Chapter 09.00A

Physical Problem for Optimization
General Engineering

Problem Statement

A 5 m long gutter is made from a flat sheet of aluminum which is 5m x 0.21m. The shape of the gutter cross-section is shown in Figure 1, and is made by bending the sheet at two locations at an angle θ (Figure 2). What are the values of s and θ, that will maximize the volume capacity of the gutter so that it drains water quickly during a heavy rainfall?

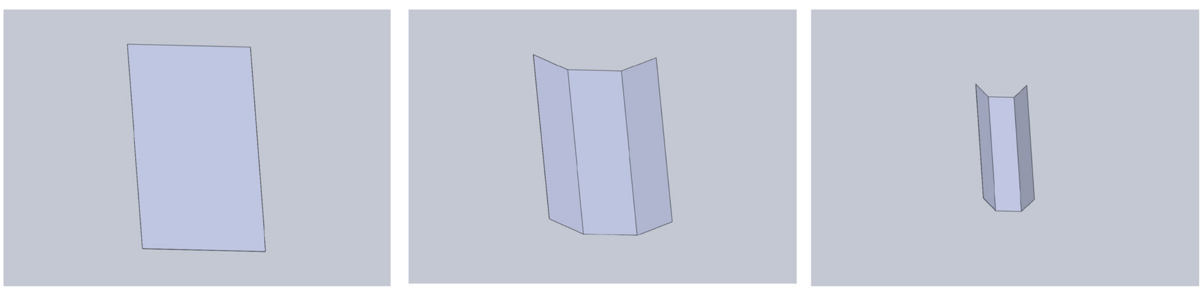

Figure 1 Sheet metal bent to form a gutter

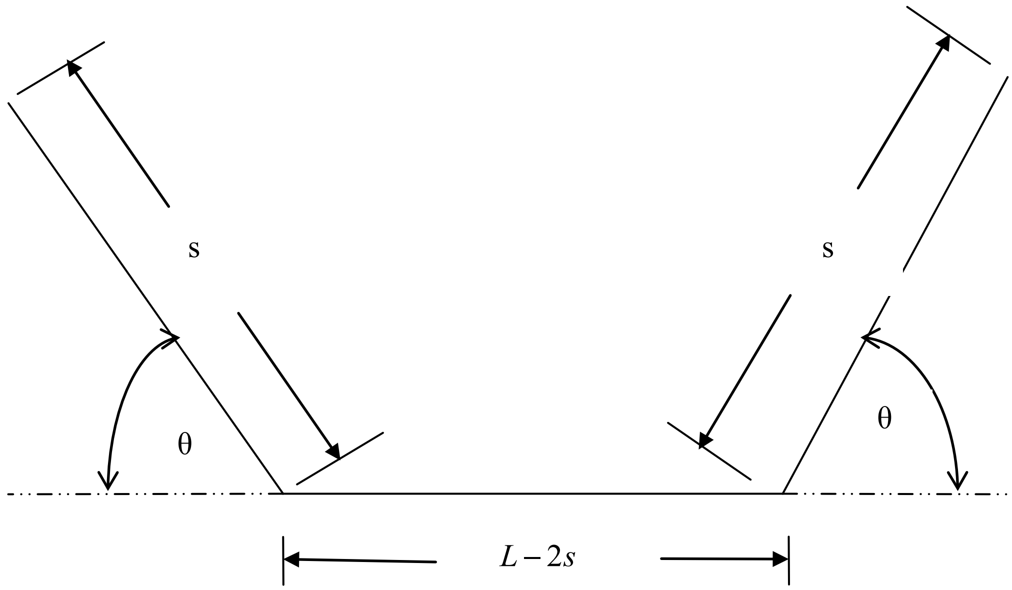

Figure 2: Parameters of the gutter

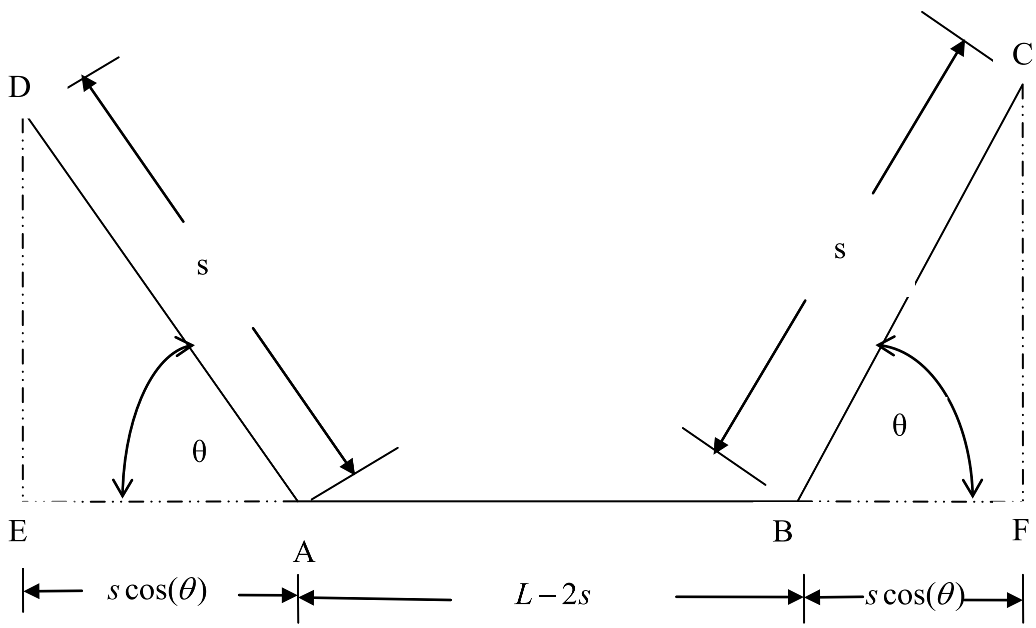

Figure 3: Labeling the parameters and points of the gutter

Solution

One needs to maximize the cross-sectional area of the gutter. The cross-sectional area G of the gutter is given by

$$G = \text{Area of trapezoid } ABCD$$
$$= \frac{1}{2}(AB + CD)(FC) \tag{1}$$

where

$$AB = L\text{-}2\,s \tag{2}$$
$$CD = AB + EA + BF$$
$$= (L\text{-}2s) + s\cos(\theta) + s\cos(\theta)$$
$$= (L\text{-}2s) + 2s\cos(\theta) \tag{3}$$
$$FC = BC\ \sin(\theta)$$
$$= s\sin(\theta) \tag{4}$$

hence

$$G = \frac{1}{2}(AB + CD)\ (DE)$$
$$G(s,\theta) = \frac{1}{2}\left[(L-2s) + (L-2s) + 2s\cos(\theta)\right] s\ \sin(\theta) \tag{5}$$

For example, for $\theta = 0°$ or $\theta = 180°$, the cross-sectional area of the gutter $G = 0$ represents a flat and overlapped sheet, respectively. For $\theta = 90°$, it represents a rectangular cross-sectional area with $G = (L-2s)\ s$.

For L=0.21m,

$$G(s,\theta) = \frac{1}{2}\left[(0.21-2s) + 2s\cos(\theta)\right] s\ \sin(\theta) \tag{6}$$

The problem is a two-dimensional optimization problem where we want to maximize A, and the value of s and θ can be chosen to do so. Since we are going to cover both one-dimensional and two-dimensional optimization problems, a one-dimensional version would be where we fix one of the two variables. One case would be where we choose

$$s = \frac{L}{3},$$

Then

$$G(\theta) = \frac{L^2}{9}\left[1 + 2\cos(\theta)\right]\ \sin(\theta) \tag{7}$$

For L=0.21m,

$$G(\theta) = 0.0049\left[1 + 2\cos(\theta)\right]\ \sin(\theta) \tag{8}$$

Questions

1) Find the optimal value of θ, such that the cross-sectional area of the gutter is maximized. Use the expression given in Equation (8).

2) Find the optimal values of s and θ, such that the cross-sectional area of the gutter is maximized. Use the expression given in Equation (6).

OPTIMIZATION	
Topic	Physical problem
Summary	A physical problem of finding the maximum area of a gutter.
Major	General Engineering
Authors	Autar Kaw
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