Chapter 07.00G

Physical Problem for Integration Mechanical Engineering

Problem Statement

To make the fulcrum (Figure 1) of a bascule bridge, a long hollow steel shaft called the trunnion is shrink fit into a steel hub. The resulting steel trunnion-hub assembly is then shrink fit into the girder of the bridge.

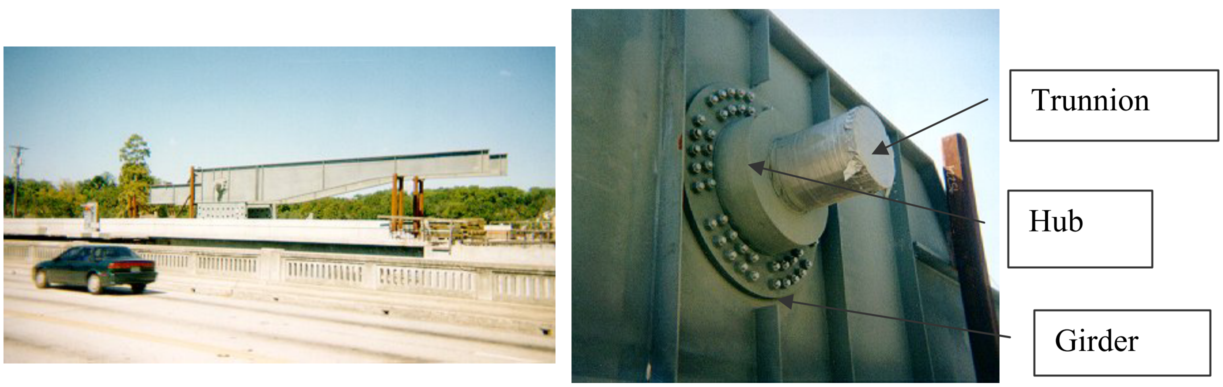

Figure 1 Trunnion-Hub-Girder (THG) assembly.

This is done by first immersing the trunnion in a cold medium such as dry-ice/alcohol mixture. After the trunnion reaches the steady state temperature of the cold medium, the trunnion outer diameter contracts. The trunnion is taken out of the medium and slid though the hole of the hub (Figure 2).

When the trunnion heats up, it expands and creates an interference fit with the hub. In 1995, on one of the bridges in Florida, this assembly procedure did not work as designed. Before the trunnion could be inserted fully into the hub, the trunnion got stuck. Luckily the trunnion was taken out before it got stuck permanently. Otherwise, a new trunnion and hub would needed to be ordered at a cost of $50,000. Coupled with construction delays, the total loss could have been more than hundred thousand dollars.

Why did the trunnion get stuck? This was because the trunnion had not contracted enough to slide through the hole. Can you find out why?

A hollow trunnion of outside diameter $12.363''$ is to be fitted in a hub of inner diameter $12.358''$. The trunnion was put in dry ice/alcohol mixture (temperature of the fluid

- dry ice/alcohol mixture is $-108°\text{F}$) to contract the trunnion so that it can be slid through the hole of the hub. To slide the trunnion without sticking, a diametrical clearance of at least $0.01''$ is required between the trunnion and the hub. Assuming the room temperature is $80°\text{F}$, is immersing it in dry-ice/alcohol mixture a correct decision?

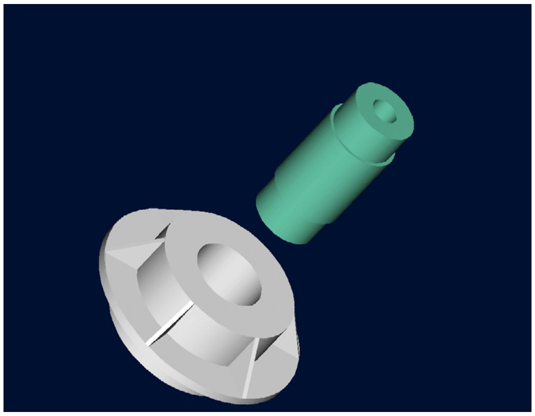

Figure 2 Trunnion slided through the hub after contracting

Solution

To calculate the contraction in the diameter of the trunnion, thermal expansion coefficient at room temperature is used. In that case the reduction, ΔD in the outer diameter of the trunnion is

$$\Delta D = D\alpha\Delta T \qquad (1)$$

where

D = outer diameter of the trunnion,

α = coefficient of thermal expansion coefficient at room temperature, and

ΔT = change in temperature,

Given

$D = 12.363''$

$\alpha = 6.817\times10^{-6}$ in/in/°F at 80°F

$$\begin{aligned}\Delta T &= T_{fluid} - T_{room}\\ &= -108-80\\ &= -188°\text{F}\end{aligned}$$

where

T_{fluid} = temperature of dry-ice/alcohol mixture,

T_{room} = room temperature,

the reduction in the trunnion outer diameter is given by

$$\begin{aligned}\Delta D &= (12.363)\left(6.47\times10^{-6}\right)(-188)\\ &= -0.01504''\end{aligned}$$

So the trunnion is predicted to reduce in diameter by 0.01504". But, is this enough reduction in diameter? As per specifications, he needs the trunnion to contract by

= trunnion outside diameter - hub inner diameter + diametral clearance

$= 12.363''-12.358''+0.01''$

$= 0.015''$

So according to his calculations, immersing the steel trunnion in dry-ice/alcohol mixture gives the desired contraction of 0.015" as we predict a contraction of $0.01504''$.

But as shown in Figure 3, the thermal expansion coefficient of steel decreases with temperature and is not constant over the range of temperature the trunnion goes through. Hence the above formula (Equation 1) would overestimate the thermal contraction.

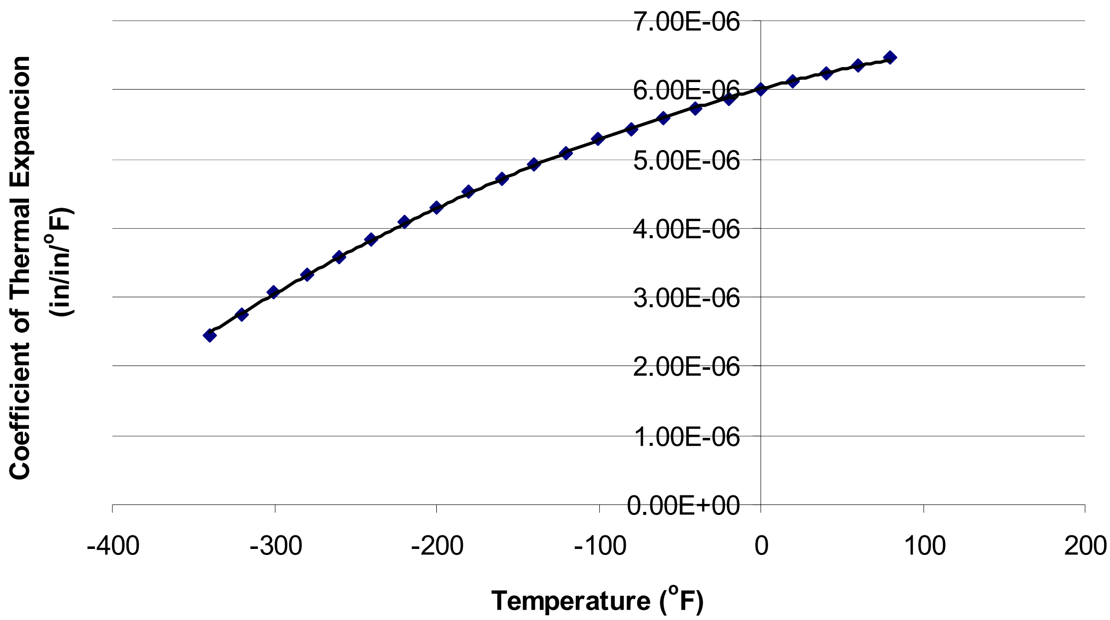

Figure 3 Varying thermal expansion coefficient as a function of temperature for cast steel.

The contraction in the diameter for the trunnion for which the thermal expansion coefficient varies as a function of temperature is given by

$$\Delta D = D \int_{T_{room}}^{T_{fluid}} \alpha dT \qquad (2)$$

Note that Equation (2) reduces to Equation (1) if the coefficient of thermal expansion is assumed to be constant. In Figure 3, the thermal expansion coefficient of a typical cast steel is approximated by a second order polynomial[1] as

$$\alpha = -1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.0150 \times 10^{-6}$$

$$\Delta D = 12.363 \int_{80}^{-108} \left(-1.2278 \times 10^{-11} T^2 + 6.1946 \times 10^{-9} T + 6.015 \times 10^{-6}\right) dT$$

QUESTIONS

1. Can you now find the contraction in the trunnion OD?
2. Is the magnitude of contraction more than 0.015" as required?
3. If that is not the case, what if the trunnion were immersed in liquid nitrogen (boiling temperature= $-321°F$)? Will that give enough contraction in the trunnion?

4. Rather than regressing the data to a second order polynomial so that one can find the contraction in the trunnion outer diameter, how would you use trapezoidal rule of integration for unequal segments? What is the relative difference between the two results? The data for the thermal expansion coefficients as function of temperature is given below.

Table 1 Instantaneous thermal expansion coefficient as a function of temperature.

Temperature	Instantaneous Thermal Expansion
°F	$\mu \text{in}/\text{in}/°\text{F}$
80	6.47
60	6.36
40	6.24
20	6.12
0	6.00
-20	5.86
-40	5.72
-60	5.58
-80	5.43
-100	5.28
-120	5.09
-140	4.91
-160	4.72
-180	4.52
-200	4.30
-220	4.08
-240	3.83
-260	3.58
-280	3.33
-300	3.07
-320	2.76
-340	2.45

[1] The second order polynomial is derived using regression analysis which is another mathematical procedure where numerical methods are employed. Regression analysis approximates discrete data such as the thermal expansion coefficient vs. temperature data as a continuous function. This is an excellent example of where one has to use numerical methods of more than one procedure to solve a real life problem.

Topic	INTEGRATION
Sub Topic	Physical Problem
Summary	A physical problem of finding if the shaft has contracted enough to be shrink fit into a hollow hub.
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