Direct Method of Interpolation

Chemical Engineering Majors

Authors: Autar Kaw, Jai Paul

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What is Interpolation ?

Given (x_0,y_0), (x_1,y_1), ... (x_n,y_n), find the value of 'y' at a value of 'x' that is not given.

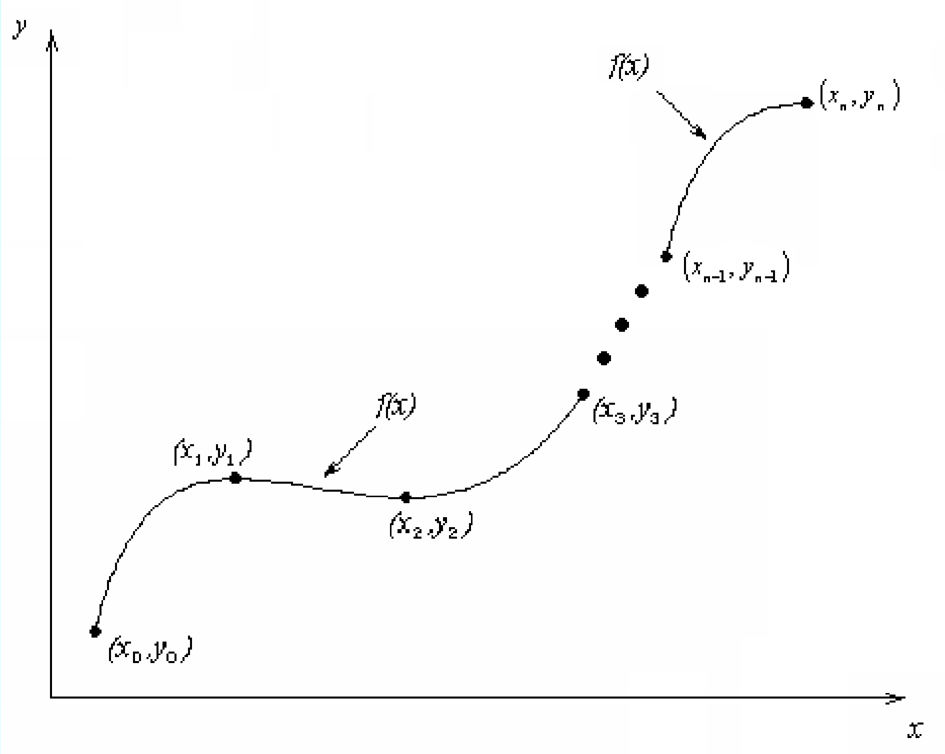


Figure 1 Interpolation of discrete.

Interpolants

Polynomials are the most common choice of interpolants because they are easy to:

- Evaluate
- Differentiate, and
- Integrate

Direct Method

Given 'n+1' data points (x_0,y_0), (x_1,y_1),.............. (x_n,y_n), pass a polynomial of order 'n' through the data as given below:

$$y = a_0 + a_1 x + ... + a_n x^n.$$

where a_0, a_1,................... a_n are real constants.

- Set up 'n+1' equations to find 'n+1' constants.
- To find the value 'y' at a given value of 'x', simply substitute the value of 'x' in the above polynomial.

Example

To find how much heat is required to bring a kettle of water to its boiling point, you are asked to calculate the specific heat of water at 61˚C. The specific heat of water is given as a function of time in Table 1. Use linear, quadratic and cubic interpolation to determine the value of the specific heat at T = 61˚C.

Table 1 Specific heat of water as a function of temperature.

Temperature, T (°C)	Specific heat, $C_p \left(\frac{\text{J}}{\text{kg}-°\text{C}}\right)$
22	4181
42	4179
52	4186
82	4199
100	4217

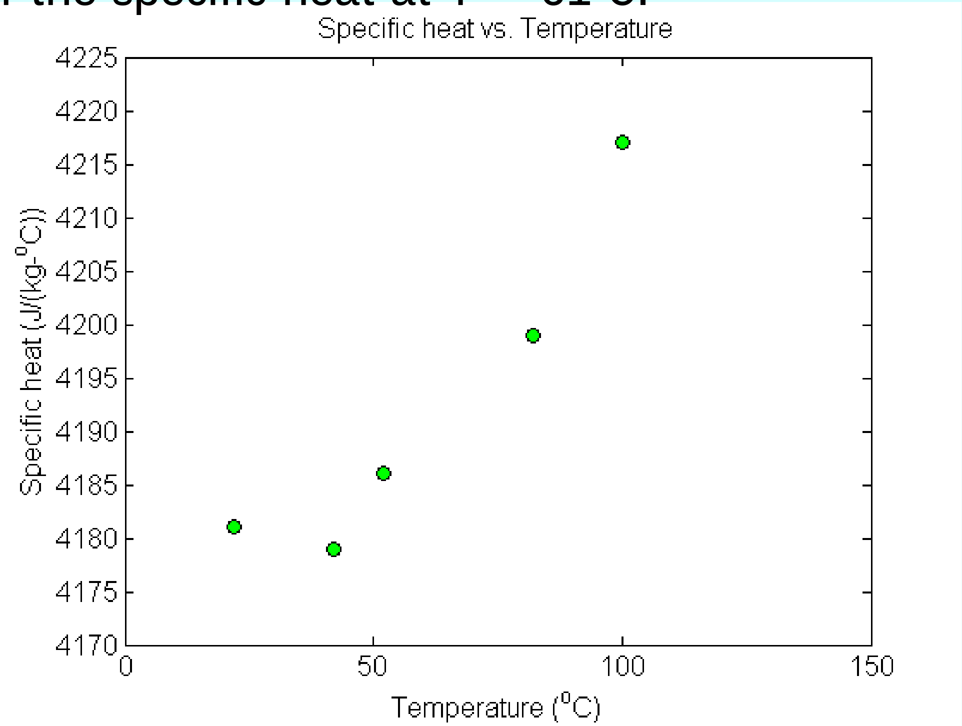


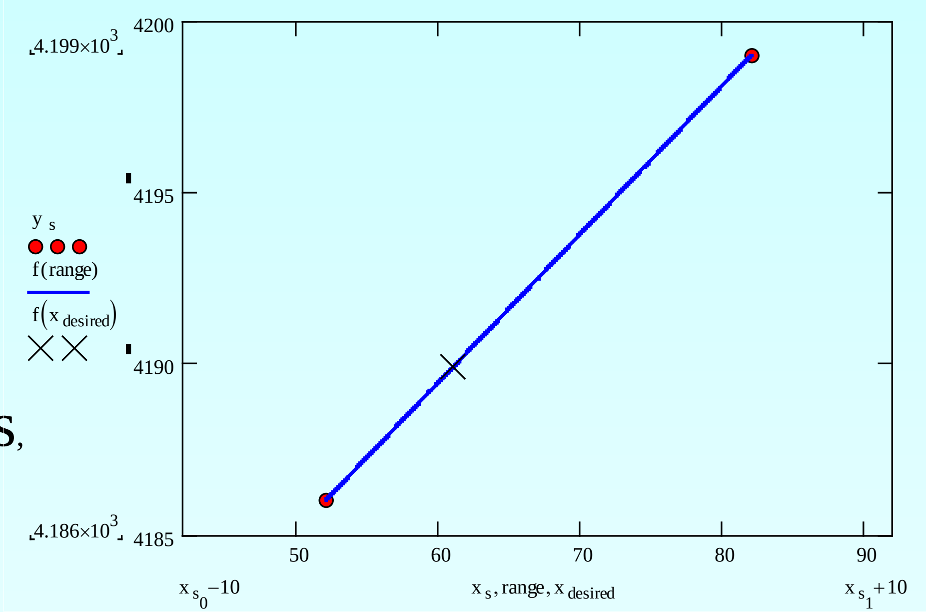
Figure 2 Specific heat of water vs. temperature.

Linear Interpolation

$$C_p(T) = a_0 + a_1 T$$

$$C_p(52) = a_0 + a_1(52) = 4186$$

$$C_p(82) = a_0 + a_1(82) = 4199$$



Solving the above two equations gives,

$$a_0 = 4163.5 \qquad a_1 = 0.43333$$

Hence

$$C_p(T) = 4163.5 + 0.43333T, \;\; 52 \le T \le 82.$$

$$C_p(61) = 4163.5 + 0.43333(61) = 4189.9 \frac{J}{kg - ^\circ C}$$

Quadratic Interpolation

$$C_p(T) = a_0 + a_1 T + a_2 T^2$$

$$Cp(42) = a_0 + a_1(42) + a_2(42)^2 = 4179$$

$$Cp(52) = a_0 + a_1(52) + a_2(52)^2 = 4186$$

$$Cp(82) = a_0 + a_1(82) + a_2(82)^2 = 4199$$

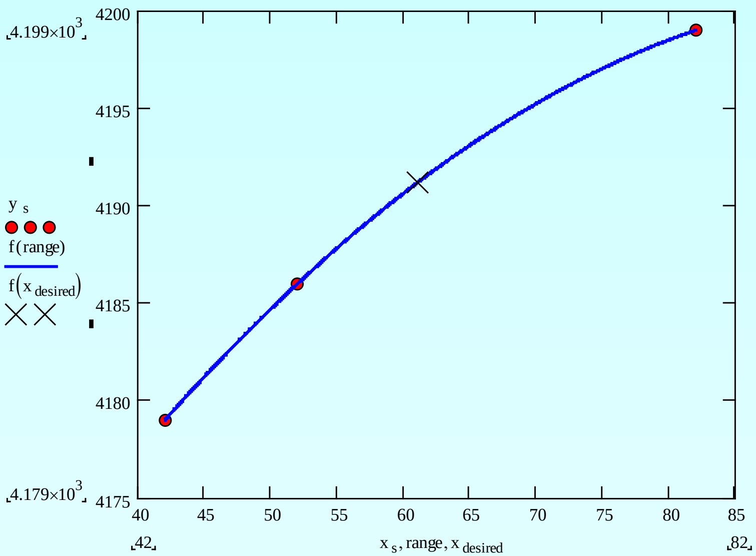
Solving the above three equations gives

$$a_0 = 4135.0 \qquad a_1 = 1.3267 \qquad a_2 = -6.6667 \times 10^{-3}$$

Quadratic Interpolation (contd)

$$C_p(T) = 4135.0 + 1.3267T - 6.6667\times10^{-3}T^2,$$
$$42 \le T \le 82$$

$$C_p(61) = 4135.0 + 1.3267(61) - 6.6667\times10^{-3}(61)^2$$
$$= 4191.2\frac{J}{kg-^\circ C}$$



The absolute relative approximate error obtained between the results from the first and second order polynomial is

$$\left|\in_a\right| = \left|\frac{4191.2 - 4189.9}{4191.2}\right|\times100 = 0.030063\%$$

Cubic Interpolation

$$C_p(T) = a_0 + a_1 T + a_2 T^2 + a_3 T^3$$

$$Cp(42) = a_0 + a_1(42) + a_2(42)^2 + a_3(42)^3 = 4179$$

$$Cp(52) = a_0 + a_1(52) + a_2(52)^2 + a_3(52)^3 = 4186$$

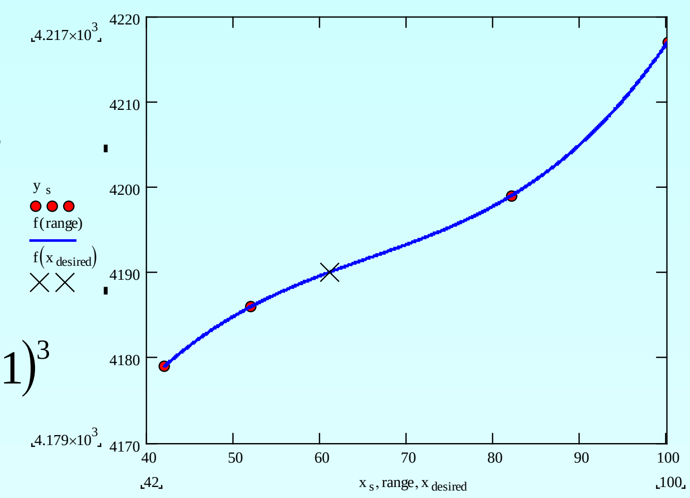
$$Cp(82) = a_0 + a_1(82) + a_2(82)^2 + a_3(82)^3 = 4199$$

$$Cp(100) = a_0 + a_1(100) + a_2(100)^2 + a_3(100)^3 = 4217$$

$$a_0 = 4078.0 \quad a_1 = 4.4771 \quad a_2 = -0.062720 \quad a_3 = 3.1849 \times 10^{-4}$$

Cubic Interpolation (contd)

$$Cp(T) = 4078 + 4.4771T - 0.06272T^2 + 3.1849 \times 10^{-4} T^3,$$
$$42 \le T \le 100$$



$$T(61) = 4078 + 4.4471(61) - 0.06272(61)^2 + 3.1849 \times 10^{-4}(61)^3$$
$$= 4191.0 \frac{J}{kg - {}^\circ C}$$

The absolute relative approximate error obtained between the results from the first and second order polynomial is

$$\left|\in_a\right| = \left|\frac{4190.0 - 4191.2}{4190.0}\right| \times 100 = 0.027295\%$$

Comparison Table

Order of Polynomial	1	2	3
$C_p(T) \quad \dfrac{J}{kg - ^\circ C}$	4189.9	4191.2	4190.0
Absolute Relative Approximate Error	----------	0.030063%	0.027295%

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/direct_method.html

THE END

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