

Binary Representation

Major: All Engineering Majors

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<http://numericalmethods.eng.usf.edu>

Transforming Numerical Methods Education for STEM
Undergraduates

Binary Representation

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How a Decimal Number is Represented

$$257.76 = 2 \times 10^2 + 5 \times 10^1 + 7 \times 10^0 + 7 \times 10^{-1} + 6 \times 10^{-2}$$

Base 2

$$(1011.0011)_2 = \left(\begin{array}{l} (1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0) \\ + (0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4}) \end{array} \right)_{10}$$
$$= 11.1875$$

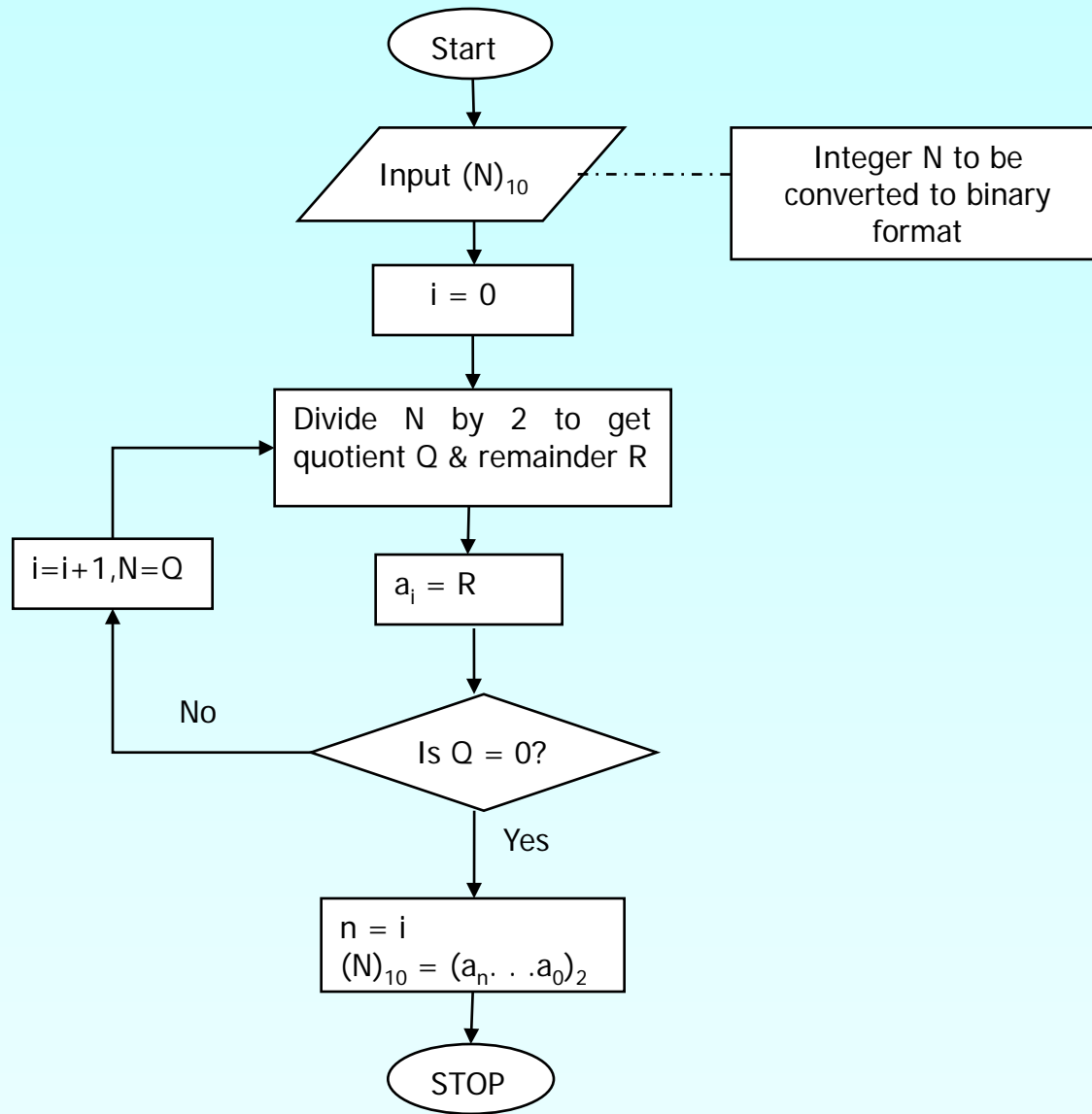
Convert Base 10 Integer to binary representation

Table 1 Converting a base-10 integer to binary representation.

	Quotient	Remainder
11/2	5	$1 = a_0$
5/2	2	$1 = a_1$
2/2	1	$0 = a_2$
1/2	0	$1 = a_3$

Hence

$$\begin{aligned}(11)_{10} &= (a_3 a_2 a_1 a_0)_2 \\ &= (1011)_2\end{aligned}$$



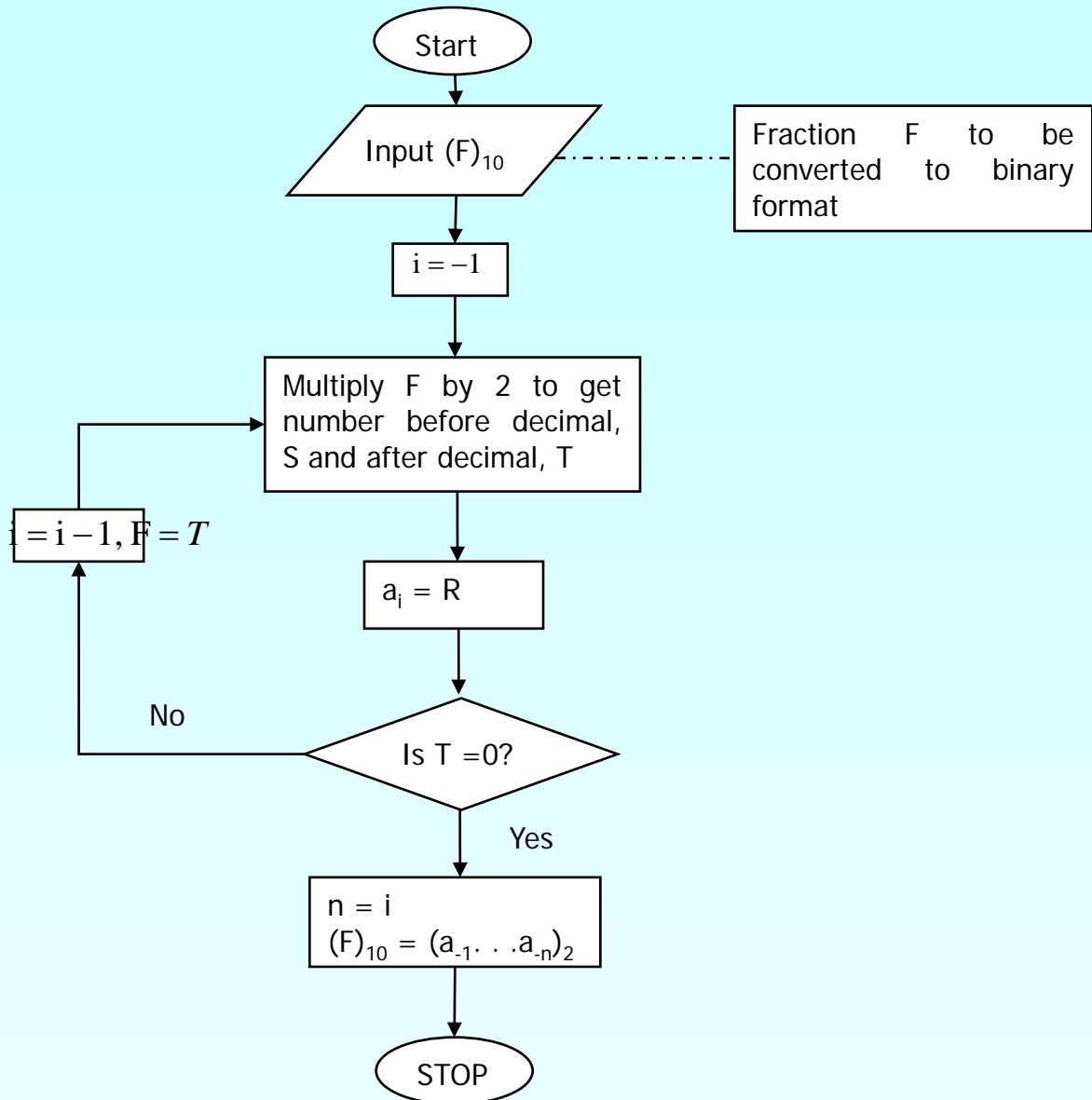
Fractional Decimal Number to Binary

Table 2. Converting a base-10 fraction to binary representation.

	Number	Number after decimal	Number before decimal
0.1875×2	0.375	0.375	$0 = a_{-1}$
0.375×2	0.75	0.75	$0 = a_{-2}$
0.75×2	1.5	0.5	$1 = a_{-3}$
0.5×2	1.0	0.0	$1 = a_{-4}$

Hence

$$\begin{aligned}(0.1875)_{10} &= (a_{-1}a_{-2}a_{-3}a_{-4})_2 \\ &= (0.0011)_2\end{aligned}$$



Decimal Number to Binary

$$(11.1875)_{10} = (\quad ?.\? \quad)_2$$

Since

$$(11)_{10} = (1011)_2$$

and

$$(0.1875)_{10} = (0.0011)_2$$

we have

$$(11.1875)_{10} = (1011.0011)_2$$

All Fractional Decimal Numbers Cannot be Represented Exactly

Table 3. Converting a base-10 fraction to approximate binary representation.

	Number	Number after decimal	Number before Decimal
0.3×2	0.6	0.6	$0 = a_{-1}$
0.6×2	1.2	0.2	$1 = a_{-2}$
0.2×2	0.4	0.4	$0 = a_{-3}$
0.4×2	0.8	0.8	$0 = a_{-4}$
0.8×2	1.6	0.6	$1 = a_{-5}$

$$(0.3)_{10} \approx (a_{-1}a_{-2}a_{-3}a_{-4}a_{-5})_2 = (0.01001)_2 = 0.28125$$

Another Way to Look at Conversion

Convert $(11.1875)_{10}$ to base 2

$$\begin{aligned}(11)_{10} &= 2^3 + 3 \\ &= 2^3 + 2^1 + 1 \\ &= 2^3 + 2^1 + 2^0 \\ &= 1 \times 2^3 + 0 \times 2^2 + 1 \times 2^1 + 1 \times 2^0 \\ &= (1011)_2\end{aligned}$$

$$\begin{aligned} (0.1875)_{10} &= 2^{-3} + 0.0625 \\ &= 2^{-3} + 2^{-4} \\ &= 0 \times 2^{-1} + 0 \times 2^{-2} + 1 \times 2^{-3} + 1 \times 2^{-4} \\ &= (.0011)_2 \end{aligned}$$

$$(11.1875)_{10} = (1011.0011)_2$$

Additional Resources

For all resources on this topic such as digital audiovisual lectures, primers, textbook chapters, multiple-choice tests, worksheets in MATLAB, MATHEMATICA, MathCad and MAPLE, blogs, related physical problems, please visit

http://numericalmethods.eng.usf.edu/topics/binary_representation.html

THE END

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