcohol mixture (temperature of dry ice/alcohol mixture is $-108°\text{F}$) to contract the trunnion so that it can be slid through the hole of the hub. To slide the trunnion without sticking, he has also specified a diametral clearance of at least $0.01"$. Assume the room temperature is $80°\text{F}$, is immersing it in dry-ice/alcohol mixture a correct decision?

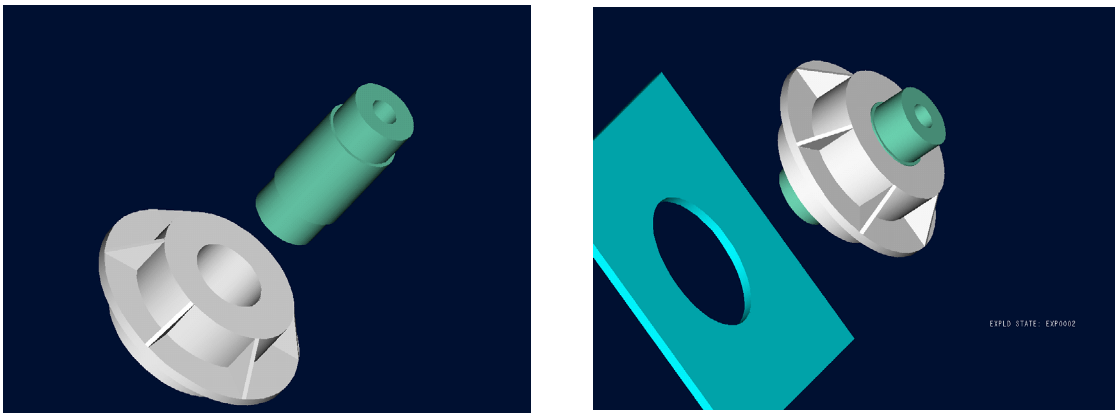

Figure 2 Trunnion slid through the hub after contracting

Solution

Looking at the records of the designer for the previous bridge where the trunnion got stuck in the hub, it was found that he used the thermal expansion coefficient at room temperature to calculate the contraction in the trunnion diameter. In that case the reduction, ΔD in the outer diameter of the trunnion is

$$\Delta D = D\alpha\Delta T \qquad (1)$$

where

$D =$ outer diameter of the trunnion,

$\alpha =$ coefficient of thermal expansion coefficient at room temperature,

$\Delta T =$ change in temperature,

Given

$$D = 12.363"$$

$$\alpha = 6.817\times 10^{-6}\ \text{in/in/°F at } 80°\text{F}$$

$$\begin{aligned}\Delta T &= T_{fluid} - T_{room}\\ &= -108 - 80\\ &= -188°\text{F}\end{aligned}$$

where

$T_{fluid} =$ temperature of dry-ice/alcohol mixture

$T_{room} =$ room temperature

The reduction in the trunnion outer diameter is given by

$$\begin{aligned}\Delta D &= 12.363\times\left(6.47\times 10^{-6}\right)(-188)\\ &= -0.01504"\end{aligned}$$

So the trunnion is predicted to reduce in diameter by 0.01504". But, is this enough reduction in diameter? As per the specifications, he needs the trunnion to contract by

= trunnion outside diameter - hub inner diameter + diametral clearance
= 12.363"−12.358"+0.01"
= 0.015"

So according to his calculations, it is enough to put the steel trunnion in dry-ice/alcohol mixture to get the desired contraction of 0.015" as he is predicting a contraction of 0.01504".

But as shown in the Figure 3, the thermal expansion coefficient of steel decreases with decrease in temperature and is not constant over the range of temperature the trunnion goes through. Hence the above formula (Equation 1) would overestimate the thermal contraction. This is the mistake he made in the calculations for the earlier bridge.

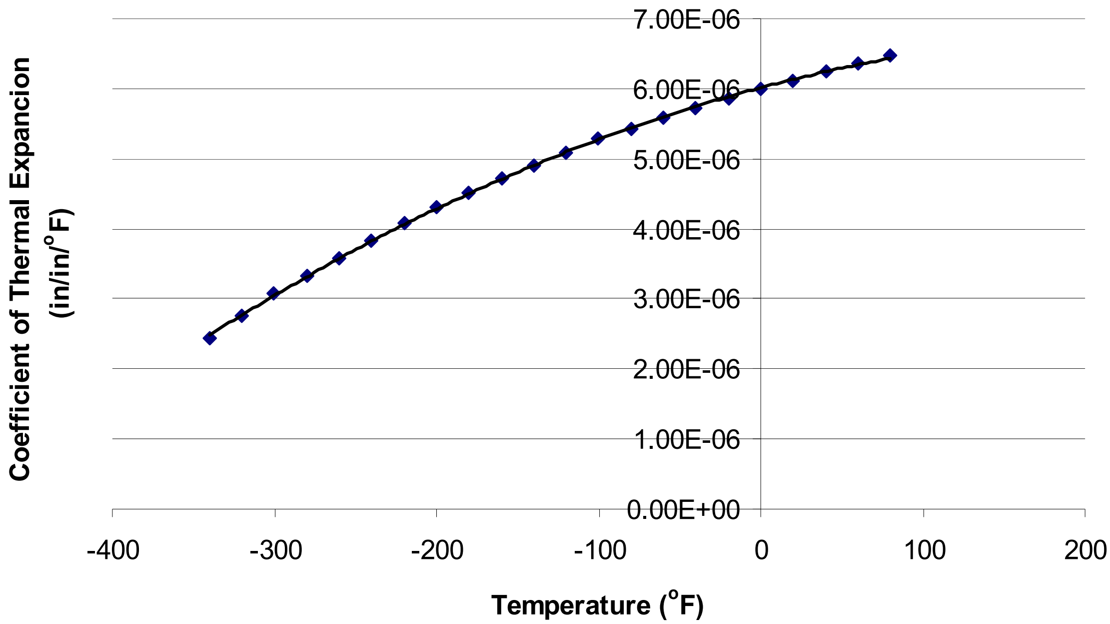

Figure 3 Varying thermal expansion coefficient as a function of temperature for cast steel.

To find contraction of a steel cylinder immersed in a bath of liquid nitrogen, one needs to know the thermal expansion coefficient data as a function of temperature. This data is given for steel in Table 1.

Table 1 Instantaneous thermal expansion coefficient as a function of temperature

Temperature, T (°F)	Coefficient of thermal expansion, (μin/in/°F)
80	6.47
40	6.24
-40	5.72
-120	5.09
-200	4.30
-280	3.33
-340	2.45

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An exercise to appreciate the way the thermal expansion coefficient changes with respect to the temperature, we can look into the slope of the thermal expansion coefficient with respect to temperature at low and high temperatures.

QUESTIONS

1. Using the data from Table 1, is the rate of change of coefficient of thermal expansion with respect to temperature more at $T = 80°\text{F}$ than at $T = -340°\text{F}$? Use any numerical differentiation technique.
2. The data given in the Table 1 can be regressed to $\alpha = a_0 + a_1 T + a_2 T^2$ to get $\alpha = 6.0217\times10^{-6} + 6.2782\times10^{-9}\text{T} - 1.2218\times10^{-11}\text{T}^2$. Compare the results with problem 1 if you use the regression curve to find the rate of change of coefficient of thermal expansion with respect to temperature at $T = 80°\text{F}$ and at $T = -340°\text{F}$.

DIFFERENTIATION	
Topic	Differentiation
Summary	A physical problem of finding how the thermal expansion coefficient varies with respect to the temperature is modeled. To find the variation, the problem would be modeled as a numerical differentiation problem.
Major	Mechanical Engineering
Authors	Autar Kaw
Date	February 16, 2010
Web Site	http://numericalmethods.eng.usf.edu