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Numerical Methods

Golden Section Search Method - Theory

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Equal Interval Search Method

- Choose an interval [a, b] over which the optima occurs
- Compute $f\left(\frac{a+b}{2}+\frac{\varepsilon}{2}\right)$ and $f\left(\frac{a+b}{2}-\frac{\varepsilon}{2}\right)$

- If $f\left(\frac{a+b}{2}+\frac{\varepsilon}{2}\right) > f\left(\frac{a+b}{2}-\frac{\varepsilon}{2}\right)$ then the interval in which the maximum occurs is $\left[\frac{a+b}{2}-\frac{\varepsilon}{2}, b\right]$ otherwise it occurs in $\left[a, \frac{a+b}{2}+\frac{\varepsilon}{2}\right]$

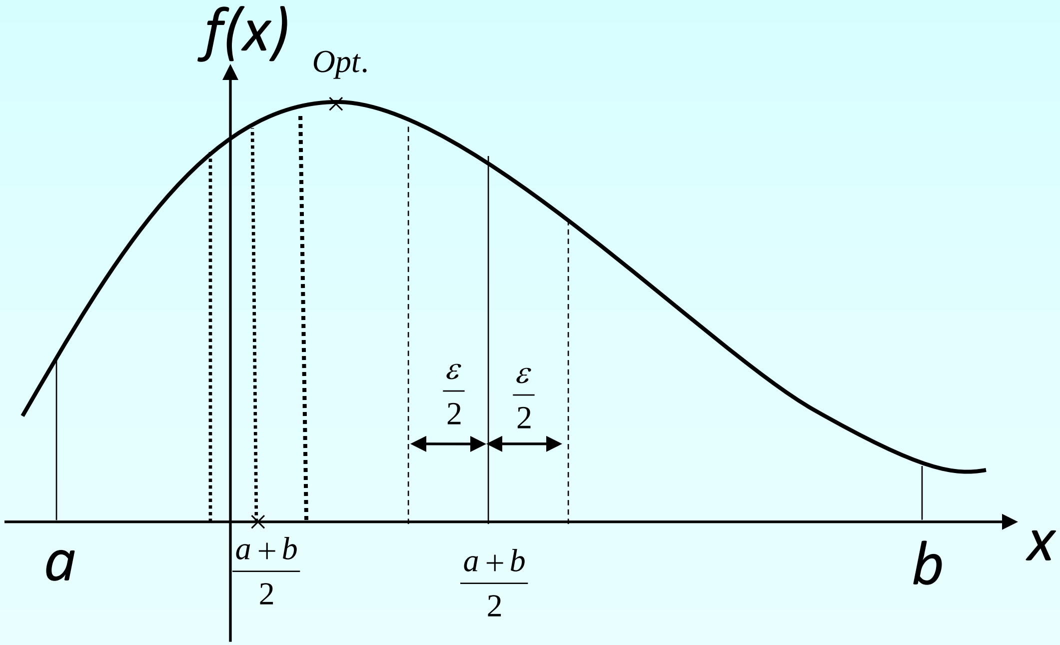


Figure 1 Equal interval search method.

Golden Section Search Method

- The Equal Interval method is inefficient when ε is small. <u>**Also, we need to compute 2 interior points !**</u>
- The Golden Section Search method divides the search more efficiently closing in on the optima in fewer iterations.

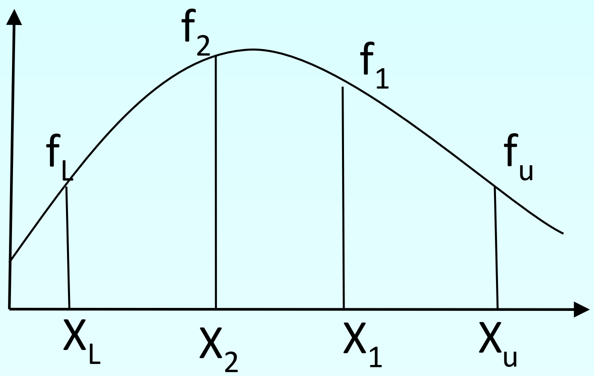
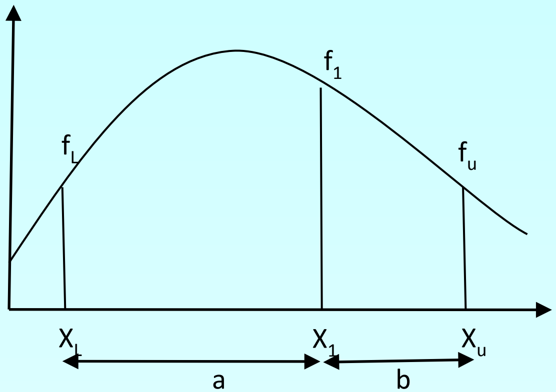


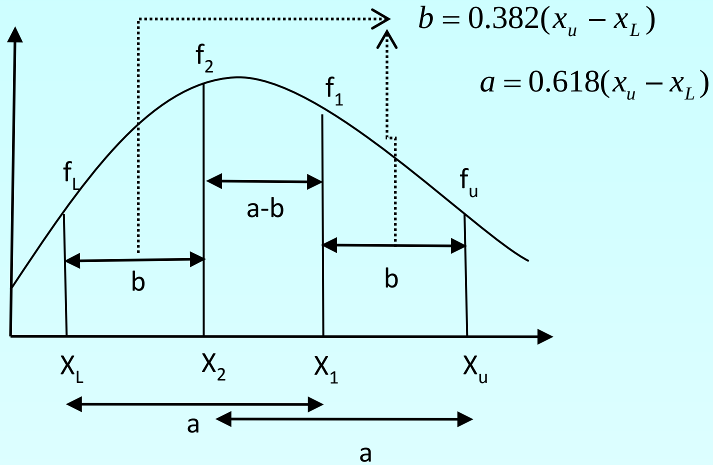
Figure 2. Golden Section Search method

Golden Section Search Method- Selecting the Intermediate Points



Determining the first intermediate point

$$X_1 = X_l + a = X_u - b$$



Determining the second intermediate point

$$\frac{a}{(a+b = X_u - X_l)} = \frac{b}{a} = 0.618(why?); hence$$

$$X_2 = X_u - a = X_l + b$$

$$a = 0.618*(X_u - X_l), and \quad b = 0.382*(X_u - X_l)$$

$$\frac{a}{b} = \frac{a+b}{a} = 1 + \frac{b}{a}$$

Let $R = \frac{b}{a}$,hence

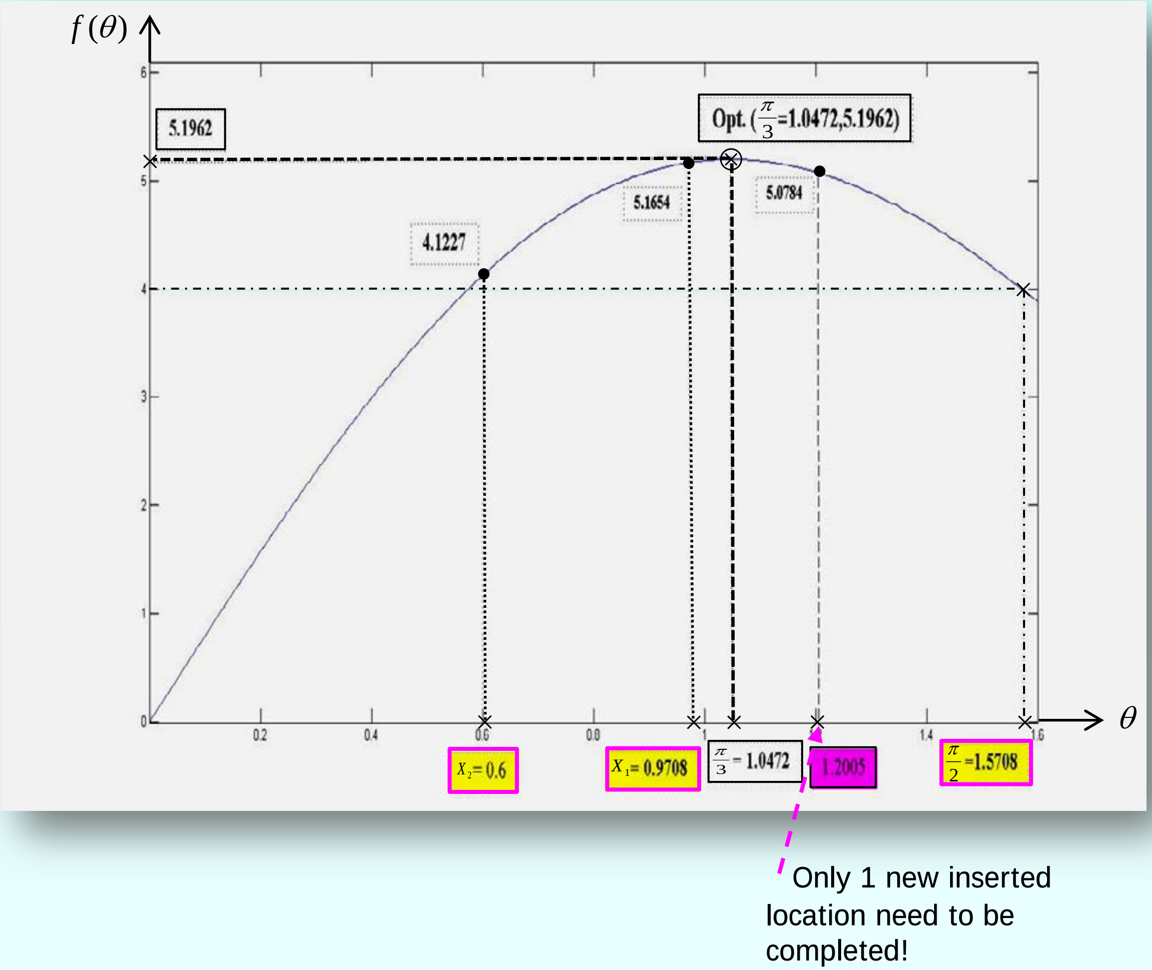
$$\frac{1}{R} = 1 + R \Rightarrow R^2 + R - 1 = 0 \Rightarrow R = \frac{(\sqrt{5}-1)}{2} \Rightarrow R = 0.61803$$

Golden Ratio=> $\frac{b}{a} = 0.618...$

Golden Section Search Method

$$f(\theta) = 4\sin\theta(1+\cos\theta)$$
$$f(\theta) = 4\sin\theta + 2\sin(2\theta)$$
$$f'(\theta) = 4\cos\theta + 4\cos(2\theta) = 0$$
$$\Rightarrow 4\cos\theta + 4[2\cos^2\theta - 1] = 0$$

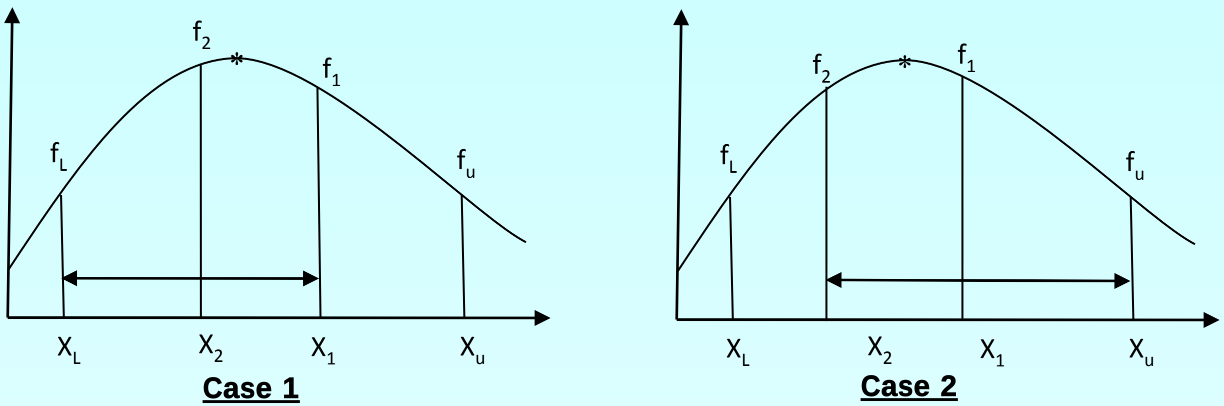
Hence, $\theta_{Opt.} = \frac{\pi}{3}$ after solving quadratic equation, with initial guess = (0, 1.5708 rad)



1^{st} = Initial Iteration

Second Iteration

Golden Section Search- Determining the new search region



- Case1:

If $f(x_2) > f(x_1)$ then the new interval is $[x_L, x_2, x_1]$

- Case2:

If $f(x_2) < f(x_1)$ then the new interval is $[x_2, x_1, x_u]$

Golden Section Search- Determining the new search region

- At each new interval ,one needs to determine only 1(not 2) new inserted location (either compute the new x_1 ,or new x_2)
- Max. $f(\theta) = 4\sin\theta(1+\cos\theta) \Leftrightarrow$ Min. $\bar{f}(\theta) = -4\sin\theta(1+\cos\theta)$
- It is desirable to have automated procedure to compute x_L and x_u initially.

Golden Section Search- (1-D) Line Search Method

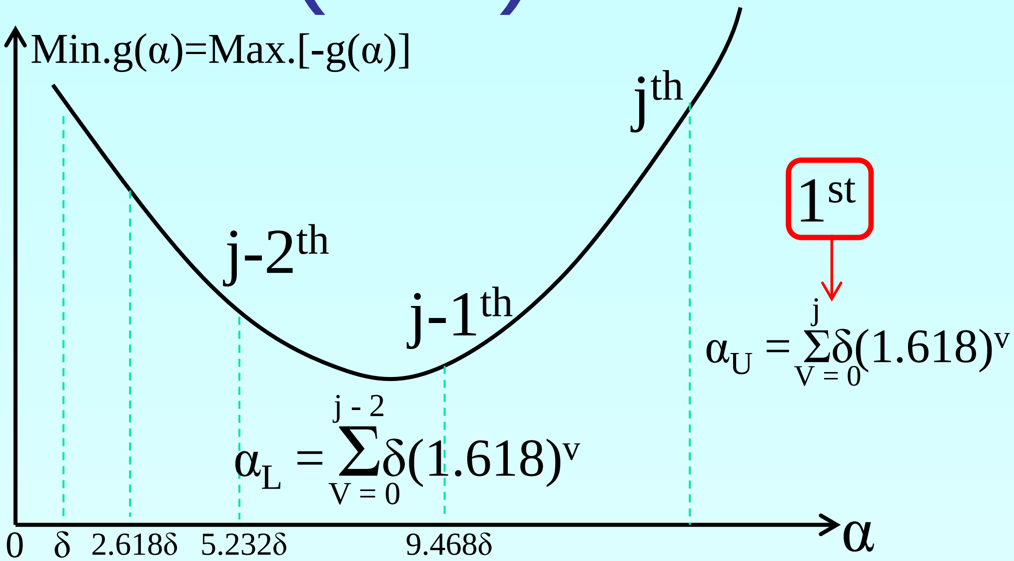
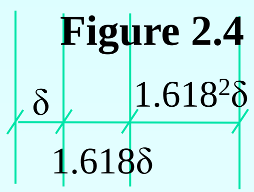


Figure 2.4 Bracketing the minimum point.



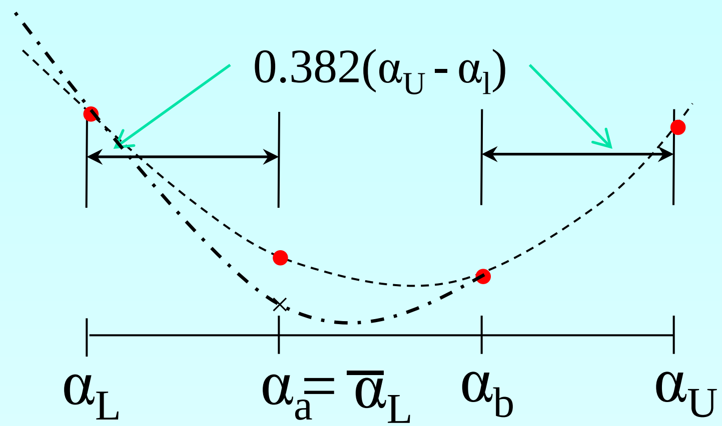


Figure 2.5 Golden section partition.

2^{nd}

$$\alpha_a = \alpha_L + 0.382(\alpha_U - \alpha_l) = \sum_{V=0}^{j-2}\delta(1.618)^v + 0.382\delta(1.618)^{j-1}\underline{(1+1.618)}$$

3^{rd}

$$\alpha_a = \sum_{V=0}^{j-2}\delta(1.618)^v + 1\delta(1.618)^{j-1} = \sum_{V=0}^{j-1}\delta(1.618)^v = \text{already known !}$$

4^{th}

Golden Section Search-
(1-D) Line Search Method

- If $g(\alpha_a) = g(\alpha_b)$, Then the minimum will be between α_a & α_b.
- If $g(\alpha_a) \rangle g(\alpha_b)$ as shown in Figure 2.5, Then the minimum will be between α_a & $\alpha_U \Rightarrow \overline{\alpha}_L = \alpha_a$ and $\overline{\alpha}_U = \alpha_U$.

Notice that: $\overline{\alpha}_U - \overline{\alpha}_L = \alpha_U - \alpha_a = \delta(1.618)^j$

And

$$\alpha_b - \overline{\alpha}_L = \alpha_b - \alpha_a = (1 - 2 \times 0.382)(\alpha_U - \alpha_L) = (0.236)(\delta[1.618]^{j-1} + \delta[1.618]^j)$$

$$= (0.236)(\delta[1.618]^{j-1} \times [1 + 1.618]) = 0.618(\delta[1.618]^{j-1}) \times \frac{1.618}{1.618}$$

$$\alpha_b - \overline{\alpha}_L = (0.382) \times (\delta[1.618]^j = 0.382(\overline{\alpha}_U - \overline{\alpha}_L)$$

Thus α_b (wrt $\overline{\alpha}_U$ & $\overline{\alpha}_L$) plays same role as α_a(wrt α_U & α_L) !!

Golden Section Search- (1-D) Line Search Method

Step 1 : For a chosen small step size δ in α, say $\delta = +10^{-2} \to 10^{-1}$, let j be the smallest integer such that $g(\sum_{V=0}^{j} \delta(1.618)^V) \rangle g(\sum_{V=0}^{j-1} \delta(1.618)^V)$

The upper and lower bound on α^i are $\alpha_U = \sum_{V=0}^{J} \delta(1.618)^V$ and $\alpha_L = \sum_{V=0}^{j-2} \delta(1.618)^V$.

Step 2: Compute $g(\alpha_b)$, where $\alpha_a = \alpha_L + 0.382(\alpha_U - \alpha_L)$, and $\alpha_b = \alpha_L + 0.618(\alpha_U - \alpha_L)$.

Note that $\alpha_a = \sum_{V=0}^{j-1} \delta(1.618)^V$, so $g(\alpha_a)$ is already known.

Step 3: Compare $g(\alpha_a)$ and $g(\alpha_b)$ and go to Step 4,5, or 6.

Step 4: If $g(\alpha_a) < g(\alpha_b)$, then $\alpha_L \le \alpha^i \le \alpha_b$. By the choice of α_a and α_b, the new points $\overline{\alpha}_L = \alpha_L$ and $\overline{\alpha}_u = \alpha_b$ have $\overline{\alpha}_b = \alpha_a$.

Compute $g(\overline{\alpha}_a)$, where $\overline{\alpha}_a = \overline{\alpha}_L + 0.382(\overline{\alpha}_u - \overline{\alpha}_L)$ and go to Step 7.

Golden Section Search-
(1-D) Line Search Method

Step 5: If $g(\alpha_a) > g(\alpha_b)$, then $\alpha_a \leq \alpha^i \leq \alpha_U$. Similar to the procedure in Step 4, put $\overline{\alpha}_L = \alpha_a$ and $\overline{\alpha}_u = \alpha_u$.

Compute $g(\overline{\alpha}_b)$,where $\overline{\alpha}_b = \overline{\alpha}_L + 0.618(\overline{\alpha}_u - \overline{\alpha}_L)$ and go to Step 7.

Step 6: If $g(\alpha_a) = g(\alpha_b)$ put $\alpha_L = \alpha_a$ and $\alpha_u = \alpha_b$ and return to Step 2.

Step 7: If $\overline{\alpha}_u - \overline{\alpha}_L$ is suitably small, put $\alpha^i = \frac{1}{2}(\overline{\alpha}_u + \overline{\alpha}_L)$ and stop. Otherwise, delete the bar symbols on $\overline{\alpha}_L, \overline{\alpha}_a, \overline{\alpha}_b$,and $\overline{\alpha}_u$ and return to Step 3.

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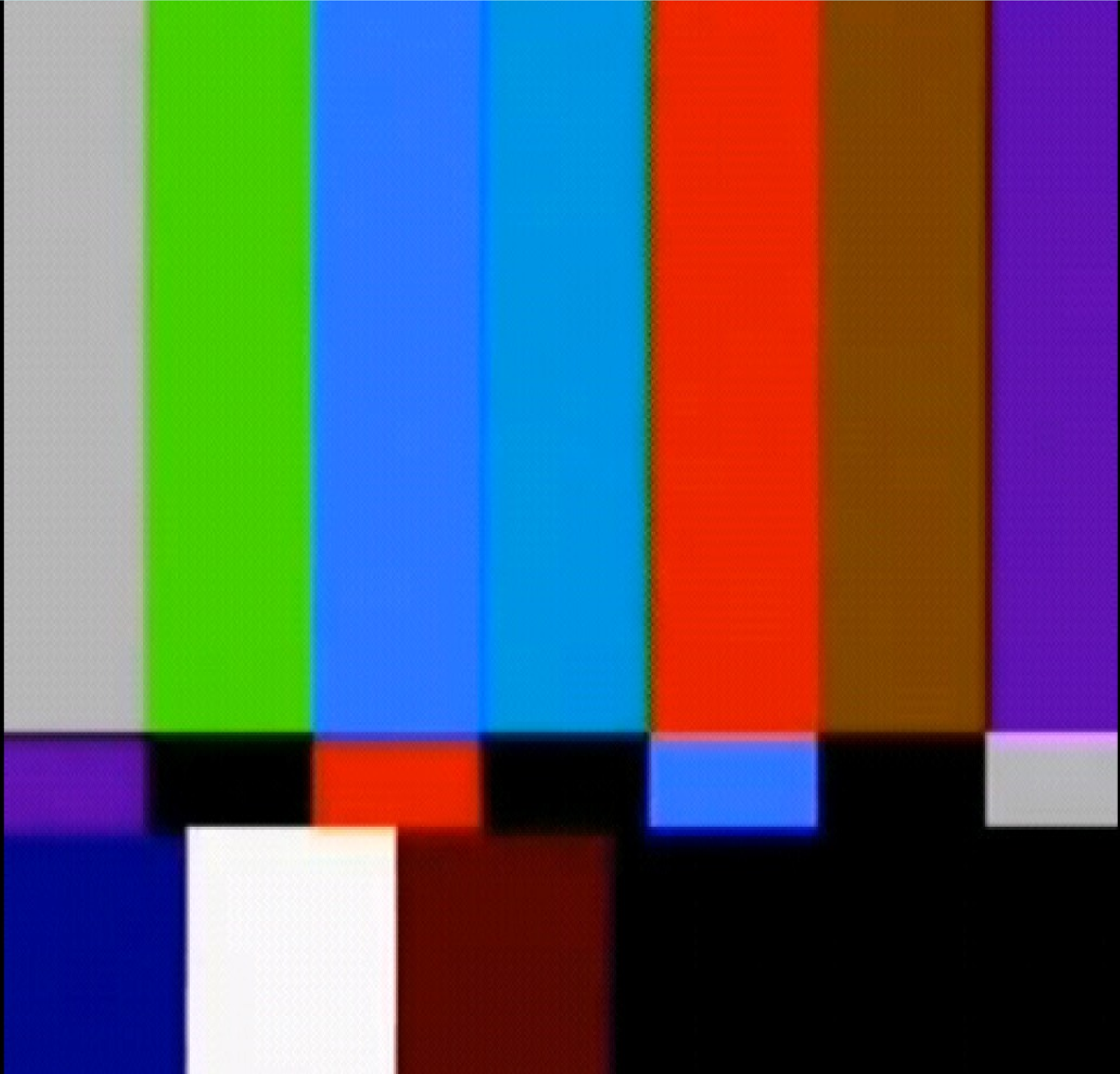

Numerical Methods

Golden Section Search Method - Example

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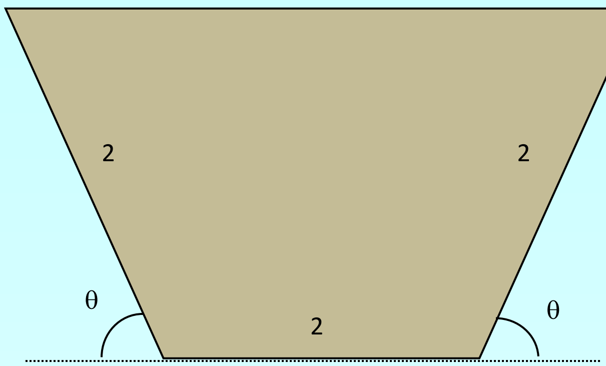
Example



The cross-sectional area A of a gutter with equal base and edge length of 2 is given by (**trapezoidal** area):

$$Max. f(\theta) = A = 4\sin\theta(1+\cos\theta) = 4\sin\theta + 2\sin(2\theta)$$

Find the angle θ which maximizes the cross-sectional area of the gutter. Using an initial interval of $[0, \frac{\pi}{2}]$ find the solution after 2 iterations.

Convergence achieved if " interval length " is within $\varepsilon = 0.05$

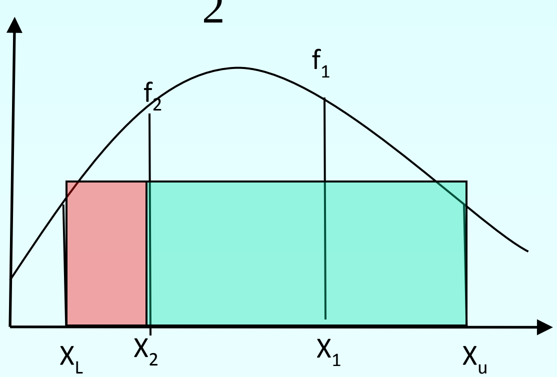
Solution

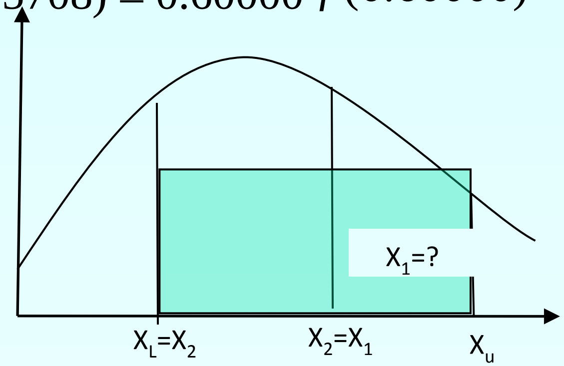
The function to be maximized is $f(\theta) = 4\sin\theta(1+\cos\theta)$

Iteration 1: Given the values for the boundaries of $x_L = 0 \; and \; x_u = \pi/2$ we can calculate the initial intermediate points as follows:

$$x_1 = x_L + \frac{\sqrt{5}-1}{2}(x_u - x_L) = 0 + \frac{\sqrt{5}-1}{2}(1.5708) = 0.97080 \quad f(0.97080) = 5.1654$$

$$x_2 = x_u - \frac{\sqrt{5}-1}{2}(x_u - x_L) = 1.5708 - \frac{\sqrt{5}-1}{2}(1.5708) = 0.60000 \quad f(0.60000) = 4.1227$$





Solution Cont

$$x_1 = x_L + \frac{\sqrt{5}-1}{2}(x_u - x_L) = 0.60000 + \frac{\sqrt{5}-1}{2}(1.5708 - 0.60000) = 1.2000$$

To check the stopping criteria the difference between x_u and x_L is calculated to be

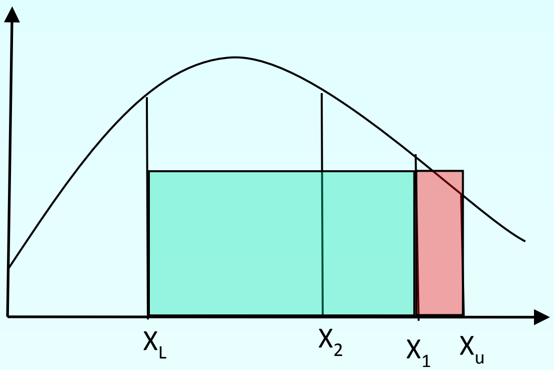
$$x_u - x_L = 1.5708 - 0.60000 = 0.97080$$

Solution Cont

Iteration 2

$$x_L = 0.60000$$
$$x_u = 1.5708$$
$$x_1 = 1.2000 \qquad f(1.2000) = 5.0791$$
$$x_2 = 0.97080 \qquad f(0.97080) = 5.1654$$

$$f(x_1) < f(x_2)$$



$$x_L = 0.60000$$
$$x_u = 1.2000$$
$$x_1 = 0.97080$$
$$x_2 = x_u - \frac{\sqrt{5}-1}{2}(x_u - x_L) = 1.2000 - \frac{\sqrt{5}-1}{2}(1.2000 - 0.6000) = 0.82918$$
$$\frac{x_u + x_L}{2} = 1.2000 + 0.6000 = 0.9000$$

Theoretical Solution and Convergence

Iteration	x_l	x_u	x_1	x_2	$f(x_1)$	$f(x_2)$	ε
1	0.0000	1.5714	0.9712	0.6002	5.1657	4.1238	1.5714
2	0.6002	1.5714	1.2005	0.9712	5.0784	5.1657	0.9712
3	0.6002	1.2005	0.9712	0.8295	5.1657	4.9426	0.6002
4	0.8295	1.2005	1.0588	0.9712	5.1955	5.1657	0.3710
5	0.9712	1.2005	1.1129	1.0588	5.1740	5.1955	0.2293
6	0.9712	1.1129	1.0588	1.0253	5.1955	5.1937	0.1417
7	1.0253	1.1129	1.0794	1.0588	5.1908	5.1955	0.0876
8	1.0253	1.0794	1.0588	1.0460	5.1955	5.1961	0.0541
9	1.0253	1.0588	1.0460	1.0381	5.1961	5.1957	**0.0334**

$$\frac{x_u + x_L}{2} = \frac{1.0253 + 1.0588}{2} = 1.0420 \qquad f(1.0420) = 5.1960$$

The theoretically optimal solution to the problem happens at exactly 60 degrees which is 1.0472 radians and gives a maximum cross-sectional area of 5.1962.

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